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Separation of Lunar Daily Geomagnetic Variations into Parts of Ionospheric and Oceanic Origin

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Summary

The mechanism for the production of lunar daily geomagnetic variations is discussed. Making reasonable assumptions, it is shown that it is possible to separate the observed variations into parts of ionospheric and oceanic origin. The method is applied to results from six observatories in the British Isles, and the ocean dynamo part is found to make an important contribution to the total lunar daily variation in all three geomagnetic elements.

Introduction

The principal mechanism for the production of lunar daily geogmagnetic variations is an ionospheric dynamo powered by tidal movements of conducting layers of the atmosphere across the lines of force of the Earth's main magnetic field. The electric currents in the ionosphere and the associated currents induced in the earth and sea, produce geomagnetic field variations. Another source of lunar daily geomagnetic variations is the 'ocean dynamo' resulting from tidal movements of the sea across the geomagnetic field (Larsen 1968; Malin 1969a). Again, the electric currents generated by this dynamo will have induced currents associated with them, both in the earth and in the ionosphere.

The ocean dynamo described here differs from the ocean effect familiar in micropulsation studies in that the latter depends only on the presence of the ocean as a region of relatively high conductivity, whereas the ocean dynamo requires also *movement* of the sea.

The interpretation of observed geomagnetic lunar variations is complicated by the presence of effects from both the ionospheric and oceanic dynamos. The purpose of the present note is to indicate a method of separating the two effects. This development of an earlier method (Malin 1969a) makes more reasonable assumptions about the nature of the ionospheric dynamo, includes the evaluation of probable errors, and permits the use of the wealth of existing data on lunar variations obtained by the Chapman & Miller (1940) method of analysis. The present method is applied to all three magnetic elements (the previous study was confined to vertical intensity) at six observatories in the British Isles.

The variations and their separation

The main lunar tidal harmonic both in the atmosphere and in the sea is the M_2 term, with a period of half a lunar day (time argument 2τ, where τ denotes mean lunar time). The conductivity of the ionosphere, κ, varies regularly with solar time, t, and season, and also irregularly because of changes originating in the Sun. The regular changes with solar time may be represented by a series of harmonics having time arguments pt, where $p = 0, 1, 2, ..., \infty$. The main geomagnetic field changes only slowly (from decade to decade) with time. At any point, the harmonics of geomagnetic variations resulting from the interaction of the M_2 atmospheric oscillations, the main geomagnetic field and κ will therefore have time arguments $2\tau \pm pt$, or, equivalently, $(n-2)\,t+2\tau$, where $n = -\infty$ to ∞. Because the conductivity of the ocean does not exhibit appreciable time variations, the geomagnetic variations resulting from the ocean dynamo will be simpler. They may be represented by a single harmonic term having time argument 2τ.

The magnetic variations resulting from the induced currents associated with the ionospheric dynamo will similarly have time arguments $(n-2)\,t+2\tau$. The magnetic effect of currents induced by the ocean dynamo may be considered in two parts; the part associated with the sea and land will have time argument 2τ (since the conductivity is effectively constant); that associated with the ionosphere will have time arguments $(n-2)\,t+2\tau$.

Table 1

Contributions to the geomagnetic lunar daily variations

Number	Source	Time argument	
	Ionospheric dynamo:		
1	direct	$(n-2)\,t+2\tau$	$n = -\infty$ to ∞
2	induced	$(n-2)\,t+2\tau$	$n = -\infty$ to ∞
	Ocean dynamo:		
3	direct	2τ	
4	induced (land and sea)	2τ	
5	induced (ionosphere)	$(n-2)\,t+2\tau$	$n = -\infty$ to ∞

The various contributions to the geomagnetic lunar variation are listed in Table 1, together with their time arguments. The totality of the geomagnetic variations, **L**, associated with the M_2 lunar tide, may be represented by Chapman's phase law:

$$\mathbf{L} = \sum_{n=-\infty}^{\infty} l_n \sin\,[(n-2)\,t+2\tau+\lambda_n]. \qquad (1)$$

Here, l_n denotes the amplitude of the n-th harmonic, and λ_n its phase, t denotes mean solar time measured from local midnight and τ denotes mean lunar time measured from local lower transit of the mean moon. The most important harmonic terms are those for which n is close to 2, and in the Chapman–Miller method (see Malin & Chapman 1970) only the harmonics for $n = 1, 2, 3$ and 4 are determined from observatory data. Recent work by Winch (private communication) has shown that some of the other harmonics may have appreciable amplitudes, but we will here consider only those for which $n = 1, 2, 3$ or 4.

The next approximation is to ignore the contribution from currents induced in the ionosphere by the ocean dynamo (contribution 5 of Table 1). Since the height-integrated conductivity of the ionosphere is 10–50 Ω^{-1} and that of the ocean is $1 \cdot 43 \times 10^4\ \Omega^{-1}$ (Larsen 1968), contribution 5 will clearly be negligible compared with the remaining ocean dynamo effects (3 and 4 of Table 1). It will subsequently be shown that the ocean dynamo and ionospheric dynamo effects are of similar magnitude, so contribution 5 is also negligible compared with the ionospheric dynamo contributions (1 and 2 of Table 1). Thus we may consider the ocean dynamo effect to be of purely internal origin, relative to the surface of the Earth, and to be purely lunar semidiurnal in period. Hence the $n = 1$, 3 and 4 harmonics are from the ionospheric dynamo, and the problem becomes one of separating the contributions of the two dynamos to the $n = 2$ term.

An important difference between the two dynamos is that the ocean dynamo operates equally well by day and by night, whereas the ionospheric dynamo is much less effective during the hours of darkness, because of the decrease in conductivity of the ionosphere in the absence of sunlight. Although the conductivity will have a minimum shortly before 120 km sunrise, the geomagnetic effect is likely to have its minimum earlier than this, mainly because the induced currents will spread into the unlit hemisphere. For this reason, and also to avoid complications due to the variation of the time of sunrise with latitude and season, it is assumed here that the contribution of the ionospheric dynamo to **L** is zero at local midnight, when the value of κ is about one thirtieth of its midday value.

The validity of this assumption may be tested for the diurnal variation of **L** observed at Irkutsk. (This observatory is at a very great distance from any ocean, so the contribution from the ocean dynamo will be negligible.) For this purpose, it is convenient to consider the **L** variations to be lunar semidiurnal, with amplitude, $l(t)$. and phase, $\lambda(t)$, functions of solar time:

$$\mathbf{L} = l(t) \sin [2\tau + \lambda(t)]. \tag{2}$$

Equation (2) is equivalent to equation (1), and by expanding (1) and (2) and equating coefficients of sin 2τ and cos 2τ we obtain:

$$l(t) \cos \lambda(t) \equiv \sum_n l_n \cos [(n-2)\, t + \lambda_n], \tag{3}$$

$$l(t) \sin \lambda(t) \equiv \sum_n l_n \sin [(n-2)\, t + \lambda_n]. \tag{4}$$

The vector probable error, ρ, of **L** is, for all values of t:

$$\rho = \left[\sum_n \rho_n^2\right]^{\frac{1}{2}}, \tag{5}$$

where ρ_n denotes the vector probable error of the n-th harmonic.

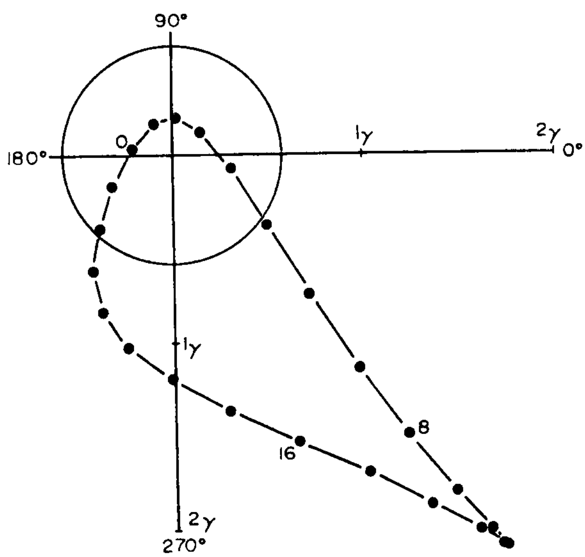

Fig. 1. Harmonic dial for the lunar semidiurnal variation of declination at Irkutsk. The 24 dial points represent the values of $l(t)$ and $\lambda(t)$ for each hour of mean solar time.

Alternatively, we may follow Chambers (1887) and Chapman (1957) in considering **L** as a semi-monthly variation whose amplitudes, $l'(t)$, and phase, $\lambda'(t)$, are functions of solar time:

$$\mathbf{L} = l'(t) \sin [\lambda'(t) - 2\nu]. \tag{6}$$

Here $\nu = t - \tau$; it is a measure of the phase of the Moon, increasing from 0 at one new moon to 2π at the next. Proceeding as before we obtain:

$$l'(t) \cos \lambda'(t) \equiv \sum_n l_n \cos [nt + \lambda_n], \tag{7}$$

$$l'(t) \sin \lambda'(t) \equiv \sum_n l_n \sin [nt + \lambda_n]. \tag{8}$$

(The semi-monthly and lunar-semidiurnal representations of **L** are closely related; a comparison of (2) and (6) shows that $l'(t) = l(t)$ and $\lambda'(t) = \lambda(t) + 2t$).

Using equations (3), (4) and (7), (8), and the observed values of l_n and λ_n for declination at Irkutsk (Malin 1969b), values of $l(t)$, $\lambda(t)$ and $l'(t)$, $\lambda'(t)$ were calculated for 24 hourly values of t, and are shown in Figs 1 and 2. (The harmonic terms for the other elements are not significantly determined at this station.) Although the minimum

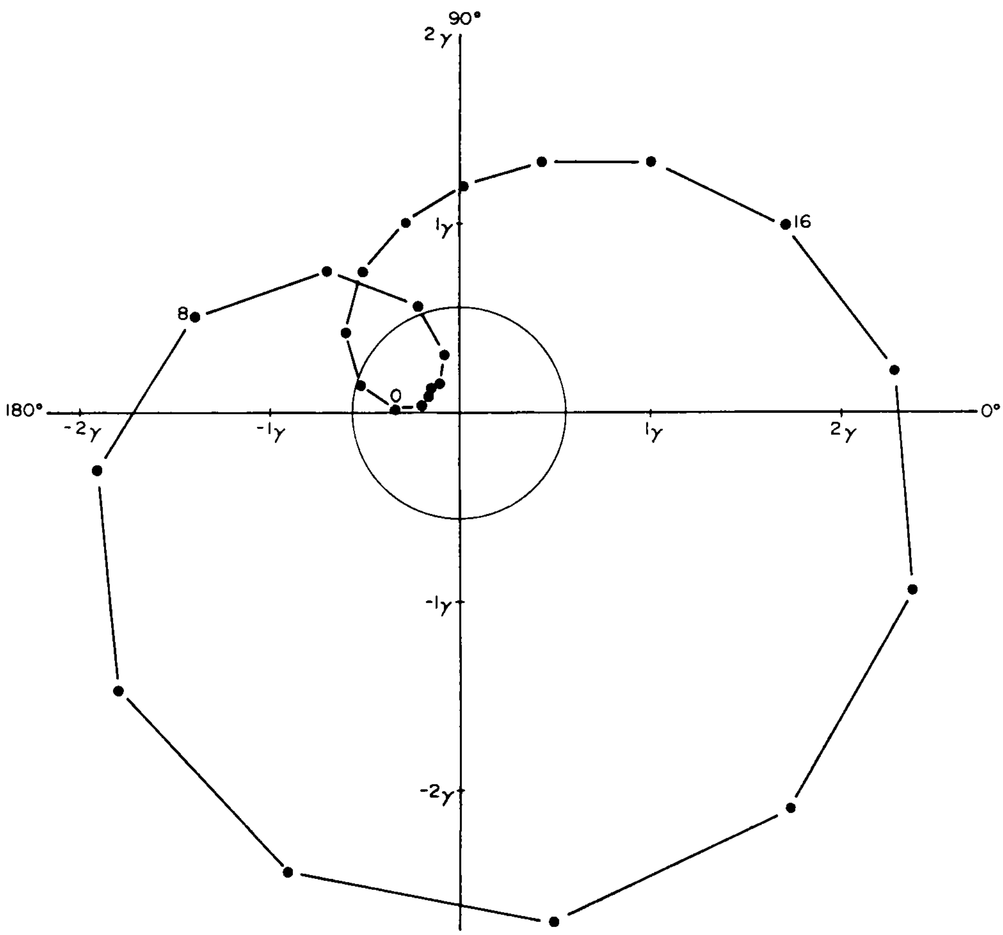

Fig. 2. Harmonic dial for the half-monthly variation of declination at Irkutsk. The 24 dial points represent the values of $l'(t)$ and $\lambda'(t)$ for each hour of mean solar time.

value of $l(t)$ occurs slightly after midnight (at 1 a.m., to the nearest hour), the midnight value of $l(t)$ is less than a tenth of the midday value, and does not differ significantly from zero.

Under the assumptions detailed above, the ocean dynamo variations, $\mathbf{L}_O$, may be represented by

$$\mathbf{L}_O = l_O \sin(2\tau + \lambda_O), \tag{9}$$

where l_O, λ_O are the midnight values of $l(t)$, $\lambda(t)$, or $l'(t)$, $\lambda'(t)$. From equations (3), (4) or (7), (8) and (5)

$$l_O \cos\lambda_O = \sum_{n=1}^{4} l_n \cos\lambda_n \tag{10}$$

$$l_O \sin\lambda_O = \sum_{n=1}^{4} l_n \sin\lambda_n \tag{11}$$

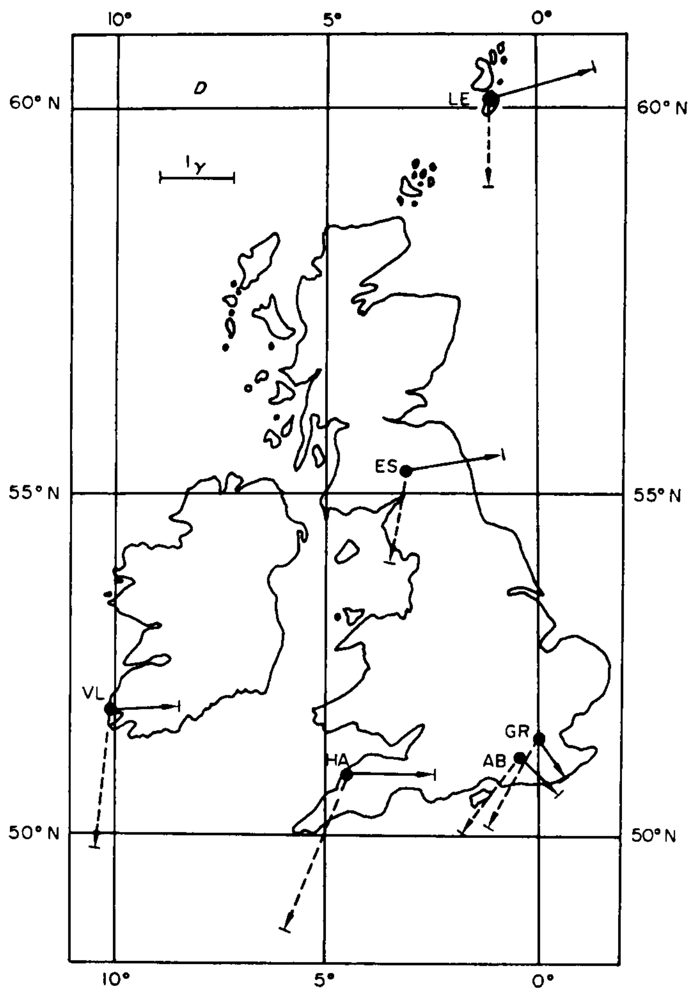

FIG. 3. Lunar semidiurnal variation of declination in the British Isles. The broken vectors represent l_2, λ_2; the solid vectors represent l_I, λ_I. The phase constant is measured anticlockwise from east.

and ρ_O, the vector probable error of $\mathbf{L}_O$ is

$$\rho_O = [\rho_1^2+\rho_2^2+\rho_3^2+\rho_4^2]^{\frac{1}{2}}. \tag{12}$$

The ionospheric dynamo part, $\mathbf{L}_I$, of the $\mathbf{L}_2$ variations is obtained by subtracting $\mathbf{L}_O$ from $\mathbf{L}_2$; thus

$$\mathbf{L}_I = l_I \sin(2\tau+\lambda_I), \tag{13}$$

where

$$l_I \cos\lambda_I = -(l_1 \cos\lambda_1 + l_3 \cos\lambda_3 + l_4 \cos\lambda_4) \tag{14}$$

and

$$l_I \sin\lambda_I = -(l_1 \sin\lambda_1 + l_3 \sin\lambda_3 + l_4 \sin\lambda_4). \tag{15}$$

The vector probable error, ρ_I, of $\mathbf{L}_I$ is given by

$$\rho_I = [\rho_1^2+\rho_3^2+\rho_4^2]^{\frac{1}{2}}. \tag{16}$$

Thus the ionospheric dynamo part of $\mathbf{L}$ (consisting of $\mathbf{L}_1$, $\mathbf{L}_I$, $\mathbf{L}_3$ and $\mathbf{L}_4$) is expressed in a form in which l_2 and λ_2 do not appear, though all the terms that do appear depend on the air motion in the ionosphere with the period of $\mathbf{L}_2$.

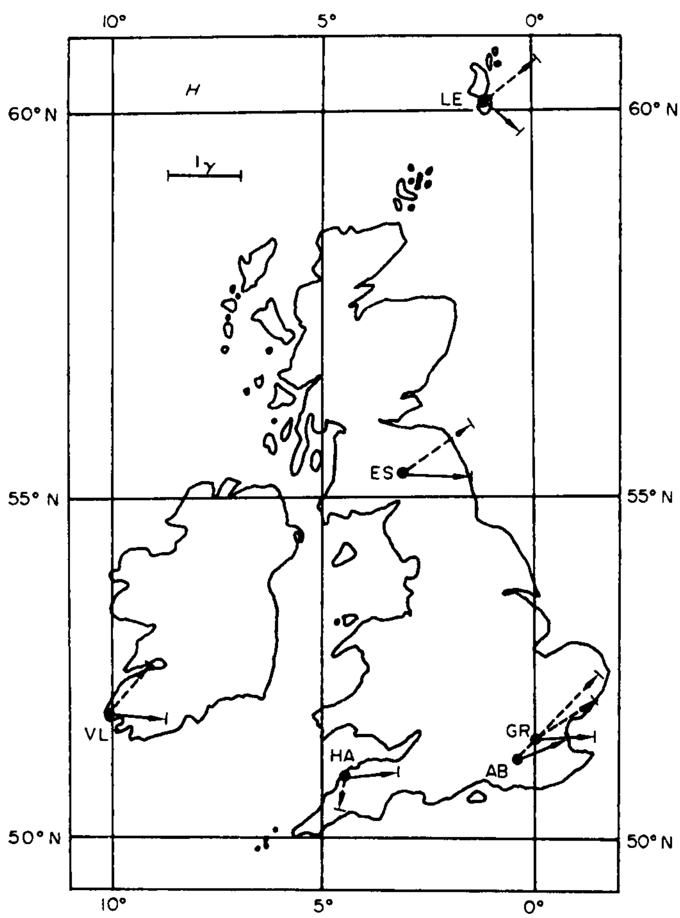

Fig. 4. Lunar semidiurnal variation of horizontal intensity in the British Isles. The broken vectors represent l_2, λ_2; the solid vectors represent l_I, λ_I. The phase constant is measured anticlockwise from east.

Application to observatories in the British Isles

The results of the previous section have been applied to the lunar daily harmonics of six observatories in the British Isles, obtained by the Chapman–Miller method. The observatories are Greenwich and Abinger (Leaton, Malin & Finch 1962), Hartland (Malin & Leaton 1967), Eskdalemuir, Lerwick and Valentia (Malin 1967). Some minor errors in the *H* harmonics for Eskdalemuir, Lerwick and Valentia, due to the placing of two days in the wrong lunar age groups, are here corrected. The oceanic and ionospheric dynamo effects are given in Table 2 for East Declination (*D*), horizontal intensity (*H*) and vertically downward intensity (*Z*). The results of an earlier analysis of *Z* (Malin 1969a), based on the assumption that the ionospheric effect is negligible between 6 p.m. and 6 a.m., are given in parenthesis in Table 2. These earlier results are in good agreement with those from the present analysis.

Table 2

	Ionospheric								Oceanic	
Element	l_1 0·01γ	λ_1 °	l_I 0·01γ	λ_I °	l_3 0·01γ	λ_3 °	l_4 0·01γ	λ_4 °	l_O 0·01γ	λ_O °
Greenwich, 1916–1925 (51°·5 N, 0°)										
D	45±12	140	68±14	303	28± 6	86	4± 5	173	121±17	210
H	52±10	181	82±15	3	37± 8	210	15± 7	82	53±15	89
Z	5±12	82	17±14	306	21± 5	126	8± 4	280	69±16	6
Abinger, 1926–1957 (51°·2 N, 0°·4 W)										
D	51± 8	147	75± 8	315	40± 3	100	14± 4	259	138±10	200
H	50± 7	186	77± 8	21	43± 3	234	13± 3	72	89± 9	67
Z	10± 9	14	12± 9	263	17± 2	125	5± 2	288	110±10	19
Eskdalemuir, 1957·5–1959·0 (55°·3 N, 3°·2 W)										
D	144±40	168	126±48	10	16±22	222	34±14	268	203±57	223
H	82±48	170	90±69	357	16±41	236	4±28	86	70±74	90
Z	100±49	149	80±54	346	5±17	231	30±15	292	75±58	100
			(71	332)					(87	93)
Hartland, 1957·5–1959·0 (51°·0 N, 4°·5 W)										
D	103±34	168	112±39	0	21±17	160	29±12	287	281±49	227
H	40±45	193	71±55	8	21±25	227	22±18	141	93±59	215
Z	72±30	161	79±32	350	20±10	164	18± 7	301	174±36	3
			(80	346)					(164	5)
Lerwick, 1957·5–1959·0 (60°·1 N, 1°·2 W)										
D	102±48	175	144±61	16	33±26	213	32±27	253	208±72	228
H	89±62	146	63±82	320	27±37	320	7±39	44	100±92	77
Z	66±114	93	62±128	253	25±36	66	66±32	46	330±139	10
			(40	281)					(285	7)
Valentia, 1957·5–1959·0 (51°·9 N, 10°·2 W)										
D	94±44	169	92±49	3	11±17	138	31±12	286	219±57	239
H	39±42	186	79±53	355	18±26	173	24±20	157	73±59	108
Z	86±39	172	77±43	12	16±13	197	34±12	318	227±47	16
			(89	5)					(202	20)

The amplitude of the ocean dynamo effect, l_O, is surprisingly large for all elements, in most cases exceeding the amplitude of $\mathbf{L}_I$, even for observatories well away from the coast. In all cases, l_O (*D*) and l_O (*Z*) are greater than l_O (*H*). Chapman & Kendall (1970) have calculated theoretical values of the ocean dynamo effect for Greenwich, Abinger, Eskdalemuir, Hartland and Valentia, which they find to be in fair agreement with the present estimates of L_O (D) and L_O (Z); however, the agreement with L_O (H) is less satisfactory. Their calculations were based on linear models of channels representing in idealized form the North Sea, English Channel, Irish Sea and Atlantic Ocean, and such a model is not suitable for the calculation of magnetic variations at Lerwick, which is on a relatively small island. The Chapman & Kendall results confirm the reality of the ocean dynamo effect and give considerable support to the present method for its determination.

The lunar semidiurnal part of the ionospheric dynamo effect, L_I, is shown in Figs 3, 4 and 5 (solid vectors), together with the total lunar semidiurnal geomagnetic harmonic (broken vectors), before removal of the ocean dynamo effect. Removal of the ocean dynamo part makes a profound difference to the L_2 variations, and, at least for H and Z, leads to a more consistent pattern over the British Isles. The L_I vectors for all elements have phase angles close to 0, corresponding to a maximum geomagnetic effect occuring 3 hours after lunar transit. (The only exception is L_I (Z) at Lerwick, where the amplitude is greatly exceeded by its vector probable error).

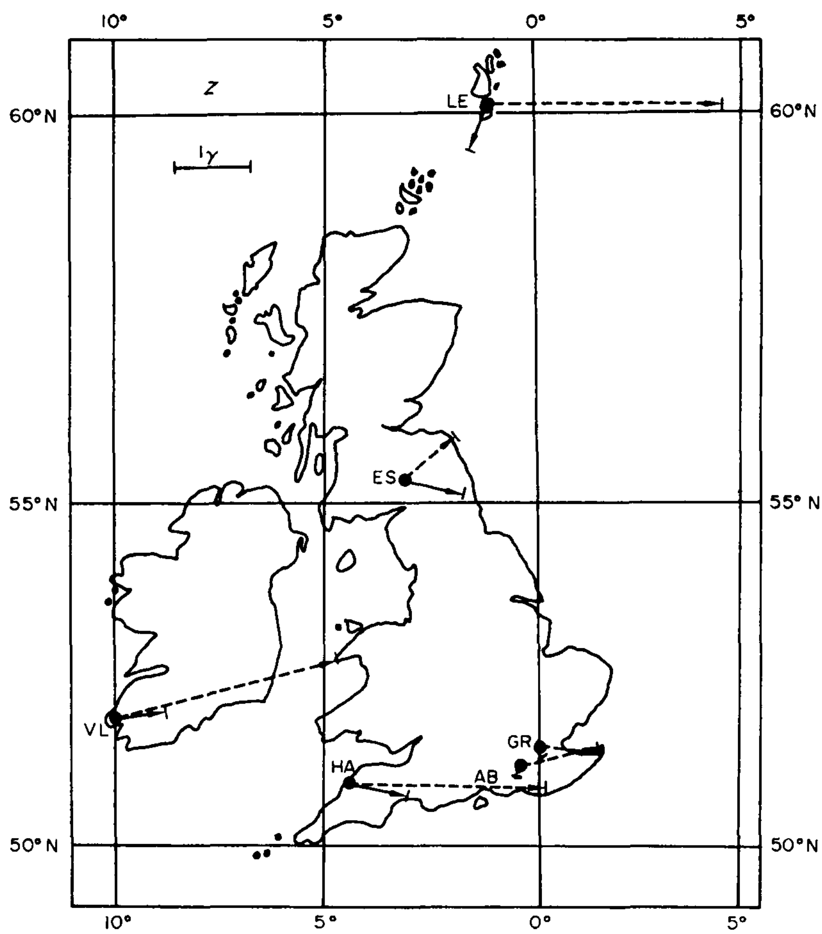

Fig. 5. Lunar semidiurnal variation of vertical intensity in the British Isles. The broken vectors represent l_2, λ_2; the solid vectors represent l_I, λ_I. The phase constant is measured anticlockwise from east.

Conclusions

Making reasonable assumptions about the mechanism for the production of geomagnetic lunar daily variations, it is possible to divide the variations into parts of oceanic and ionospheric origin. The method may be readily applied to results of lunar analyses obtained by the Chapman–Miller method, without recourse to the observatory hourly mean data. The ionospheric dynamo effect is expressed without explicit mention of the observed values of l_2 and λ_2; this must cast doubt on the validity of ionospheric current systems deduced from studies of $\mathbf{L}_2$ data alone.

The ocean dynamo effect at six British Isles observatories is found to be comparable with and, in many cases, to exceed the annual mean of the ionospheric dynamo effect, for all elements and even for inland stations. It is clearly important to remove this part prior to any study of the global morphology of the ionospheric dynamo variations. In addition, study of the ocean dynamo effect might be expected to lead to a better understanding of the pelagic tides.

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