Sequence-based Rendezvous for Dynamic Spectrum Access

Luiz A. DaSilva
Bradley Dept. of Electrical and Computer Engineering
Virginia Tech
Arlington, VA, USA
ldasilva@vt.edu

Igor Guerreiro
Wireless Telecommunications Research Group
Federal University of Ceará
Fortaleza, CE, Brazil
igor@gtel.ufc.br

***Abstract*— In the context of dynamic spectrum access (DSA), *rendezvous* refers to the ability of two or more radios to meet and establish a link on a common channel. In decentralized networks, this is often accomplished by each radio visiting potential channels in random fashion, in a process that we call blind random rendezvous. In this work, we propose the use of sequences that determine the order with which radios visit potentially available channels. Through sequence-based rendezvous, it is possible to: (i) establish an upper bound to the time to rendezvous (TTR); (ii) establish a priority order for channels in which rendezvous occurs; (iii) reduce the expected TTR as compared to random rendezvous. We provide an example of a family of sequences and derive the expected time-to-rendezvous using this method. We also describe how the method can be adopted when one or more primary users are detected in the channels of interest.**

Keywords - cognitive radios; rendezvous; dynamic spectrum access; multi-channel MAC

I. INTRODUCTION

Dynamic and opportunistic utilization of available spectrum requires that radios be capable of finding one another to establish a link and bootstrap communications, in a process that is referred to as *rendezvous*.

The rendezvous process can be aided by a server or base station or performed in a completely distributed fashion among all cognitive radios. In the former, radios often rely on a common signal, such as a beacon broadcasting time and frequency information. In the latter, probe signals and probe acknowledgements are exchanged among radios on a selection of available channels. One important decision is whether or not to dedicate one or more channels for the exchange of control information. The use of a common control channel simplifies the rendezvous process but may result in a bottleneck for communications, as well as create a single point of failure. In this paper, we consider the problem how to perform rendezvous when all channels can be used for both data and control information; this is what we call *blind rendezvous*.

This material is based on work partially sponsored by DARPA through Air Force Research Laboratory (AFRL) Contract FA8750-07-C-0169. The views, conclusions and recommendations contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of any of the sponsoring agencies or the U.S. Government.

A basic solution for blind rendezvous, adopted in some dynamic spectrum access systems, is for each radio to randomly visit all potential communication channels in search of its peers. For two radios adopting blind random rendezvous, the expected time to rendezvous (TTR) increases as O(N), where N is the number of possible channels. There is, in this case, no upper bound on the actual time required for rendezvous.

In contrast to that, we propose the use of non-orthogonal sequences to attain rendezvous, while still not requiring any synchronization between radios. Our proposed scheme provides a method for rendezvous that: (a) provides an upper bound for the TTR; (b) establishes a priority order for channels in which rendezvous occurs; (c) may reduce the expected TTR as compared to random rendezvous. Reduced TTR leads to reduced channel access delay, while the existence of an upper bound enables deterministic service guarantees regarding link establishment time. Whether and how well each of these properties is achieved depends on the design of the sequence.

A reasonable parallel to the method we propose is the use of frequency hopping spread spectrum techniques. In frequency hopping, radios are assigned hopping sequences. If these sequences are orthogonal (or nearly orthogonal), two radios have zero (or close to zero) probability of occupying the same channel simultaneously. The design of sequences with good orthogonality properties is the topic of [1]. In contrast, for purposes of rendezvous, we propose the use of non-orthogonal sequences so as to maximize the probability that two radios looking for each other will eventually be searching on the same channel.

This paper is organized as follows. In the next section, we summarize some of the approaches to rendezvous found in the literature. We then describe the sequence-based rendezvous that we propose and derive the expected time to rendezvous achieved by our method for a particular family of sequences, comparing it to blind random rendezvous. In the next section, we describe how this method can be applied when one or more incumbent users are detected and quantify the effects of incumbent users on the expected time to rendezvous. The last section summarizes our main conclusions and outlines additional areas for future research.

II. APPROACHES TO RENDEZVOUS

We can broadly classify rendezvous mechanisms into aided (or infrastructure-based) and unaided (infrastructure-less). Aided rendezvous is accomplished with help from a server, which periodically broadcasts information regarding available channels and may even serve as a clearinghouse for link establishment and the scheduling of transmissions, typically using a well-known control channel.

For example, [2] proposes an architecture in which some frequencies are set aside for use as spectrum information channels. Clients dedicate a wireless interface to scan these channels, where the base stations broadcast information regarding spectrum availability, interference conditions, etc. Clients can use those same control channels to request the use of dedicated spectrum to their traffic (or, alternatively, clients may directly proceed to the data channels that they now know to be available).

In unaided rendezvous, each cognitive radio must find other nodes in the network on its own. Unaided rendezvous may also avail itself of a dedicated control channel, which all radios visit periodically to bootstrap their connectivity to other nodes in the network, or to set up links in new channels.

While the use of a dedicated control channel simplifies the initial step of determining in which frequency to look for neighbors, it incurs additional overhead and creates a single point of failure; the common control channel may also become a bottleneck for communications.

An alternate approach is not to dedicate a channel for control, but rather to attempt rendezvous in one of the same channels that can be used for the exchange of data. Such an approach is taken, for instance, by [3]. The question then, from the point of view of each individual radio, is how to visit the potentially available channels so as to maximize the probability of encountering another radio that also wishes to establish communications.

Let us take a set of N potential channels. In the blind rendezvous mechanism, each radio will visit these channels at random: at a particular instant, a radio will be occupying one of these channels with probability 1/N. When two radios occupy the same channel (and one is transmitting a probe or beacon while the other is listening for such a probe), rendezvous occurs. Blind random rendezvous is adopted, for instance, in the implementation described in [4]. While this approach is not unreasonable when dealing with a small number of channels, the time to rendezvous is unbounded. The solution we propose here provides an upper bound for the TTR and, for some selected sequences, may reduce the expected TTR as compared to blind random rendezvous.

It is worth noting that rendezvous techniques have also been proposed for implementation at the physical layer. The approach proposed by [5] is to embed cyclostationary signatures into all transmitted signals. These signatures can then be detected in a short amount of time by radios seeking to join the network.

III. SEQUENCE-BASED RENDEZVOUS

We propose the use of pre-defined sequences by each radio to determine the order in which potential channels are to be visited. These sequences are constructed in such a way to minimize the maximum and/or the expected time-to-rendezvous even when radios are not synchronized to each other. For instance, consider radio 1 starting to look for a peer at time t_1 and radio 2 doing the same at time t_2. In our method, each radio follows a pre-defined sequence in visiting the potentially-available channels in search of each other. The properties of the time to rendezvous depend on the sequence selected.

We provide a concrete example by describing one method for building these sequences below. Consider again a set of N potentially-available channels, numbered 1 through N. A visiting sequence $a = (a_1, a_2, a_3, \ldots)$ describes the order in which a radio visits channels in search of other radios with which to rendezvous. We are particularly interested in sequences that are periodic and that, for fairness reasons, contain in each period the same number of instances of each channel.

One method for building such a sequence is to select a permutation of the N channels (there are N! such permutations) and building the sequence as illustrated in Fig. 1. The selected permutation appears (N+1) times in the sequence: N times the permutation appears contiguously, and once the permutation appears interspersed with the other N permutations.

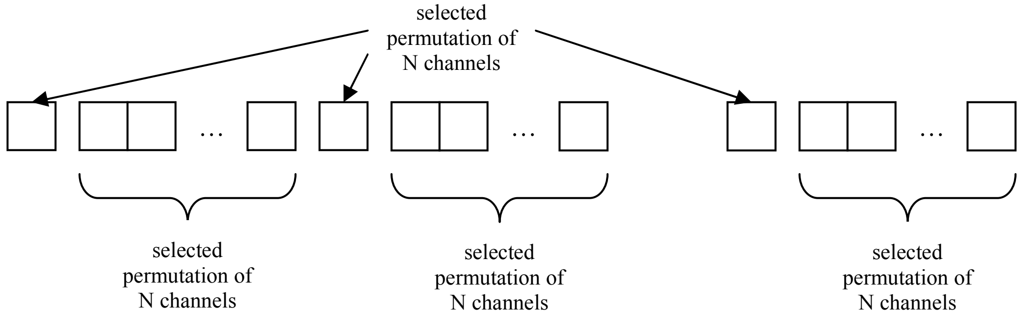

Figure 1. Building a sequence for sequence-based rendezvous.

An example may make things more clear. Take N = 5, and select at random a permutation of these 5 channels, say the permutation (3, 2, 5, 1, 4). The method described above to form a sequence would yield a sequence described by (only one period is shown):

3, 3, 2, 5, 1, 4, 2, 3, 2, 5, 1, 4, 5, 3, 2, 5, 1, 4, 1, 3, 2, 5, 1, 4, 4, 3, 2, 5, 1, 4

Note that the original permutation appears 6 times in the sequence, including once interspersed among the other 5 appearances of the permutation (underlined for easier visualization). This sequence would then repeat ad infinitum.

For later derivations, it will be convenient to express the basic sequence in matrix form. For the example above, the matrix would be:

$$\begin{bmatrix} 3 & 3 & 2 & 5 & 1 & 4 \\ 2 & 3 & 2 & 5 & 1 & 4 \\ 5 & 3 & 2 & 5 & 1 & 4 \\ 1 & 3 & 2 & 5 & 1 & 4 \\ 4 & 3 & 2 & 5 & 1 & 4 \end{bmatrix}.$$

We are able to derive the expected TTR for two radios following *any* sequence constructed in this manner. Further, we are able to show that there is an upper bound on the time it will take the two radios to find each other (note that blind rendezvous admits no such upper bound). These properties are explored in the next section.

IV. EXPECTED TIME TO RENDEZVOUS

For the case where two radios sweep the frequency spectrum by visiting channels in random order (what we characterize as blind random rendezvous), we can express the expected time-to-rendezvous as $E[TTR] = k \cdot N$. Here k is a constant that represents the probability that one of the radios is transmitting and the other receiving, in a given time slot (often taken as a constant) and N represents the number of available channels for rendezvous. Without loss of generality, we will omit the constant in the following discussions.

We have derived a closed-form expression for the expected TTR using our proposed sequence-based rendezvous technique, with all radios adopting the same pre-selected sequence. In this context, lack of synchronism between radios must be taken into account. In other words, there may be some delay between the time radio A starts looking for a peer and the time radio B starts doing the same. Figure 2 provides an example. We denote the lag between the times when each of the two radios starts looking for each other by t_d.

It is worth noting that not any sequence will yield a finite E[TTR]. There are some sequences for which rendezvous may never be achieved for some values of t_d. We avoid these, concentrating on selection of sequences, like the example in the previous section, for which E[TTR] can be shown to be finite.

Figure 2. Secondary users A and B perform blind, sequence-based rendezvous.

A convenient way to represent the TTR as a function of the delay t_d, when sequence-based rendezvous is applied, is using a matrix in which each entry is related to a delay value. Thus, let **T** be a N-by-(N+1) matrix containing the time-to-rendezvous as a function of t_d; we call this the Time-to-Rendezvous Matrix. To obtain a closed-form expression for time to rendezvous, we exploit patterns in this matrix. These patterns are described in the Appendix, and the general form of the matrix is shown below.

$$\mathbf{T} = \begin{pmatrix} t_R(0) & t_R(1) & \cdots & t_R(N) \\ t_R(N+1) & t_R(N+2) & \cdots & t_R(2N+1) \\ \vdots & \vdots & \ddots & \vdots \\ t_R(N^2-1) & t_R(N^2) & \cdots & t_R(N^2+N-1) \end{pmatrix}_{N\times N+1}$$

The resulting closed-form expression for expected value E[TTR] for the family of sequences presented in the previous section can be derived, as shown in the Appendix, as:

$$E[TTR] = \frac{N^4 + 2N^2 + 6N - 3}{3N(N+1)} \qquad (1)$$

The closed-form expression above was also validated using simulation. As mentioned before, using this family of sequences rendezvous is not equally likely to occur in any of the N channels. If we use the permutation (1, 2, …, N) as the basis for forming the sequence, channel 1 is favored over the remaining channels. (Also note that we can, without loss of generality, rename the channels so as to order them from most to least preferred.) This ability to prioritize channels for rendezvous can be useful if there is reason to believe that some channels are more prone to be occupied by primary users than others, or if some channels have better propagation characteristics than others.

It is then possible to derive, for the same family of sequences, the probability that rendezvous occurs in the most favored channel (the "best" channel) and the probability that it occurs in the least favored channel (the "worst" channel), given respectively by:

$$P[best\_channel] = \frac{3N-1}{N(N+1)} \qquad (2)$$

$$P[worst\_channel] = \frac{1}{N(N+1)} \qquad (3)$$

Fig. 3 plots these two probabilities.

Again using the same family of sequences, we can express the conditional expectation of TTR, conditioned on rendezvous occurring in the "best" and "worst" channels, as:

$$E[TTR \mid best\_channel] = \frac{N^3 - 2N^2 + 9N - 4}{2(3N-1)} \qquad (4)$$

$$E[TTR \mid worst\_channel] = N^2 \qquad (5)$$

Note that the TTR for sequence-based rendezvous using these sequences is upper bounded by N^2.

Fig. 4 plots the expected TTR as a function of the number of channels N, as well as the conditional expectation of TTR. The squares shown in the figure correspond to the expected TTR conditioned on rendezvous occurring in each channel between the "best" and the "worst."

It should be noted that, while the family of sequences described here provide the advantages of an upper-bounded TTR and the prioritization of channels for rendezvous, it does not improve on the E[TTR] of blind, random rendezvous. It is, however, possible to devise sequences that do reduce E[TTR]. As an existence proof, Table I shows some specific sequences for which the average TTR is lower than N [6].

Up to now, we have not considered the appearance of an incumbent user on one of the channels. In the next section, we describe a methodology for using sequence-based rendezvous when the presence of an incumbent is detected on one or more channels, and we quantify the effect of incumbents on the time-to-rendezvous.

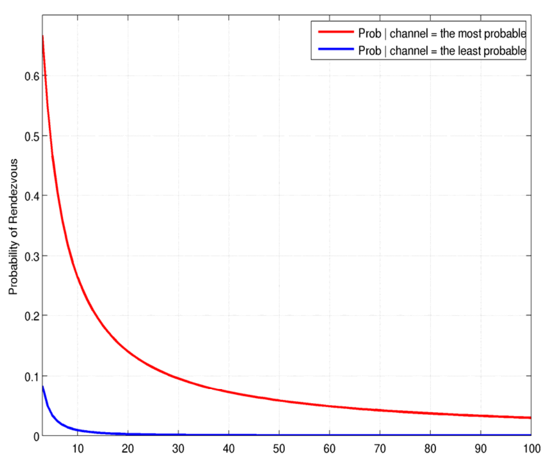

Figure 3. Probability that rendezvous occurs in the most and the least preferred channels.

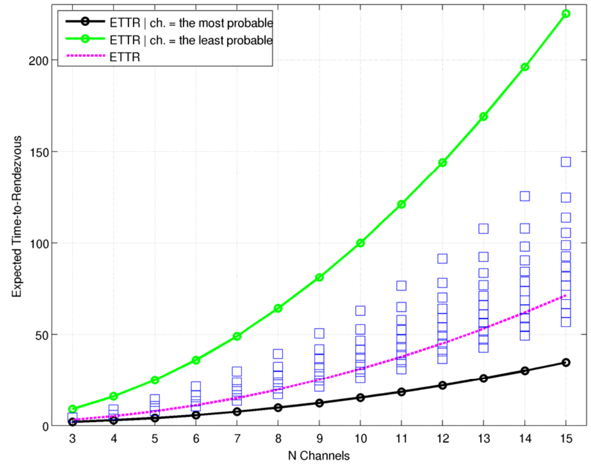

Figure 4. Expected time to rendezvous (middle curve) and conditional expectation of time to rendezvous, given that rendezvous occurs in the "best" (most probable) and "worst" (least probable) channel. The squares in the graph correspond to the conditional expectations for each channel between the "best" and the "worst."

TABLE I. EXAMPLE SEQUENCES (ONE PERIOD SHOWN) WITH E[TTR] < N.

N	Sample sequence (one period)	Max TTR	E[TTR]
3	1 1 2 3 2 2 1 3 3 3 1 2	8	2.75
4	1 1 1 2 3 4 2 2 2 1 3 4 3 3 3 1 2 4 4 4 4 1 2 3	13	3.96
5	2 3 5 4 1 1 2 5 4 3 4 5 3 2 1 4 2 5 3 1 3 4 5 1 2 3 4 2 5 1	11	4.23

V. AVOIDING PRIMARY USERS

When licensed channels are used opportunistically by secondary users, some considerations have to be included in the development of dynamic spectrum access algorithms. One of them is the presence of primary users in one or more of these channels. Primary (also referred to as incumbent) users are always given priority in using the spectrum. Secondary users are required to periodically sense for the presence of incumbents and to vacate the channel within a short period of time when such users are detected.

We now describe how a sequence-based rendezvous algorithm can be followed when primary users are detected in one or more of the channels. As long as at least one channel is available (not occupied by a primary), the sequence-based method will guarantee that rendezvous will eventually occur. The complete algorithm is described next.

After selecting or being assigned a sequence, each radio visits channels in that sequence and senses for the presence of a primary user. When a primary user is detected in a given channel, all instances of that channel are removed from the sequence. The radio continues visiting channels in the order of the modified sequence. The process is summarized in Fig. 5.

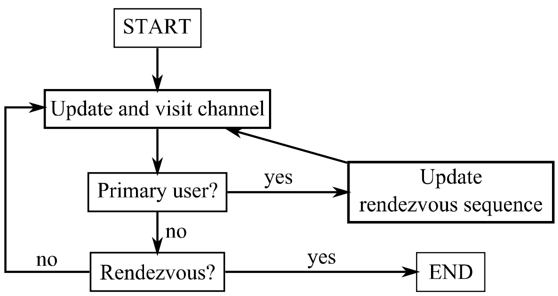

Figure 5. Rendezvous process in the presence of incumbent users.

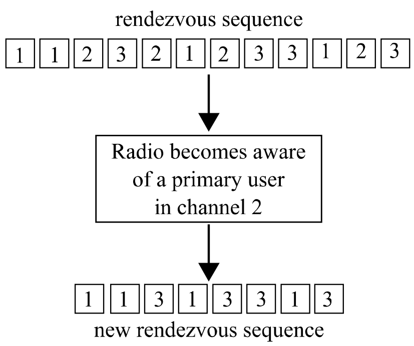

Figure 6. Process of removing a channel occupied by a primary user.

The main addition with respect to the method as described in the previous sections is the block "update rendezvous sequence" shown in the flow diagram. When a radio visits the n-*th* channel, it verifies whether there is primary user on that channel. If so, its rendezvous sequence must be updated. That is, that channel will be removed from its sequence. After that, the radio hops to the next channel based on its new sequence, as shown in Fig. 6. If there is no primary user in that channel, the radio resumes its discovery process.

Note that we do not assume that two radios searching for each other will detect the presence of a primary simultaneously (as, in practice, this is unlikely to happen). Regardless of when an incumbent is detected, the sequence update process will eventually lead to rendezvous, provided that both radios are capable of sensing the same incumbents. We can also reset the entire process at some point to account for incumbents' eventually vacating the channel again.

Intuitively, the process of removing some channels from the sequence due to the presence of an incumbent reduces the number of channels to visit and leads to lower expected time to rendezvous. We quantify this effect through simulation. We use MATLAB simulations to consider all sequences of N channels that can be constructed by the method above and (taking into account all possible values of delay between the time each of the two radios starts to attempt rendezvous) calculate the average time to rendezvous conditioned on the presence of incumbents on one or more channels. The outcomes are shown in Fig. 7.

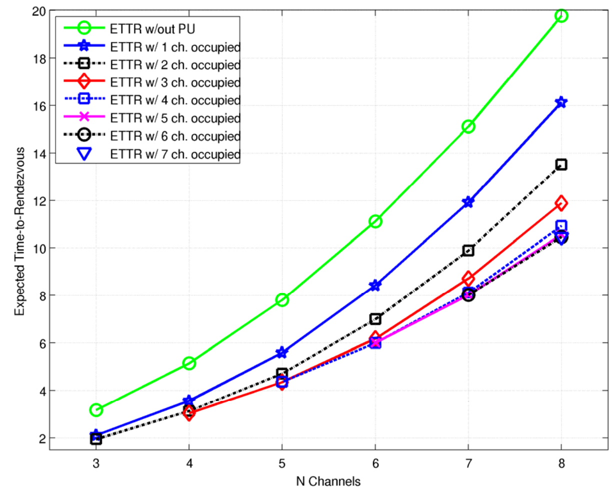

Figure 7. Expected TTR with one more channels occupied by a primary user.

VI. CONCLUSIONS AND FUTURE WORK

In this work, we propose the use of sequences that dictate the order in which two radios will visit a set of N channels of interest when attempting to rendezvous with each other. We derive a closed-form expression for expected time to rendezvous using such sequences and show that it has an upper bound. We also derive expressions for the probability that rendezvous occurs in the "best" and "worst" channels, as well as the conditional expectation of TTR given that rendezvous occurs in each of those channels.

While we describe how to construct a sequence with some desirable rendezvous properties, no claim is made as to the optimality of this family of sequences. In particular, we know these sequences do not minimize average TTR. We continue to work on the study of sequences that achieve optimal expected and/or maximum TTR.

REFERENCES

[1] D. V. Sarwate, "Optimum PN Sequences for CDMA Sequences," *IEEE 3rd Intl. Symp. on Spread Spectrum Techniques and Applications*, vol. 1, pp. 27-35, 1994.

[2] M. M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, "DIMSUMNet: New Directions in Wireless Networking Using Coordinated Dynamic Spectrum Access," *Proc. of the 6th IEEE Intl. Symp. on a World of Wireless Mobile and Multimedia Networks (WoWMoM'05)*, pp. 78-85, 2005.

[3] B. Horine and D. Turgut, "Link Rendezvous Protocol for Cognitive Radio Networks," in *Proc. IEEE DySPAN*, pp. 444-447, 2007.

[4] M. D. Silvius, F. Ge, A. Young, A. B. MacKenzie, and C. W. Bostian, "Smart Radio: Spectrum Access for First Responders," *Proc. of SPIE*, vol. 6980, Wireless Sensing and Processing III, Apr. 2008.

[5] P. D. Sutton, K. E. Nolan, and L. E. Doyle, "Cyclostationary Signatures in Practical Cognitive Radio Applications," *IEEE JSAC*, vol. 26, no. 1, Jan. 2008.

[6] R. W. Thomas and R. K. Martin, personal correspondence.

APPENDIX

In this section, we derive the closed form expression for the expected time to rendezvous presented in Section IV as Eq. 1.

We start by constructing a matrix **T**, as defined in Section IV, for an arbitrary sequence constructed as described in Section III. Analyzing the structure of the matrix **T**, there are sequences of 1's and 2's whose number of elements can be described as functions of N. Besides, there is an entry that equals N^2. Moreover, there are two additional numerical sequences denoted here as M_1 and M_2. Thus, **T** can be shown as follows:

$$\mathbf{T} = \begin{pmatrix} 1 & 1 & & & N^2 \\ 2 & 1 & & & \\ \vdots & \vdots & & \textit{filled with } M_1 \textit{ and } M_2 & \\ 2 & 1 & N & & \\ 2 & 1 & & & \\ 2 & 1 & & & 2 \end{pmatrix}_{N \times N+1}$$

Let $\|.\|$ be the norm operator, here defined as the summation of all elements of some sub-structure of a matrix. Hence, the matrix **T** can be summarized as follows:

- $\|$sequence of 1's$\|$ = N + 1
- $\|$sequence of 2's$\|$ = 2N
- an entry equals N^2
- M_1 and M_2

The intended metric is the expected value E[TTR] and it is obtained by calculating $\|\mathbf{T}\|$ and dividing by the number of elements in the matrix. Thus:

$$\|\mathbf{T}\| = N^2 + 3N + 1 + \|M_1\| + \|M_2\| \quad . \tag{6}$$

Eq. 6 depends on M_1 and M_2 and therefore these sub-sequences must be described.

A. Describing M_1

The sequence M_1 looks like a simple arithmetic series with some gaps in the middle represented by K_1, as follows:

$$\|M_1\| = \sum_{i=3}^{N^2-N} i - K_1 \tag{7}$$

where K_1 has a pattern that can be described as:

$$\|K_1\| = \sum_{j=1}^{N-2} \left(\sum_{k=jN+1}^{j(N+1)+1} k \right) \quad . \tag{8}$$

An example may be useful to illustrate its behavior. Considering N = 5, M_1 is shown as:

$$\boxed{3\ \ 4\ \ 5}\ \underbrace{6\ \ 7}_{}^{gap}\ \boxed{8\ \ 9\ \ 10}\ \underbrace{11\ \ 12\ \ 13}^{gap}\ \boxed{14\ \ 15}\ \underbrace{16\ \ 17\ \ 18\ \ 19}^{gap}\ \boxed{20}$$

In general, this sequence is represented by:

$$\|M_1\| = \sum_{i=1}^{N^2-N} i - 3 - \sum_{j=1}^{N-2} \left(\sum_{k=jN+1}^{j(N+1)+1} k \right) \quad . \tag{9}$$

After some algebra, Eq. 9 can be represented as a closed-form function of N. This step will be demonstrated later.

B. Describing M_2

The next step is to find a representation of M_2 as a function of N. This sequence can be represented in terms of rows and columns as follows:

$$\begin{pmatrix} N+2 & 2N+3 & & N^2-N-1 \\ N+2 & 2N+3 & \cdots & N^2-N-1 \\ N+2 & \vdots & & \\ \vdots & 2N+3 & & \\ N+2 & & & \end{pmatrix}_{N-1 \times N-2} \quad .$$

add N+1

Each row represents an arithmetic progression with the first term equal to N+2. The last row has only one term but the predecessor rows until the second one increase progressively their number of terms by one. Moreover, there are N-1 rows and the first one has N-2 terms. Thus, $\|M_2\|$ is represented by:

$$\|M_2\| = \sum_{j=1}^{N-2} \{ (N-j) \cdot [N + 2 + (N+1) \cdot (j-1)] \} \quad . \tag{10}$$

An example might be useful to illustrate its behavior. Considering N = 5, M_2 appears as:

$$\begin{pmatrix} 7 & 13 & 19 \\ 7 & 13 & 19 \\ 7 & 13 & \\ 7 & & \end{pmatrix}$$

Next, the expected time-to-rendezvous expression for sequence-based rendezvous will be represented as a closed-form equation in N.

C. Closed-form expression

The intended metric E[TTR] can now be derived as follows:

$$
\begin{aligned}
E\{t_R\} &= \frac{\|T\|}{N \cdot (N+1)} \\
&= \frac{f_1(N) - f_2(N)}{N \cdot (N+1)}
\end{aligned}
\quad . \qquad (11)
$$

The expression $\|\mathbf{T}\|$ may be broken into two intermediate functions $f_1(N)$ and $f_2(N)$ to simplify algebraic manipulation. The function $f_1(N)$ can be described as follows:

$$
\begin{aligned}
f_1(N) &= \left(N^2 + 3N - 2\right) + \left(\sum_{i=1}^{N^2-N} i\right) \\
&= \frac{N^4}{2} - N^3 + 2N^2 + \frac{5N}{2} - 2
\end{aligned}
\qquad (12)
$$

and the same thing can be done to describe $f_2(N)$, as follows:

$$
\begin{aligned}
f_2(N) &= \sum_{j=1}^{N-2} \left\{ \left(\sum_{k=jN+1}^{j(N+1)+1} k \right) - (N-j) \cdot \left[N + 2 + (N+1) \cdot (j-1) \right] \right\} \\
&= \frac{N^4}{6} - N^3 + \frac{4N^2}{3} + \frac{N}{2} - 1
\end{aligned}
\qquad (13)
$$

After replacing $f_1(N)$ and $f_2(N)$ in Eq. 11 by Eq. 12 and Eq. 13, we finally obtain:

$$
E\{t_R\} = \frac{N^4 + 2N^2 + 6N - 3}{3N \cdot (N+1)} \quad . \qquad (14)
$$