Separation of Gravitational-Wave and Cosmic-Shear Contributions to Cosmic Microwave Background Polarization

Michael Kesden,* Asantha Cooray,† and Marc Kamionkowski‡

California Institute of Technology, Mail Code 130-33, Pasadena, California 91125

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Inflationary gravitational waves (GW) contribute to the curl component in the polarization of the cosmic microwave background (CMB). Cosmic shear—gravitational lensing of the CMB—converts a fraction of the dominant gradient polarization to the curl component. Higher-order correlations can be used to map the cosmic shear and subtract this contribution to the curl. Arcminute resolution will be required to pursue GW amplitudes smaller than those accessible by the Planck surveyor mission. The blurring by lensing of small-scale CMB power leads with this reconstruction technique to a minimum detectable GW amplitude corresponding to an inflation energy near 10^{15} GeV.

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Observation of acoustic oscillations in the temperature anisotropies of the cosmic microwave background (CMB) [1] strongly suggests an inflationary origin for primordial perturbations [2]. It has been argued that a new "smoking-gun" signature for inflation would be the detection of a stochastic background in inflationary gravitational waves (IGWs) [3]. These IGWs produce a distinct signature in the CMB in the form of a contribution to the curl, or magneticlike, component of the polarization [4]. Since there is no scalar, or density-perturbation, contribution to these curl modes, curl polarization was considered to be a direct probe of IGWs.

There is, however, another source of a curl component. Cosmic shear (CS)—weak gravitational lensing of the CMB due to large-scale structure along the line of sight—results in a fractional conversion of the gradient mode from density perturbations to the curl component [5]. The amplitude of the IGW background varies quadratically with the energy scale $E_{\rm infl}$ of inflation, and so the prospects for detection also depend on this energy scale. In the absence of CS, the smallest detectable IGW background scales simply with the sensitivity of the CMB experiment—as the instrumental sensitivity is improved, smaller values of $E_{\rm infl}$ become accessible [3,6]. More realistically, however, the CS-induced curl introduces a noise from which IGWs must be distinguished. If the IGW amplitude (or $E_{\rm infl}$) is sufficiently large, the CS-induced curl will not be a problem. However, as $E_{\rm infl}$ is reduced, the IGW signal becomes smaller and will get lost in the CS-induced noise. This confusion leads to a minimum detectable IGW amplitude [7].

In addition to producing a curl component, CS also introduces distinct higher-order correlations in the CMB temperature pattern. Roughly speaking, lensing can stretch the image of the CMB on a small patch of sky and thus lead to something akin to anisotropic correlations on that patch of sky, even though the CMB pattern at the surface of last scatter had isotropic correlations. By mapping these effects, the CS can be mapped as a function of position on the sky [8]. The observed CMB polarization can then be corrected for these lensing deflections to reconstruct the intrinsic CMB polarization at the surface of last scatter (in which the only curl component would be that due to IGWs). In this Letter we evaluate how well this subtraction can be accomplished and study the impact of CS on experimental strategies for the detection of IGWs.

To begin, we review the determination of the smallest detectable IGW amplitude in the absence of CS. Following Ref. [6], we consider a CMB-polarization experiment of some given instrumental sensitivity quantified by the noise-equivalent temperature (NET) s, angular resolution $\theta_{\rm FWHM}$, duration $t_{\rm yr}$ in years, and a fraction of the sky covered $f_{\rm sky}$. We then make the null hypothesis of no IGWs and determine the largest IGW amplitude $\mathcal{T}$, defined as $\mathcal{T} = 9.2V/m_{Pl}^4$, where $V = E_{\rm infl}^4$ is the inflaton-potential height that would be consistent at the 1σ level with the null detection. We then obtain the smallest detectable IGW amplitude $\sigma_{\mathcal{T}}$ from

$$\sigma_{\mathcal{T}}^{-2} = \sum_{l>180/\theta} (\partial C_l^{BB,{\rm GW}}/\partial\mathcal{T})^2 (\sigma_l^{BB})^{-2}, \quad (1)$$

where $C_l^{BB,{\rm GW}}$ is the IGW contribution to the curl power spectrum, and

$$\sigma_l^{BB} = \sqrt{\frac{2}{f_{\rm sky}(2l+1)}}\,(C_l^{BB} + f_{\rm sky} w^{-1} e^{l^2\sigma_b^2}) \quad (2)$$

is the standard error with which each multipole moment C_l^{BB} can be determined. Here, $w^{-1} = 4\pi(s/T_{\rm CMB})^2/(t_{\rm pix}N_{\rm pix})$ [9] is the variance per unit area on the sky for polarization observations when $t_{\rm pix}$ is the time spent on each of $N_{\rm pix}$ pixels with detectors of NET s, and $\theta \simeq 203 f_{\rm sky}^{1/2}$ is roughly the width (in degrees) of the survey. In restricting the sum to $l > 180/\theta$, we have assumed that no information from modes with wavelengths larger than the survey size can be obtained; in fact, some information can be obtained, and our results should thus be viewed as conservative [7].

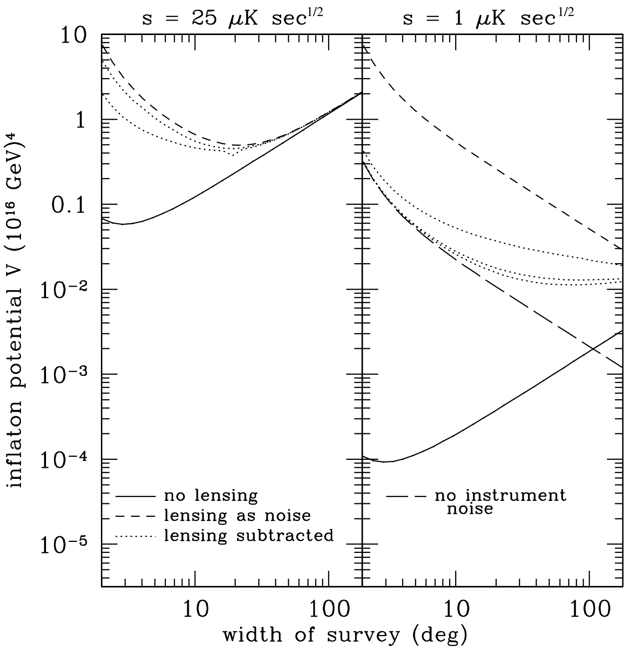

FIG. 1. Minimum inflation potential observable at 1σ as a function of a survey width for a one-year experiment. The left panel shows an experiment with NET $s = 25\ \mu\mathrm{K\,sec}^{1/2}$. The solid curve shows results, assuming no CS, while the dashed curve shows results, including the effects of an unsubtracted CS; we take $\theta_{\mathrm{FWHM}} = 5'$ in these two cases. The dotted curves assume that the CS is subtracted with $\theta_{\mathrm{FWHM}} = 10'$ (upper curve) and $5'$ (lower curve). Since the dotted curves are close to the dashed curve, it shows that these higher-order correlations will not be significantly useful in reconstructing the primordial curl for an experiment similar to Planck surveyor mission's sensitivity and resolution. The right panel shows results for hypothetical improved experiments. The dotted curves show results with CS subtracted and assuming $s = 1\ \mu\mathrm{K\,sec}^{1/2}$, $\theta_{\mathrm{FWHM}} = 5'$, $2'$, and $1'$ (from top to bottom). The solid curve assumes $\theta_{\mathrm{FWHM}} = 1'$ and $s = 1\ \mu\mathrm{K\,sec}^{1/2}$, and no CS, while the dashed curve treats CS as an additional noise. The long-dashed curve assumes CS subtraction with no instrumental noise ($s = 0$).

The second term in Eq. (2) is due to instrumental noise, and the first term is due to cosmic variance. In the absence of CS, and for the null hypothesis of no IGWs, we set $C_l^{BB} = 0$, and the results for the smallest detectable IGW amplitude are shown as the solid curves in Fig. 1 for an experiment with detectors of comparable sensitivity to the Planck surveyor mission (left) and a hypothetical experiment (right) with better sensitivity. The smallest detectable IGW amplitude $\mathcal{T}$ scales as $s^2 t_{\mathrm{yr}}^{-1}$. For large survey widths, it scales as θ, but at survey widths smaller than $\sim 5°$ it increases because information from the larger-angle modes in the IGW-induced curl power spectrum is lost (cf. the IGW power spectrum in Fig. 2).

It is now easy to see how inclusion of CS affects these results. As discussed above, lensing of the gradient polarization at the surface of last scatter due to density perturbations leads to a CMB curl component with a power spectrum,

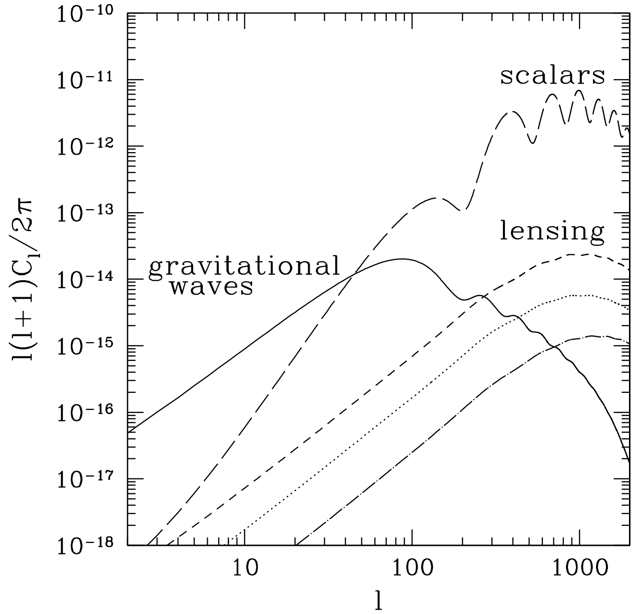

FIG. 2. Contributions to the CMB polarization power spectra. The long-dashed curve shows the dominant polarization signal in the gradient component due to scalar perturbations. The solid line shows the maximum allowed curl polarization signal from the gravitational-wave background, which will be smaller if the inflationary energy scale is smaller than the maximum value allowed by COBE of 3.47×10^{16} GeV. The dashed curve shows the power spectrum of the curl component of the polarization due to CS. The dotted curve is the CS contribution to the curl component that comes from structures out to a redshift of 1; this is the level at which low-redshift lensing surveys can be used to separate the CS-induced polarization from the IGW signal. The dot-dashed line is the residual when lensing contribution is separated with a no-noise experiment and 80% sky coverage.

$$\tilde{C}_l^{BB} = \frac{1}{2}\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[\mathbf{l}_2\cdot\mathbf{l}_1]^2(1-\cos 4\phi_{l_1})C_{l_2}^{\phi\phi}C_{l_1}^{EE}, \tag{3}$$

where $\mathbf{l}_2 = \mathbf{l} - \mathbf{l}_1$ here and throughout, C_l^{EE} is the power spectrum of the gradient component of polarization, and $C_l^{\phi\phi}$ is the power spectrum of the projected lensing potential [10]. The latter is defined in terms of the potential fluctuations, Φ, along the line of sight such that

$$\phi(\hat{\mathbf{n}}) = -2\int_0^{r_0} dr\, \frac{d_A(r_0 - r)}{d_A(r)d_A(r_0)}\,\Phi(r, \hat{\mathbf{n}}r), \tag{4}$$

where r is the comoving radial distance, or conformal look-back time, with r_0 at the last scattering surface, and $d_A(r)$ is the comoving angular diameter distance. The CS-induced curl power spectrum is shown as the dashed curve in Fig. 2.

By the time these measurements are made, the cosmological parameters that determine this lensed curl power spectrum should be sufficiently well determined that this power spectrum can be predicted with some confidence. In that case, the CS-induced curl component can be treated simply as a well-understood noise for the IGW background. The smallest detectable IGW amplitude can then be calculated as above, but now inserting the lensed power spectrum, Eq. (3), in Eq. (2) for C_l^{BB}. The results are shown as the short-dashed curves in Fig. 1. When lensing is included, the results no longer scale simply with $f_{\rm sky}$, s, or $t_{\rm yr}$, as there is now a trade-off between the instrumental-noise and CS-noise terms in Eq. (2). The left panel shows that the IGW sensitivity for an experiment with NET similar to Planck's should not be affected by CS. This is because the IGW amplitudes that could be detectable by such experiments are still relatively large compared with the expected CS signal, especially at the larger angles that will be best accessed by Planck. However, CS will affect the ability of experiments more sensitive than Planck to detect unambiguously IGWs, as shown in the right panel. CS also shifts the preferred survey region to larger areas, as the IGW power spectrum peaks at larger angles than the CS power spectrum (cf. Fig. 2). Finally, note that if the CS curl is treated as an unsubtracted noise, it leads, assuming a no-noise polarization map, to the smallest detectable IGW amplitude, corresponding to an inflaton-potential height, $V^{1/4} \sim 4 \times 10^{15}$ GeV.

Now we arrive at the main point of this Letter; i.e., how well can the CS-induced curl be subtracted by mapping the CS as a function of position on the sky? One possibility is that the primordial polarization pattern might be reconstructed from that observed by using CS maps obtained with correlations of galaxy ellipticities [11]. However, the source galaxies for these CS surveys are at redshifts $z \sim 1$, while only a small fraction of the CS-induced curl comes from these redshifts, as indicated in Fig. 2. An alternative possibility is to use higher-order correlations in the CMB [8] to map the CS-induced curl all the way back to the surface of last scatter.

CS modifies the temperature and polarization pattern, giving rise to anisotropic correlations on small scales, where the image of the CMB surface of last scatter is sheared by weak lensing. According to Ref. [12], the quantity, $\nabla \cdot [T(\hat{\mathbf{n}})\nabla T(\hat{\mathbf{n}})]$, provides the best quadratic estimator, given a temperature map, of the deflection angle at position $\hat{\mathbf{n}}$ on the sky. In Fourier space, we can write this quadratic estimator for the deflection angle as

$$\hat{\alpha}(\mathbf{l}) = \frac{N_l}{l} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} (\mathbf{l} \cdot \mathbf{l}_1 C_{l_1}^{\rm CMB} + \mathbf{l} \cdot \mathbf{l}_2 C_{l_2}^{\rm CMB}) \times \frac{T(l_1)T(l_2)}{2C_{l_1}^{\rm tot} C_{l_2}^{\rm tot}}, \qquad (5)$$

where $C_l^{\rm CMB}$ is the unlensed CMB power spectrum and $C_l^{\rm tot} = C_l^{\text{lensed-CMB}} + f_{\rm sky} w^{-1} e^{l^2\sigma_b^2}$ includes all contributions to the CMB temperature power spectrum. The ensemble average over CMB realizations, $\langle \hat{\alpha}(\mathbf{l})_{\rm CMB} \rangle$, is equal to the deflection angle, $l\phi(\mathbf{l})$, when

$$N_l^{-1} = \frac{1}{l^2} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} \frac{(\mathbf{l} \cdot \mathbf{l}_1 C_{l_1}^{\rm CMB} + \mathbf{l} \cdot \mathbf{l}_2 C_{l_2}^{\rm CMB})^2}{2C_{l_1}^{\rm tot} C_{l_2}^{\rm tot}}. \qquad (6)$$

It can also be shown [12] that N_l is the noise power spectrum associated with the reconstructed deflection angle power spectrum,

$$\langle \hat{\alpha}(\mathbf{l}) \hat{\alpha}(\mathbf{l}') \rangle = (2\pi)^2 \delta_D(\mathbf{l} + \mathbf{l}')(l^2 C_l^{\phi\phi} + N_l). \qquad (7)$$

Here the ensemble average is taken independently over realizations of both the CMB and the intervening large-scale structure. In addition to these temperature estimators for the deflection angle, we also use analogous ones constructed from the polarization, as discussed in Ref. [13], although we do not reproduce those formulas here. The total noise in the estimator for the deflection angle can then be constructed by summing the inverses of the individual noise contributions. We thus determine the variance with which each Fourier mode of ϕ can be reconstructed.

With the deflection angle obtained this way as a function of position on the sky, the polarization at the CMB surface of last scatter can be reconstructed (details to be presented elsewhere [14]). In the ideal case, there would be no error in the CS reconstruction leading to no residual lensing-induced curl component. Realistically, however, there will be some error in the CS reconstruction from measurement error. Even in the absence of measurement error, there will be a transfer of CMB power from large scales to small scales by cosmic shear. This additional small-scale power, generated well after recombination, swamps the primordial small-scale power, which is silk damped and thus blurs the cosmic-shear signal we discuss here. There will therefore be some residual curl component in the CMB with a noise power spectrum, $C_l^{\phi\phi,\rm noise} = N_l/l^2$. This leads to a finite limit on lensing extraction and, subsequently, a limit for the amplitude of IGW contribution that can be separated from CS with this reconstruction.

The lensing reconstruction from CMB data allows only the extraction of $C_l^{\phi\phi}$ to a multipole of $\lesssim$1000 [13], but there is substantial contribution to the CS-induced curl component from lensing at smaller angular scales. We thus replace N_l/l^2 by $C_l^{\phi\phi}$ when the former exceeds the latter at large l. This provides an estimate to the noise expected in the reconstructed curl component that follows from implementing a filtering scheme where high-frequency noise in the CS reconstruction is removed to the level of the expected CS signal. The dot-dashed curve in Fig. 2 shows the residual CS-induced curl component that remains after subtraction.

We can now anticipate the smallest IGW amplitude detectable by a CS-corrected polarization map by simply using this residual noise power spectrum in Eq. (3). The results are shown as dotted curves in Fig. 1. The left panel shows the results as a function of survey size for an experiment with NET similar to Planck, while the right-hand panel shows the results for experiments with better sensitivity and resolution. Since the dotted curves are just below the dashed curve in the left-hand panel of Fig. 1, we learn that Planck's sensitivity will not be sufficient to warrant an effort to reconstruct the primordial curl and we would do just as well to simply treat the CS-induced curl as a noise component of known amplitude. We can expect to improve the discovery reach for IGWs by increasing the sensitivity and resolution. The right-hand panel of Fig. 1 shows results for a hypothetical experiment with $s = 1\ \mu\mathrm{K\,sec}^{1/2}$ and angular resolutions of $5'$, $2'$, and $1'$. We now see there is a significant difference between the dashed curve and the dotted lines, suggesting increasing improvement with increasing resolution.

Finally, to indicate the limits of this type of cosmic-shear reconstruction, the long-dashed curve in Fig. 1 shows the results assuming perfect detectors (i.e., $s = 0$). If there were no CS-induced curl, then we would have sensitivity to an arbitrarily small IGW amplitude, but the existence of a CS-induced curl provides an ultimate limit of $V^{1/4} \simeq 4 \times 10^{15}$ GeV, as discussed above. Correction for the effects of CS with the CS map inferred from higher-order correlations would allow us to access lower IGW amplitudes. If there is no instrumental-noise limitation, the sensitivity to an IGW signal is maximized by covering as much sky as possible, and the lowest accessible inflaton potential, $\sim 10^{15}$ GeV, is obtained with a nearly all-sky experiment.

To conclude, we have studied the IGW amplitudes accessible by mapping the curl component of the CMB polarization, taking into account the effects of a CS-induced curl that is either modeled as an unsubtracted noise or subtracted with a CS map obtained with higher-order correlations. We find that the CS reconstruction is unlikely to improve the IGW discovery reach of Planck. To go beyond Planck, however, a CS map will need to be constructed with temperature and polarization maps of higher sensitivity and resolution than Planck. An ultimate limit of roughly $V^{1/4} \sim 10^{15}$ GeV to the detectable IGW amplitude using the techniques considered here comes from the existence of finite CMB power on small angular scales. There are several possible ways this lower limit may be improved. We have used only the lowest-order temperature-polarization correlations to reconstruct the CS. The inclusion of the complete temperature-polarization four-point correlation functions and higher-order correlations may possibly improve the CS reconstruction. We have neglected reionization in our analysis. Reionization will boost the large-angle CMB polarization [15], improving the detectability of IGWs. Another improvement in this limit may be achieved by relaxing our assumption that power in the CMB drops exponentially at small scales. If the excess small-scale power recently detected by Cosmic Background Imager [16] comes from redshifts higher than the redshifts at which lensing is peaked, then there will be more "primordial" small scale with which to reconstruct the CS. In this case, it is imaginable that a far more precise CS map can be reconstructed, but this might require even better angular resolution and sensitivity.

During the preparation of this paper, we learned of another very recently completed work by Knox and Song [17] that performs a very similar calculation and reaches similar conclusions.

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*Email address: kesden@caltech.edu
†Email address: asante@caltech.edu
‡Email address: kamion@tapir.caltech.edu

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