Sequential fusion:
sub-barrier fusion enhancement due to neutron transfer

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From the analysis of appropriate experimental data within a simple theoretical model it is shown that the intermediate neutron transfer channels with positive Q-values really enhance the fusion cross section at sub-barrier energies. The effect of sequential fusion was found to be very large especially for fusion of weakly bound nuclei. New experiments are proposed, which may shed additional light on the effect of neutron transfer in fusion processes.

Introduction

Neutron transfer cross sections are known to be rather large at near-barrier energies of heavy-ion collisions and there is a prevailing view that coupling with the transfer channels should play an important role in sub-barrier fusion of heavy nuclei. If, however, the sub-barrier fusion enhancement caused by rotation of statically deformed nuclei and/or by vibration of the nuclear surfaces is well studied in many experiments and well understood theoretically, the role of neutron transfer is not so clear. There are two reasons for that. First, in the experimental study of the effect of the valence neutrons we need to compare with each other fusion cross sections of different combinations of nuclei, which among other things have different collective properties, and it is not so easy to single out the role of neutron transfer from the whole effect of sub-barrier fusion enhancement. Second, it is very difficult for many reasons to take into account explicitly the transfer channels within the consistent channel coupling (CC) approach used successfully for the description of collective excitations in the near-barrier fusion processes. As a result, we are still far from good understanding of the subject. Moreover, there is no consensus on the extent to which the intermediate neutron transfer is important in fusion reactions.

However, recently more and more experimental evidence appears for additional enhancement of the sub-barrier fusion cross section due to neutron transfer with positive Q-values, both in reactions with stable nuclei and especially in reactions with weakly bound radioactive projectiles. Correlations in the near-barrier transfer and fusion cross sections were studied in Ref. [1]. It was found that the loss of transfer flux (mainly in the channels with positive Q-values) goes essentially into fusion. Another example of this type is shown in Fig. 1, where the fusion cross sections for the ^{40}Ca+^{48}Ca and ^{48}Ca+^{48}Ca combinations [2] are plotted as a function of the center-of-mass energy divided by the Coulomb barrier. For the more neutron rich ^{48}Ca+^{48}Ca combination, one could expect higher sub-barrier fusion enhancement

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compared to the ^{40}Ca+^{48}Ca reaction. The experiment gives the opposite result, which means that the neutron excess itself does not play a crucial role in sub-barrier fusion. Moreover, while the cross sections for ^{48}Ca+^{48}Ca can be well reproduced by CC calculations including inelastic excitations to the 2^+ and 3^- states of both nuclei, the cross sections of ^{40}Ca+^{48}Ca at deep sub-barrier energies were found much larger than the calculated ones [2]. The authors assumed that just the coupling with neutron transfer channels with positive Q-values gives this additional enhancement for the ^{40}Ca+^{48}Ca combination.

Some years ago Stelson et al. [3),4)] proposed an empirical distribution of barriers technique and found that many experimental data may be well described by a flat distribution of barriers with the lower-energy cutoff, which corresponds to the energy at which the nuclei come sufficiently close together for neutrons to flow freely between the target and projectile (neck formation). A simple phenomenological model for a CC calculation was proposed by Rowley et al. [5], in which the coupling with neutron transfer channels was simulated by a parameterized coupling matrix. It was found that sequential transfers with negative Q-values can lead to a broad barrier distribution consistent with a neck formation. For positive Q-values, however, the results revealed an "anti-necking" configuration. Later, using the same scheme and assuming a dominance of neutron transfers with $Q = 0$ Rowley fitted very well the fusion cross section for the ^{40}Ca+^{96}Zr reaction [6]. The Refs. [7),8)] should be mentioned here, in which the problem of coupling with transfer channels in fusion reactions was also studied. Time-dependent wave packet method was used in Ref. [9] for simultaneous description of near barrier fusion and transfer reactions with weakly bound nuclei. Nevertheless, the problem of developing a consistent microscopic approach with predictive power, which could clarify unambiguously the role of neutron transfer in sub-barrier fusion processes, remains open. It is especially important for forthcoming experiments with radioactive beams of accelerated neutron-rich fission fragments. In Ref. [10] the model of *sequential fusion* has been proposed to describe the effect of intermediate neutron transfer in sub-barrier fusion reactions.

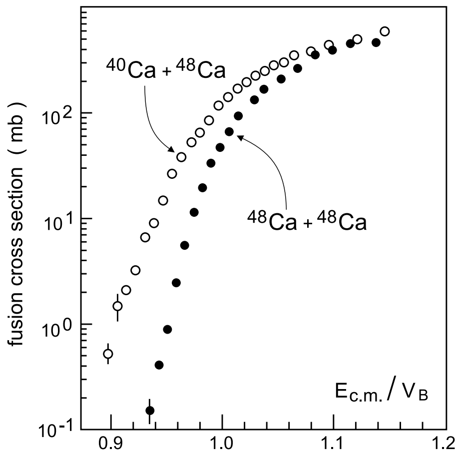

Fig. 1. Fusion cross sections for ^{40}Ca+^{48}Ca (open circles) and ^{48}Ca+^{48}Ca (filled circles) as a function of the reduced center-of-mass energy [2].

§1. Sequential fusion

Rather accurate description of sub-barrier fusion cross sections may be obtained within the semi-empirical approach [11),12)], in which the quantum penetrability of the Coulomb barrier is calculated using the concept of barrier distribution arising due to the multi-dimensional character of the real nucleus-nucleus interaction: $T(E,l) = \int f(B) P_{HW}(B;E,l)dB$. Here

$$P_{HW} = \{1 + \exp(\frac{2\pi}{\hbar\omega_B(l,E)}[B + \frac{\hbar^2}{2\mu R_B^2(l)} l(l+1) - E])\}^{-1} \qquad (1{\cdot}1)$$

is the usual Hill-Wheeler formula [13] for the estimation of the quantum penetration probability of the one-dimensional potential barrier with the barrier height modified to include a centrifugal term, $\hbar\omega_B(l,E)$ is defined by the width of the parabolic barrier, and R_B is the position of the barrier. Empirical barrier distribution function $f(B)$, which satisfies the normalization condition $\int f(B)dB = 1$, may be found from the multi-dimensional nucleus-nucleus interaction $V_{12}(r; \vec{\beta}_1, \theta_1, \vec{\beta}_2, \theta_2)$, where $\vec{\beta} = \{\beta_\lambda\}$ are the deformation parameters of the projectile and target ($\lambda = 2, 3, ...$) and $\theta_{i=1,2}$ are the orientations of statically deformed nuclei [11), 12)].

Assume now that the projectile $a = (b + n)$ has a valence neutron, which moves much faster than the relative motion of colliding nuclei $a + A$. This neutron can be transferred from the projectile a to the target A before the nuclei overcome the Coulomb barrier and come in contact. If the neutron is transferred with the Q-value less than zero, the relative motion of the nuclei $b = a - n$ and $B = (A + n)$ becomes slower and at initial sub-barrier energy these nuclei will re-separate giving a contribution to the cross section of normal transfer process, which dominates at sub-barrier energies. If, however, the valence neutron is transferred to the states with positive Q-values (if they are), then the nuclei b and B get a gain in kinetic energy and may overcome the Coulomb barrier with larger probability giving a contribution to the fusion cross section. This process can be called *sequential fusion.*

It is evident that the incoming flux may penetrate the multi-dimensional Coulomb barrier in the different intermediate neutron transfer channels. Denote by $\alpha_k(E,l,Q)$ the probability for the transfer of k neutrons at the center-of-mass energy E and relative motion angular momentum l in the entrance channel to the final state with $Q \leq Q_0(k)$, where $Q_0(k)$ is a Q-value for the ground state to ground state transfer reaction. Then the total penetration probability may be written as

$$T(E,l) = \int f(B) \frac{1}{N_{tr}} \sum_k \int_{-E}^{Q_0(k)} \alpha_k(E,l,Q) P_{HW}(B; E+Q, l) dQ dB, \qquad (1{\cdot}2)$$

where $N_{tr} = [\sum_k \int \alpha_k(E,l,Q)dQ]$ is the normalization constant and $\alpha_0 = \delta(Q)$.

In collision of heavy nuclei for the transfer probability one may use a semiclassical approximation (see, for example, [14)]). Assuming predominance of sequential neutron transfer mechanism, which means multiplication of transfer probabilities, one get $\alpha_k(E,l,Q) \sim e^{-2\kappa D(E,l)}$, where $D(E,l)$ is the distance of closest approach of the two nuclei and $\kappa = \kappa(\epsilon_1) + \kappa(\epsilon_2) + ... + \kappa(\epsilon_k)$ for sequential transfer of k neutrons, $\kappa(\epsilon_i) = \sqrt{2\mu_n \epsilon_i/\hbar^2}$ and ϵ_i is the separation energy of the i-th transferred neutron. Experiments show that the transfer probability becomes very close to unity at a short distance between the two nuclei, when their surfaces are rather overlapped. Denote this distance by $D_0 = d_0(A_1^{1/3} + A_2^{1/3})$ and will use for parameter d_0 the value of about 1.40 fm [14]. It is also well known that in heavy ion few-nucleon transfer reactions the final states with $Q \approx Q_{opt}$ are populated with largest probability due to mismatch of incoming and outgoing waves. For neutron transfer Q_{opt} is close to zero. The Q-window may be approximated by the Gaussian $exp(-C[Q-Q_{opt}]^2)$ with the constant $C = R_B\mu_{12}/\kappa\hbar^2(2E-B)$ [15], where μ_{12} is the reduced mass of the two nuclei in the entrance channel. Finally, the transfer probability may be estimated in the following way

$$\alpha_k(E,l,Q) = N_k e^{-C[Q-Q_{opt}]^2} e^{-2\kappa[D(E,l)-D_0]}, \quad (1\cdot3)$$

where $N_k = \{[\int_{-E}^{Q_0(k)} exp(-C[Q-Q_{opt}]^2)dQ\}^{-1}$ and the second exponent has to be replaced by 1 at $D(E,l) < D_0$.

¿From Eq.(1·2), one can see that in the reactions with negative values of all $Q_0(k)$ there is no *additional* enhancement of the total penetration probability of the Coulomb barrier $T(E,l)$ due to the neutron transfer in the entrance channel, because the "partial" penetration probability $P_{HW}(B;E+Q,l)$ becomes smaller for negative Q-values. It means that neutron transfers with zero and/or negative Q-values (most probable processes) play their role and lead to some regular fusion probability. If, however, $Q_0(k)$ are positive for some channels, in spite of the lower transfer probability to the states with positive Q-values compared to $Q = 0$, the penetration probability may significantly increase due to a gain in the relative motion energy for $Q > 0$. In other words, an intermediate neutron transfer to the states with $Q > 0$ is, in a certain sense, an "energy lift" for the two interacting nuclei. This looks quite different from the well-known fusion enhancement due to surface vibrations or rotation of nuclei leading to decrease of potential barrier in some channels. However, having in mind the driving potential of di-nuclear system depending in addition on neutron transfer (or mass asymmetry), the mentioned above gain in the relative motion energy may be interpreted in usual way as a decrease of the driving potential in some neutron transfer channels.

§2. Analysis of experimental data

Using for the 2^+ and 3^- excited states of ^{40,48}Ca and for the ion-ion potentials the same parameters as in [2] we repeated the CC calculations for the ^{48}Ca+^{48}Ca and ^{40}Ca+^{48}Ca fusion reactions. As in [2], the calculated cross section was found to be lower compared to the experimental data for ^{40}Ca+^{48}Ca at deep sub-barrier energies, whereas quite satisfactory agreement was obtained for ^{48}Ca+^{48}Ca. The semi-empirical calculation of the fusion cross section [11),12] gives the same result (see Fig. 2). Contrary to the ^{48}Ca+^{48}Ca combination, where the values of $Q_0(k)$ are negative in all the neutron transfer channels, for the ^{40}Ca+^{48}Ca reaction $Q_0(2n) =$ +2.6 MeV and $Q_0(4n)$ = +3.9 MeV. It means that in its intermediate channels (^{42}Ca+^{46}Ca) and (^{44}Ca+^{44}Ca) the system has a gain in energy, which may increase the penetration probability of the Coulomb barrier. Indeed, as can be seen from Fig. 2, the neutron transfer leads to a noticeable increase in the fusion cross section at sub-barrier energies and gives much better agreement with the experiment. Looking at the barrier distribution functions (bottom panel of Fig. 2) one may see that the neutron transfer does not simply smooth this function but makes it very asymmetric with a long high-energy tail.

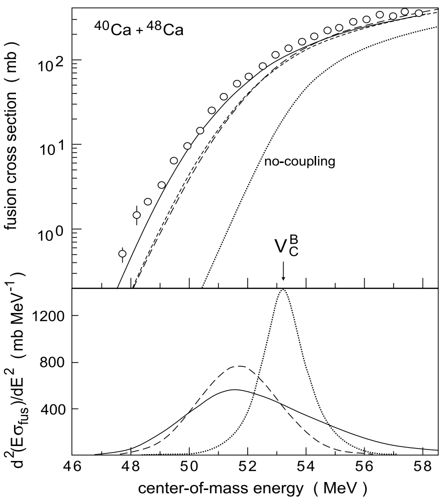

Fig. 2. Fusion cross section [2] (top panel) and barrier distribution functions for $^{40}Ca+^{48}Ca$. The short and long dashed curves correspond to CC [16] and semi-empirical [11), 12] calculations without neutron transfer. The solid line shows the effect of 2n ($Q_0 = +2.6$MeV) and 4n ($Q_0 = +3.88$MeV) transfer in the entrance channel.

Even higher neutron transfer Q_0-values (+0.51 Mev, +5.53 MeV, +5.24 MeV, and +9.64 MeV for 1, 2, 3, and 4 neutron transfer channels, respectively) are in the $^{40}Ca+^{96}Zr$ reaction. The near-barrier fusion cross sections for this reaction have been measured in [17] in comparison with the $^{40}Ca+^{90}Zr$ combination and a great difference between the two combinations has been found (see Fig. 3). Using the "proximity" ion-ion potential (which gives the corresponding Coulomb barriers B_0 = 99 MeV and B_0 = 100 MeV for $^{40}Ca+^{90}Zr$ and $^{40}Ca+^{96}Zr$ spherical nuclei), the quadrupole and octupole vibration properties of ^{40}Ca and $^{90,96}Zr$ (see, for example, [17]), one can reproduce quite well the experimental fusion cross sections for $^{40}Ca+^{90}Zr$ without any coupling with transfer channels. We failed to do the same in the case of $^{40}Ca+^{96}Zr$ [*]. However, if the neutron transfer is taken into account by means of formulae (1·2) and (1·3), the calculated cross sections agree quite well with the experiment (see Fig. 3). The effect hear comes mainly from one and two-neutron transfer channels and it is much larger than in the case of $^{40}Ca+^{48}Ca$ because the transfer probability at sub-barrier energies sharply decreases with increasing the number on transferred neutrons.

Trying to find experimentally the neutron transfer effect in fusion processes, one should be careful in the choice of the two combinations to be compared in order to avoid additional changes in the fusion cross sections, which may originate from some other effects. In this connection such combinations as $^{18}O+^{58}Ni$ and $^{16}O+^{60}Ni$ leading to the same compound nucleus are very interesting because the vibration properties of ^{58}Ni (2^+, 1.45 MeV, $\beta_2 = 0.183$) and of ^{60}Ni (2^+, 1.33 MeV, $\beta_2 = 0.207$) are very close and the ion-ion interaction potentials have to be also very close. In contrast with $^{16}O+^{60}Ni$, the neutron transfer Q_0-values are positive and rather large in the $^{18}O+^{58}Ni$ reaction: $Q_0(1n) = +0.96$MeV and $Q_0(2n) = +8.20$MeV. Unfortunately, the fusion cross sections for these two combinations have been measured only at near-barrier energies [19]. Nevertheless, the effect of one- and two-neutron transfer in the entrance channel of the $^{18}O+^{58}Ni$ fusion reaction is large and well visible (see Fig. 4).

[*] Note that in Ref. [18] the authors were able to reproduce the $^{40}Ca+^{96}Zr$ fusion cross section in the framework of a semiclassical model without assumption about intermediate neutron transfer.

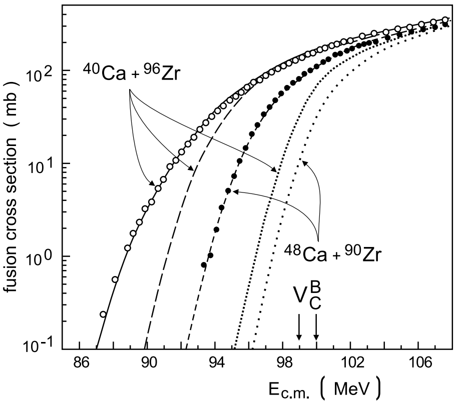

Fig. 3. Fusion of ^{40}Ca+^{96}Zr (open circles) and ^{40}Ca+^{90}Zr (filled circles)[17]. The no-coupling limits are shown by the dotted curves. The dashed curves show the semi-empirical calculations without neutron transfer, whereas the solid line was obtained with accounting for neutron transfer in the entrance channel of the ^{40}Ca+^{96}Zr reaction.

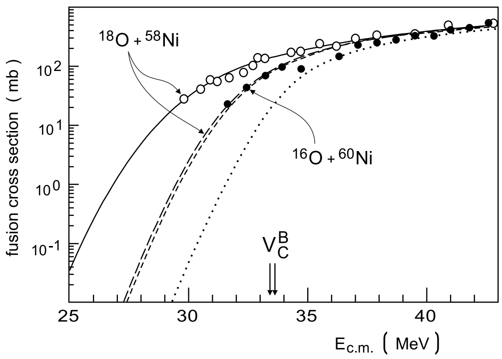

Fig. 4. Fusion excitation functions for ^{18}O+^{58}Ni and ^{16}O+^{60}Ni[19], open and filled circles, correspondingly. The no-coupling limit is shown by the dotted curve (it is practically the same for both cases). The dashed curves show the calculations without neutron transfer and the solid line was obtained with the formulas (1·2) and (1·3).

§3. Fusion of weakly bound nuclei

A stronger effect from the intermediate neutron transfer with positive Q-values one may expect in fusion reactions of radioactive weakly bound projectiles with stable target nuclei. Inspiring experiments of such kind have been already performed using the ^{6}He beam[20)–22)] demonstrating in general terms an enhancement of the fusion probability for ^{6}He compared to ^{4}He. However, again it is rather difficult to interpret unambiguously the results of these experiments. In the fusion-fission reactions (like ^{6}He+^{238}U[22]) one has to distinguish the processes of complete and incomplete fusion of the projectile. Comparing the evaporation residue (ER) cross sections in the ^{6}He+^{209}Bi and ^{4}He+^{209}Bi fusion reactions[21], one has to take into account that different compound nuclei are obtained in these reactions with different excitation energies and different decay properties. To avoid additional ambiguities one may propose to measure the ER cross sections in reactions, in which the same compound nucleus is formed, such as ^{6}He+A→C and ^{4}He+(A-2)→C, for example. In that case any difference in the ER cross sections may originate only from the difference in the entrance channels of the two reactions.

The promising reactions of such type are ^{6}He+^{206}Pb (see schematic Fig. 5) and ^{4}He+^{208}Pb with formation of the same α-decayed ^{212}Po compound nucleus. In the first combination there are intermediate neutron transfer channels with very large positive Q-values: $^{6}\mathrm{He}+^{206}\mathrm{Pb} \rightarrow ^{5}\mathrm{He}+^{207}\mathrm{Pb}(Q_0 = 4.9\mathrm{MeV}) \rightarrow ^{4}\mathrm{He}+^{208}\mathrm{Pb}(Q_0 = 13.1\mathrm{MeV}) \rightarrow ^{212}\mathrm{Po}$. Of course, as mentioned above, the probability for neutron transfer to the ground states are rather small, but the total possible gain in energy is very high as compared with the height of the Coulomb barrier (which is about 20

MeV) and has to reveal itself in the fusion probability of ^{6}He compared to ^{4}He.

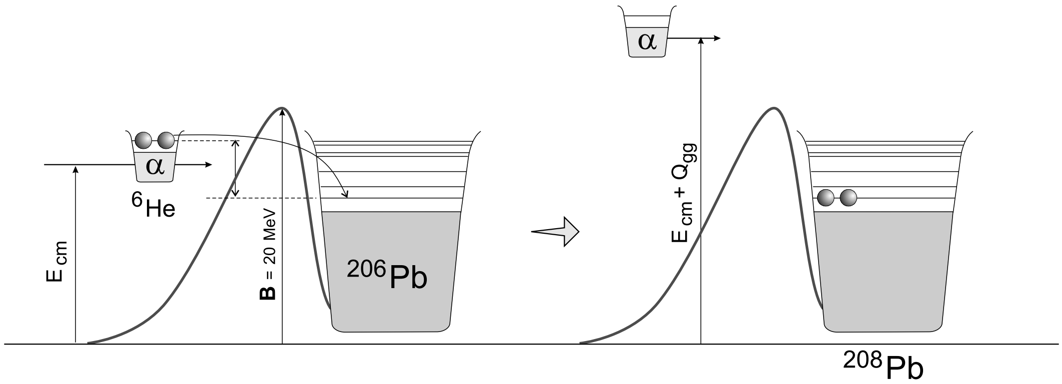

Fig. 5. Schematic picture of sequential fusion of ^{6}He with ^{206}Pb at sub-barrier energy.

To calculate the ER cross sections for these combinations we used the Woods-Saxon type potentials for ^{4}He+^{208}Pb ($V_0 = -96.44$ MeV, $R_V = 8.15$ fm, $a_V = 0.625$ fm [23])) and for ^{6}He+^{206}Pb ($V_0 = -109.5$ MeV, $R_V = 7.83$ fm, $a_V = 0.811$ fm, proposed in Ref. [24]) for low-energy ^{6}Li scattering), which give the corresponding fusion barriers $B_0 = 20.6$ MeV (at $R_B = 10.8$ fm) and $B_0 = 19.4$ MeV (at $R_B = 11.2$ fm). The vibration properties of ^{208}Pb ($3^-, 2.61$ MeV, $\beta_3 = 0.16$) and ^{206}Pb ($2^+, 0.80$ MeV, $\beta_2 = 0.04$) were also taken into account to find the barrier distribution function $f(B)$, though it plays a minor role here. The calculated ER cross sections for both reactions are shown in Fig. 6. As can be seen the effect of the intermediate neutron transfer channels in the ^{6}He+^{206}Pb fusion reaction is very large and may enhance the fusion cross section by several orders of magnitude at deep sub-barrier energies. We ignored here the influence of the break-up channel on fusion of ^{6}He which may reduce slightly the fusion cross section.

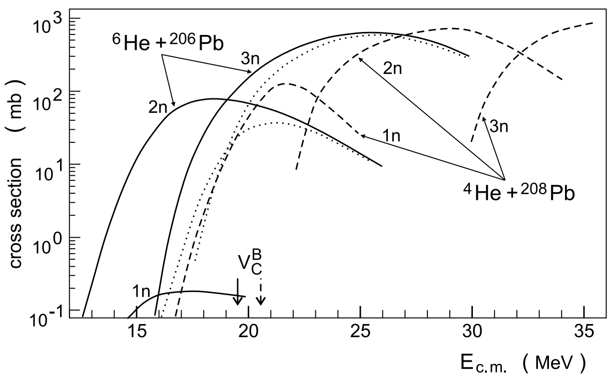

Fig. 6. Excitation functions for the production of evaporation residues in the ^{6}He+^{206}Pb (solid curves) and ^{4}He+^{208}Pb (dashed curves) reactions. Dotted curves show the 2n and 3n evaporation channels in the ^{6}He+^{206}Pb fusion reaction calculated ignoring the neutron transfer channels.

Many other combinations of stable and unstable nuclei should reveal a noticeable enhancement of the sub-barrier fusion cross sections due to intermediate neutron transfer with positive Q-values. They are ^{12,14}C+^{42,40}Ca, ^{16,18}O+^{42,40}Ca, ^{40,48}Ca+^{124,116}Sn, ^{9,11}Li+^{208,206}Pb, and many others, which have positive Q_0-values of the 1n and/or 2n transfer channels for one combination and negative or zero Q_0-values for another one. Direct comparison of the corresponding experimental fusion cross sections has to display immediately such an enhancement.

Additional enhancement in sub-barrier fusion of weakly bound neutron rich nuclei can be used, in principle, for synthesis of new super-heavy nuclei in reactions with radioactive beams. There are, for example, ^{26}F+^{248}Cm→^{271}Db+3n (Q_{1n-4n} = 3.7/5.2/5.7/3.8 MeV), ^{30}Na+^{248}Cm→^{275}Bh+3n (Q_{1n-4n} = 2.6/4.0/4.9/3.8 MeV), ^{34}Mg+^{248}Cm→^{279}Hs+3n ($Q_{1n-4n} = -0.1/3.7/2.5/5.7$ MeV), ^{37}Si+^{248}Cm→^{282}110 +3n (Q_{1n-4n} = 2.5/2.2/4.2/2.4 MeV). However, the corresponding cross sections for ER production are rather small (due to a low survival probability of easily fissile heavy compound nuclei) and beam intensity has to be higher than 10^8 pps in all these reactions.

Note, in conclusion, that the method proposed is rather simplified. However, it takes into account approximately the main effect of intermediate neutron transfer with positive Q-values, agrees reasonably with experiment and has a predictive power. No doubts, that more sophisticated consideration of neutron transfer in sub-barrier fusion processes is needed. By many reasons it is rather difficult to perform with high accuracy. Three-body time-depending Schrödinger equation and/or transport theories could be used for that.

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