Sequential Voting Procedures in Symmetric Binary Elections

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We explore sequential voting in symmetric two-option environments. We show that the (informative) symmetric equilibria of the simultaneous voting game are also equilibria in any sequential voting structure. In unanimity games, (essentially) the whole set of equilibria is the same in all sequential structures. We also explore the relationship between simultaneous and sequential voting in other contexts. We illustrate several instances in which sequential voting does no better at aggregating information than simultaneous voting. The inability of the sequential structure to use additional information in voting models is distinct from that in the herd-cascade literature.

I. Introduction

Theoretical research on voting has focused on the case in which the electorate votes simultaneously; in this paper we explore sequential voting models. We wish to analyze the relative effectiveness of se-

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quential voting vis-à-vis simultaneous voting at aggregating private information and preferences.[1] We allow for any sequence of voters in the voting phase, but, as is standard in this literature, we restrict attention to elections with only two options.[2] (Our results immediately extend to a sequence of votes over binary agendas.) We view this as a preliminary step in investigating more generally the effects of sequential voting structures on equilibrium outcomes, both when the timing is exogenous and when it is determined by the voters themselves, as in the case of states choosing their primary dates.

One would expect that sequential voting structures would facilitate the revelation and aggregation of private information: in sequential voting, earlier voters can convey (partially) the content of their information to later voters through their votes. On the one hand, this observation naturally poses the question whether allowing voters to choose when to vote leads to an efficient structure. On the other hand, it raises the concern that the outcome of sequential elections would be biased toward the preferences or, as in the herd-cascade literature, toward the private information of early voters. Our first result suggests that the situation is surprisingly more subtle but simple: in a symmetric environment with incomplete information (which is what we consider throughout), any symmetric equilibrium of the simultaneous voting game in which players use their information—which is precisely the equilibrium on which the information aggregation literature has focused—is in fact a sequential equilibrium in *any* sequential voting game. This result has two notable implications. On the negative side, it completely demolishes any hope of obtaining strong conclusions about endogenous timing in this context: given any sequential structure, there is an equilibrium in which the players select that structure and vote according to the symmetric simultaneous equilibrium. Essentially, the sequential structure is ignored. On the positive side, it extends the successful-aggregation results of Feddersen and Pesendorfer (1997) to any sequential voting environment.

Naturally, this conclusion raises the issue of asymmetric equilibria: while no sequential structure necessarily improves on simultaneous voting, as the symmetric equilibrium in the latter remains an equilib-

[1] The question of how well voting mechanisms aggregate private information and related questions have been explored in McKelvey and Ordeshook (1985), Ordeshook and Palfrey (1988), Austen-Smith (1990), Cukierman (1991), Lohmann (1994), Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997, 1998), Fey (1996), McLennan (1996), and Wit (1997).

[2] We do not analyze the effect of cheap talk. In particular, the sequential voting procedures that we consider do not include straw votes. While cheap talk may play an important role in the transmission of information, our purpose is to focus on information conveyed in the actual voting.

rium, perhaps asymmetric equilibria make a sequential structure more suitable for information transmission. We identify several cases in which they do not.

First, we identify a class of asymmetric equilibria that are also independent of the sequential structure. An interesting feature of these equilibria is the presence of seemingly cascading behavior: early voters vote informatively and later voters vote for the same options regardless of their signals. Such appearance is of course deceptive since these equilibria are equivalent in outcome to simultaneous voting.

Second, in unanimous voting games, the set of (essentially) all equilibria coincide regardless of the sequential structure.[3] While unanimity rules are rare, this result is otherwise quite general (within the class of two-option environments).

The third case is rather special but further demonstrates the subtlety of the timing issue when voters are strategic. Consider a pure common-values environment (where the value of a candidate is the same for all voters), so that the equilibria can be ranked in terms of the (common) welfare of the players. If there are only two signals, then there is a best equilibrium in monotonic strategies that is the same regardless of the sequential structure. (A monotonic strategy is one in which the probability that a voter votes in favor of a particular option is higher when her private signal concerning the value of that option is better.) In the case of simultaneous voting, if the best equilibrium uses information (i.e., not all players vote for the same option independently of their signals), it is also (generically) the only strict equilibrium. This raises doubts about the common practice of focusing on the symmetric equilibrium since the strict and Pareto-dominating equilibrium seems at least as natural a candidate to focus on as the symmetric one. The restriction to monotonic strategies, however, occurs without loss of generality only for simultaneous voting: sequential elections can do better than simultaneous elections if nonmonotonic strategies are used.

The main point underlying our results is well known: strategic voters condition their actions on being pivotal. Therefore, for the sequential structure to make a difference, it needs to reveal more information than is contained in the event that the voter is pivotal. The results demonstrate some strong implications of this observation.

In Section II, we discuss related literature. The model and results are stated in Section III. Proofs are in the Appendix.

[3] This result strengthens the surprising conclusions of Feddersen and Pesendorfer (1998), which focused on unanimity rules (e.g., jury voting) in the case of simultaneous voting.

II. Related Literature

The result that sequential voting may not confer informational advantages may seem similar to the herd-cascade literature (Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992). However, the strategic considerations and results in the two models are *very* different. The behavior of agents deciding simultaneously in the herd-cascade model will be different from their behavior when they decide sequentially; by contrast, our results show that in voting models the lack of an informational advantage for the sequential structure arises precisely when the equilibrium does not change. The reasoning is also very different: in the herd-cascade models, each decision maker's action determines the outcome for that person; in voting models, each voter knows that the outcome will be determined by the electorate as a whole. Thus, in the herd-cascade models the future choices of other players are irrelevant to the present decision maker, and the sequential structure gives no advantage since everyone relies on the information revealed by the first few decision makers. By contrast, the information of future voters can (and typically will) affect the outcome for present voters.

While Bikhchandani et al. interpret evidence concerning elections as consistent with informational cascades, our paper suggests that this interpretation may not be appropriate. Coordination of primary dates by certain states is obviously ''an attempt to avoid the consequences of sequential voting'' (1992, p. 1010), but we are skeptical that informational cascades are the real issue. After all, there exist unappealing equilibria that fail to aggregate information in both the simultaneous and sequential environment, and there seems to be no reason to think that they are more likely to occur in the sequential environment. Although our paper does not bear directly on the issue of primaries because of the restriction to two-candidate elections, it does suggest that in voting models the strategic issues are very different from those in the herd-cascade models.[4]

To our knowledge, three other papers consider sequential voting. Sloth (1993) shows that in the context of perfect information, the subgame-perfect equilibria of roll-call voting games, in which players vote one after another, are closely related to sophisticated equilibria of (agenda) games in which (on each issue) the electorate votes simultaneously. Thus her interests and ours are quite different since she focuses on sequential voting as a refinement in a perfect-information environment. Much closer to our work, Fey (1996) and Wit (1997) have independently examined a special case of the two-

[4] However, see the discussion of Fey (1996) and Wit (1997) below.

option, two-signal, common-value environment. They show that there exist two equilibria in this case: one in which everyone votes for an option if and only if that option is preferred according to his or her private signal, and a herd-cascade equilibrium in which essentially the same strategy is adopted until a two-vote lead for an option develops, after which everyone votes for the leading option. However, as we discuss in Section III*C*, optimal equilibria can exhibit features similar to the herd-cascade equilibrium. For the case of common values and two signals, the optimal monotonic equilibrium derived in our paper—which we show to be the same in the simultaneous and sequential games—maximizes the lead required, thereby generating the most information possible.

III. The Model and Results

We assume that there are n people who vote on two options, the status quo, N or "no," and the alternative, Y or "yes." The alternative is adopted if and only if the number of votes in favor is n_p or greater, $n_p > n/2$. The value of the alternative for each voter, v_i, is drawn from a set V_i, and the value of the status quo is normalized to zero. We assume that $v = (v_1, \ldots, v_n)$ is not observable and that each voter observes a private signal x_i drawn from a set of signals X_i. For notational simplicity, V_i and X_i are assumed to be finite. We let $X = \prod_{i=1}^{n} X_i$ and $x = (x_1, \ldots, x_n)$. The joint probability distribution of v and x is denoted by $f(v, x)$. The expected value of v_i conditional on x is $E_i(v_i|x)$. We assume symmetry throughout, for which the following notation is useful: given an n-tuple z, define $T^{ij}z$ to be the n-tuple obtained from z by exchanging z_i and z_j.

Axiom 1. *Symmetry.*—(1) $X_i = X_j$ and $V_i = V_j$ for any $i, j = 1, \ldots, n$. (2) $f(v, x) = f(T^{ij}v, T^{ij}x)$ for any $i, j = 1, \ldots, n$ and any (v, x).

Axiom 2. *Full support.*—For any v, if $f(v, x) > 0$ for some x, then $f(v, x') > 0$ for any x'.

We consider voting games with T periods, $T \leq n$. A player votes in only one period and knows the previous votes at the time of voting, but several players can vote simultaneously in the same period. Note that the one-period voting game describes simultaneous voting, and n-period voting games describe roll-call voting in which each player votes in a different period. Let I_t be the set of players voting in period t; n_t be the total number of players who vote in periods $1, \ldots, t-1$; and $t(i)$ be the period in which player i votes. For simplicity we assume that I_t is nonempty for every $t \leq T$. A strategy for a player is thus a function that maps the player's signal and the observed history of votes into the probability of voting for the alternative. For a player i voting in $t(i) > 1$, it is a function s_i: $\{Y,$

$N\}^{n_{t(i)}} \times X_i \rightarrow [0, 1]$. For a player i voting in period 1, it is a function s_i: $X_i \rightarrow [0, 1]$. For notational convenience we shall sometimes denote the strategy of a voter in period 1 by $s_i(h, x_i)$, where $h \in \{Y, N\}^{n_1}$, with the convention that $\{Y, N\}^{n_1}$ contains only the empty history. A history consisting of m observations of Y is denoted by $(Y)^m$. We say that a strategy s_i of player i is *informative* if the player uses her private information, that is, if for some history $h \in \{Y, N\}^{n_{t(i)}}$ and some pair of signals x_i, $x_i' \in X_i$, the voting differs, that is, $s_i(h, x_i) \neq s_i(h, x_i')$.

A. *Symmetric Equilibria*

Our first result imposes no further restrictions beyond the two-option and symmetric structure presented above and is thus quite general. Theorem 1 states that any symmetric equilibrium of the simultaneous voting game that uses informative strategies is an equilibrium of *any* sequential voting game. As noted, one implication of this result is that if players are allowed to choose their own timing, then any sequential structure is an equilibrium. Also, it shows that it is possible that no informational benefits will be realized when one moves from a simultaneous voting structure to a sequential one, since the symmetric equilibrium can be played in both. In relation to earlier work, theorem 1 extends the aggregation result of Feddersen and Pesendorfer (1997) to any sequential structure. They showed that in a simultaneous election with a large number of voters, symmetric and undominated equilibria successfully aggregate information in that the outcome of the election would be unchanged if the private information became common knowledge.

THEOREM 1. Consider a symmetric strategy profile of the simultaneous voting game, say $s = (s_1, \ldots, s_n)$, with $s_i = s^*$ for all i, in which the strategy of each player is informative. The profile s is an equilibrium of the simultaneous voting game if and only if every T-period voting game has a sequential equilibrium $s^T = (s_1^T, \ldots, s_n^T)$ such that, for all i, $s_i^T(h, x_i) = s^*(x_i)$ for any $h \in \{Y, N\}^{n_{t(i)}}$ and any $x_i \in X_i$.

The intuition for theorem 1 is that in a symmetric equilibrium of the simultaneous game, the voters choose their optimal action conditional on the event that they are pivotal (otherwise their vote is irrelevant). That is, they vote as though they know that $n_p - 1$ yes votes and $n - n_p$ no votes have occurred. Since the equilibrium is symmetric and since the voters are identical ex ante, no *useful* information is gained from knowing the identity of those who have voted yes and those who have voted no. In sequential voting, that is the only information gained when the voters adopt a symmetric strategy profile. Hence, the symmetric equilibrium of the simultaneous game remains an equilibrium in the sequential game. Of course, along the play of the symmetric equilibrium in the sequential game, information about the value of the alternative is revealed and later voters are better informed than earlier voters. However, these gains in information are of no use since voters evaluate payoffs conditional on the pivotal event. The symmetry assumption on the distribution of the players' valuations and signals is crucial for this conclusion. If, for example, different voters have a different quality of information, then knowing exactly who has voted yes and who has voted no does convey additional information. In this case, the symmetric equilibria of the simultaneous voting game need not be equilibria in the sequential game.[5]

The intuition for theorem 1 suggests that only voters who vote informatively need to be playing the same strategy, and in fact this result is more general than stated since it applies to certain natural asymmetric environments and equilibria as well. For example, it extends to the case in which there are known ''partisans'' whose preferences are either to always vote yes or to always vote no, and a third symmetric group. Then, a strategy profile in which all those in the third group adopt the same informative strategy is an equilibrium in the simultaneous voting game if and only if it is a sequential equilibrium in all sequential voting games.

Theorem 1 can also be extended to asymmetric equilibria in which some voters adopt the same partisan strategy, voting uninformatively in favor of (or against) the alternative, and all other voters use the same informative strategy. In this case, however, an additional issue arises concerning the off-equilibrium-path beliefs in the sequential equilibrium. (This issue is irrelevant when the uninformative voting results from partisan preferences since such partisans will not deviate regardless of previous votes.) Therefore, we state two results. First, these asymmetric equilibria are Nash equilibria in the sequential game if and only if they are equilibria in the simultaneous game. Second, under a mild monotonicity condition, they are sequential equilibria in a sequential game in which informative voters go first if and only if they are equilibria of the simultaneous game.

Given a proper subset of signals Z, define $E(v_i;\ x_i,\ n_1,\ n_2,\ Z)$ to be the conditional expected value of v_i if i's signal is x_i, n_1 signals of different voters are in Z, and n_2 are in the complement of Z.

[5] Similarly, in models with more than two options, the symmetric equilibria of the simultaneous move game will not typically be equilibria in sequential versions of the game: as there is more than one way to be pivotal, voters will reassess which way they are likely to be pivotal as the election unfolds.

AXIOM 3. *Monotonicity.*—Suppose that, for some n_1, n_2, $E(v_i;\, x_i,\, n_1,\, n_2,\, Z) \geq 0$ for any x_i in Z and $E(v_i;\, x_i,\, n_1,\, n_2,\, Z) \leq 0$ for any x_i in the complement of Z. Then $E(v_i;\, x_i,\, n_1,\, n_2,\, Z)$ is nondecreasing in n_1 and nonincreasing in n_2 for any $x_i \in X_i$.[6]

THEOREM 2. Consider a strategy profile of the simultaneous voting game $s = (s_1, \ldots, s_n)$ for which there exists $k > 1$ such that (i) for all $i, j < k$, $s_i = s_j$; (ii) for all $i, j \geq k$, $s_i = s_j$ and $s_i(x_i) = s_i(x_i')$ for all $x_i, x_i' \in X_i$; and (iii) the alternative passes with probability strictly between zero and one.

1. The profile s is an equilibrium of the simultaneous voting game if and only if every T-period voting game has a Nash equilibrium $s^T = (s_1^T, \ldots, s_n^T)$ such that, for all i, $s_i^T(h, x_i) = s_i(x_i)$ for any $h \in \{Y, N\}^{n_{t(i)}}$ and any $x_i \in X_i$.
2. Under monotonicity, the profile s is an equilibrium of the simultaneous voting game if and only if every T-period voting game in which $t(i)$ is nondecreasing in i has a sequential equilibrium $s^T = (s_1^T, \ldots, s_n^T)$ such that, for all i, $s_i^T(h, x_i) = s_i(x_i)$ for any $h \in \{Y, N\}^{n_{t(i)}}$ and any $x_i \in X_i$.

As mentioned, the intuition behind theorem 2 is partially similar to that behind theorem 1 in that informative voting in simultaneous structures is sequentially rational in sequential structures. The assumption of monotonicity ensures that uninformative voting is sequentially rational as well. First note that uninformative voting never takes place before informative voting. Then, if an uninformative Y voter chooses N, later uninformative voters are pivotal if a higher number of Y votes by informative voters is realized. Monotonicity ensures that Y is, a fortiori, still optimal. Note that this argument might not apply if informative voting occurs after uninformative voting. If the number of informative voters choosing Y is higher than in the pivotal event in equilibrium, the optimal informative strategy can change unless out-of-equilibrium beliefs are chosen appropriately. Further conditions are needed to ensure that such beliefs can be obtained in a sequential equilibrium. Insofar as Nash equilibria are concerned, these considerations are of course irrelevant, and the result above holds independently of monotonicity and the ordering of voters.

Theorem 2 also shows that evidence of cascades in sequential elections can be deceptive: the equilibria in theorem 2 are equivalent in outcome to equilibria of sequential elections in which voters' behavior is such that whenever a critical number of Y (N) is realized, any subsequent voter opts for Y (N) independently of signals. Fur-

[6] Axiom 3 is satisfied when signals are affiliated.

thermore, as we shall see in subsection *C*, these equilibria can be optimal.

Remark 1.—In our model it is assumed that voters cannot abstain. However, the equilibria in theorems 1 and 2 remain equilibria when the voters' choice set is enlarged to include abstention. The reason is that an abstention is equivalent to a yes vote if, when a voter abstains, the number of votes in favor necessary to pass the alternative decreases and equivalent to a no vote if it is unchanged.

B. Unanimity Elections

In this subsection, we study elections, such as jury voting, in which an option is passed only if all voters are in favor. Our third result states that the set of (essentially) *all* equilibria in a unanimity voting game, one in which $n_p = n$, are the same regardless of the sequential structure.

THEOREM 3. Consider a strategy profile of a unanimity simultaneous voting game, $s = (s_1, \ldots, s_n)$, such that, for some vector of signals $x = (x_1, \ldots, x_n) \in X$, the alternative passes with positive probability, that is, $s_i(x_i) > 0$ for all i. The profile s is an equilibrium of the simultaneous voting game if and only if every T-period voting game has a sequential equilibrium $(s_1^T, \ldots, s_n^T)$ such that, for all i and $h = (Y)^{n_{t(i)}}$, $s_i^T(h, x_i) = s_i(x_i)$ for any $x_i \in X_i$.

To understand theorem 3, note that in a unanimity voting game, the players' behavior after histories in which someone voted no is irrelevant. The theorem says that given any equilibrium of any T-period game, the relevant portion of the strategies, those following histories of only yes votes, constitutes an equilibrium for any other sequential structure. Asymmetric equilibria also coincide since in a unanimity game there is only one way to be pivotal. Hence, voters behave as though they know that everyone else has chosen yes, which is exactly what they know in the sequential game.

One should observe that the argument above does not require the symmetry axiom, and in fact the same proof as provided below formally demonstrates that theorem 3 holds regardless. Hence, this result is very general. In relation to jury voting, it yields a remarkable conclusion: regardless of anything that occurs prior to voting, the procedure determining the order in which the vote is taken is of no consequence.

Remark 2.—Sloth (1993) has noted that the sequential structure serves as a refinement to rule out ''implausible'' equilibria of the simultaneous voting game. Since we are not concerned with sequential voting as a method of eliminating such equilibria, we rule them out a priori. Thus, in theorem 1, we consider equilibria in informative strategies to rule out equilibria in which, for example, everyone always votes no, which need not be a sequential equilibrium for roll-call voting. For similar reasons, in theorem 2 we consider equilibria in which the alternative passes with probability strictly between zero and one, and in theorem 3 we consider equilibria for which the alternative passes with strictly positive probability. The full-support axiom also serves this role: it rules out perfect correlation in order to avoid situations in which ''uninteresting'' profiles are equilibria in the simultaneous voting game, but not in the roll-call voting game. (For example, if there are two states of the world—one in which everyone prefers the alternative and the second in which everyone prefers the status quo—it is an equilibrium in the simultaneous game for everyone to vote the opposite of his or her preference, but this need not be a sequential equilibrium of the roll-call game.)

C. Common Values

We now consider the case in which the electorate has a common value of the alternative, so that the *only* purpose of the election is to aggregate private information. The advantage of this case is that one can make unambiguous welfare comparisons among the different sequential structures. It might be objected that cheap talk, if feasible, would easily solve (as for any game of common interest) the information aggregation problem in the common-value case. While we consider this criticism valid in circumstances in which communication is available and inexpensive, this special case serves as an illustration of the subtleties of information aggregation. It is a simple and natural environment that has been considered previously in the voting literature (see, e.g., Fey 1996; Wit 1997; McLennan 1998).

We focus on the special case in which there are only two signals, ''good'' and ''bad,'' $X_i = \{G, B\}$, and denote the expected value of the alternative by $e(g, b)$ if g voters have signal G and b voters have signal B, where $g + b \leq n$. We assume that e is increasing in g and decreasing in b.[7]

[7] Fey (1996) and Wit (1997) consider the special case of this model when the signals are independently and identically distributed across voters; Wit also restricts attention further to the case in which the value of the alternative is either one or minus one and the probability that i receives a good signal when the alternative is one equals the probability of a bad signal when it is minus one.

We say that a strategy s_i is monotonic if $s_i(h, G) \geq s_i(h, B)$ for all $h \in \{Y, N\}^{n_{t(i)}}$, and we denote the set of monotonic strategy profiles by M. A strategy profile is M-optimal if it maximizes the expected value of the election over all profiles in M.

We now describe an M-optimal equilibrium for all T-period voting games. This will be an asymmetric, pure-strategy, history-independent equilibrium in which some players vote uninformatively (i.e., their choice does not depend on their private signal) and the remaining players vote perfectly informatively (i.e., they vote Y if their private signal is G and N if they observe B).

It is well known that the number of perfectly informative voters (Y if G, N if B) will in general be less than the total number of players. Consider simultaneous voting for simplicity, and assume, for example, that $e(n_p - 1, n - n_p + 1) > 0$. In this case, when all but one player vote perfectly informatively, that one player will know that if he is pivotal, $n_p - 1$ players have voted Y and observed G, whereas $n - n_p$ have voted N and observed B. Then, even if he observes B, he should vote in favor of the alternative.

In fact, the same argument implies that when $e(g, n - n_p + 1) > 0$, for $g < n_p - 1$, if $n - n_p + g$ vote perfectly informatively, then the remaining $n_p - g$ should vote yes uninformatively. More precisely, if players $1, \ldots, n - n_p + g$ vote perfectly informatively, then it is optimal for any other player—regardless of what other players, $n - n_p + g + 1, \ldots, n$, do (as long as they use monotonic strategies)—to vote Y uninformatively. No equilibrium in simultaneous voting can have more than $n - n_p + g$ perfectly informative voters. If the number of perfectly informative voters is $n - n_p + g'$, $g' > g$, the expected value of an informative pivotal voter having a bad signal is $e(g' - 1, n - n_p + 1)$, which is positive by hypothesis. Hence, when $e(n_p - 1, n - n_p + 1) \geq 0$, define γ to be the smallest g for which $e(g, n - n_p + 1) \geq 0$. That there can be $n - n_p + \gamma$ perfectly informative voters in equilibrium follows from theorem 4 below. In fact, this theorem states that if $e(n_p - 1, n - n_p + 1) \geq 0$, then it is an M-optimal strategy profile in any T-period game for $n_p - \gamma$ voters to vote Y independently of their signal and for $n - n_p + \gamma$ to vote perfectly informatively.

Similarly, if $e(n_p, n - n_p) < 0$, define β to be the smallest b such that $e(n_p, b) \leq 0$; below we show that in this case it is an M-optimal strategy profile for $n_p + \beta - 1$ to vote perfectly informatively and the remainder to vote N. Finally, if $e(n_p, n - n_p) \geq 0$ and $e(n_p - 1, n - n_p + 1) < 0$, it is M-optimal for everyone to vote perfectly informatively. This last case is one in which the threshold n_p has been determined optimally.

THEOREM 4. Consider a T-period voting game.

1. If $e(n_p - 1, n - n_p + 1) \geq 0$, then the history-independent pure-strategy profile s, defined as follows, is M-optimal: (a) For $i \leq n - n_p + \gamma$, $s_i(h, G) = 1$ and $s_i(h, B) = 0$ for any $h \in \{Y, N\}^{n_{t(i)}}$. ($b$) For $i > n - n_p + \gamma$, $s_i(h, x_i) = 1$ for any $x_i \in \{G, B\}$ and any $h \in \{Y, N\}^{n_{t(i)}}$.
2. If $e(n_p, n - n_p) < 0$, then the history-independent pure-strategy profile s, defined as follows, is M-optimal: (a) For $i \leq n_p - 1 + \beta$, $s_i(h, G) = 1$ and $s_i(h, B) = 0$ for any $h \in \{Y, N\}^{n_{t(i)}}$. ($b$) For $i > n_p - 1 + \beta$, $s_i(h, x_i) = 0$ for any $x_i \in \{G, B\}$ and any $h \in \{Y, N\}^{n_{t(i)}}$.
3. If $e(n_p, n - n_p) \geq 0$ and $e(n_p - 1, n - n_p + 1) < 0$, then the history-independent pure-strategy profile s, defined as follows, is M-optimal: For all i, $s_i(h, G) = 1$ and $s_i(h, B) = 0$ for any $h \in \{Y, N\}^{n_{t(i)}}$.

The M-optimal strategies satisfy some additional properties.

Theorem 5. In a simultaneous voting game, if as a result of the M-optimal strategy profile of theorem 4 the alternative passes with probability strictly between zero and one, then the strategies identified therein are generically a strict equilibrium and constitute the unique strict equilibrium.

It is not difficult to show that the strategy profile in theorem 4 is also a sequential equilibrium in any T-period game. Consider part 1. Clearly, only deviations by uninformative voters need to be considered. Following such deviations, assume that the posterior beliefs about signals are identical to the equilibrium ones. Then a subsequent uninformative voter is pivotal only if more Y votes by perfectly informative players are realized than in equilibrium. Thus, a fortiori, voting Y independently of the signal observed is optimal. The arguments for parts 2 and 3 are analogous. The theorem then says that in this environment there exists a history-independent equilibrium (i.e., an equilibrium in all sequential structures) that is a best strategy profile among all monotonic strategy profiles.

This equilibrium is equivalent in outcome to a herding-like equilibrium in which the perfectly informative voters vote first, and after a critical number of N or Y votes, everyone votes N or Y. However, the critical number for either N or Y is the same as the number required to determine the outcome. In particular, the equilibrium in case 1 is equivalent to everyone's voting Y after γ Y votes and everyone's voting N after $n - n_p$ N votes.

To see that the restriction of monotonic strategies is necessary, we present an example in which $n = 3$, $n_p = 2$, and the expected value of the alternative is positive if and only if three positive signals are observed (i.e., $e(g, b) > 0$ if and only if $g = 3$ and $b = 0$). Consider the following two-period game: in the first period one player votes Y if she observes B and votes N if she observes G, and in the next period the two players vote N if the first player voted Y and otherwise vote perfectly informatively (Y on observing G, N on observing B). This is an equilibrium that obtains the best possible outcome: the alternative passes if and only if it is better than the status quo. It is easy to see that no strategy profile in the simultaneous game can achieve this outcome.

Finally, to see that the restriction to two signals is also necessary, we present a three-signal example in which a sequential procedure is strictly better than the simultaneous one. Consider the following specification: $n = 3$; $n_p = 2$; $X_i = \{G, M, B\}$; $V_i = \{1, -0.2, -L\}$, where L will be a large positive number; the prior probability of each value is 1/3, that is, $f(v_i) = 1/3$ for $v_i \in V_i$; and the signals are conditionally independent, $f(x_1, x_2, x_3|v_i) = f(x_1|v_i)f(x_2|v_i)f(x_3|v_i)$, where $f(x_j|v_i)$ is as follows. If $v_i = -L$, only the lower two signals, M and B, are possible, and they are equally likely; if $v_i = -0.2$, all three signals are equally likely; and if $v_i = 1$, the upper two signals, M and G, are assigned an equal probability of .5. (The zero probability events could be changed to ϵ probability events, for ϵ small enough, without changing the subsequent analysis.)

If anyone observes B, the best decision is for the alternative to fail; so in the simultaneous vote the best equilibrium is one in which every player votes N on observing B. While there are equilibria in which two or more players always vote N, such equilibria are dominated by the strategy profile in which everyone votes Y if and only if he or she observes G (which yields $\frac{1}{3}[4(\frac{1}{8}) - \frac{7}{5}(\frac{1}{27})] > 0$). (It is irrelevant whether this strategy profile is an equilibrium because if it is not, then there must be an equilibrium that dominates this profile since the optimal profile is an equilibrium.) So the only possibilities are those in which one player votes N always and all three players vote informatively. Denote a strategy by the signals on which the player votes Y; for example, GM is the strategy of voting Y if and only if G or M is observed, and ϕ is the strategy of voting N always. If player 1 plays ϕ, then players 2 and 3 play (GM, GM), (GM, G), or (G, G). In the second case, a fortiori in the last one, player 1 is not playing optimally on observing G. In the second case, when she observes G and is pivotal, the signal profile, x, is in $P = \{GBG, GGM, GGB, GMM, GMB\}$. The sign of the expectation of v_i conditional on P is the same as the sign of

$$\tfrac{1}{3}(-\tfrac{1}{5})(\tfrac{1}{3})^3 + [\tfrac{1}{3}(\tfrac{1}{2})^3 - \tfrac{1}{3}\tfrac{1}{5}(\tfrac{1}{3})^3] + \tfrac{1}{3}(-\tfrac{1}{5})(\tfrac{1}{3})^3$$
$$+ [\tfrac{1}{3}(\tfrac{1}{2})^3 - \tfrac{1}{3}\tfrac{1}{5}(\tfrac{1}{3})^3] + \tfrac{1}{3}(-\tfrac{1}{5})(\tfrac{1}{3})^3 = \tfrac{1}{3}(2\tfrac{1}{8} - \tfrac{1}{27}) > 0.$$

If players 2 and 3 play (GM, GM), then player 2 is not playing optimally on observing M. The event that she observes M and is pivotal, say P', includes the signal profile *BMM*; whereupon if L is large enough, the expected value conditional on this event is negative. So all three players must vote informatively in equilibrium. However, at most one player plays *GM*: if two players do so, then the third player knows that when she is pivotal, at least one other player observed B; so this third player will always vote N, which contradicts that it is an equilibrium in informative strategies. So two players must adopt G, in which case the best reply for the third is *GM*: the event that this player (say player 1) observes M and is pivotal is $P'' = \{MGM, MMG, MBG, MGB\}$, and the sign of the expected value of the alternative conditional on P'' is the same as the sign of $2\frac{1}{3}[(\frac{1}{8} - \frac{1}{5} \times \frac{1}{27}) - \frac{1}{5} \times \frac{1}{27}] > 0$. Finally, if, say, player 1 chooses *GM* and player 3 chooses G, then the best reply for player 2 is G. This must be the case because there is a best profile that is a pure-strategy profile and that is an equilibrium, and this is the only profile that we have not ruled out. (It can also be easily verified directly that for player 2 playing G is a best reply against player 1 playing *GM* and player 3 playing G: the event that player 2 observes M and is pivotal is P''', which equals, up to a permutation of signals, $P' - \{GMG, MMG\} \cup \{GMB\}$. So if the expected value given P' is negative, so is the expected value given P'''. The expected value conditional on the event that player 2 observes G and is pivotal is the same as the expected value conditional on P, which is positive.)

Thus we conclude that the best strategy profile in the simultaneous voting game is *GM*, G, G (or a permutation thereof). However, in a sequential game in which players 2 and 3 vote after player 1, there is a better profile: player 1 plays *GM* and players 2 and 3 play ϕ after player 1 votes N (indicating the signal B, which implies that $v_i < 0$), and they play G after player 1 votes Y.

IV. Conclusion

This paper illustrates some important considerations in the aggregation of information for sequential elections. In particular, we demonstrate weak and strong forms of equivalence for simultaneous and sequential elections. This contrasts with two opposing intuitions. First, because voters later in the sequence are better informed at the time they vote (this is true), sequential structures must enable better information aggregation (this is false). Second, because of herding, sequential voting is worse at aggregating information.

We wish to emphasize two possible extensions capable of undermining the results above. First, if the electorate decides on more than two options, early voters may be able to restrict the set of candidates for future voters. This extension is especially apt for the analysis of primary elections; we hope to explore this model in future work. Second, if voters are endowed ex ante with differential information (some voters can be better informed than others), knowing which voters voted in favor and which against can affect the choice of a later voter. It can be shown that, in a common-value, two-signal environment (as in Sec. III*C* above), if the player's signals are completely ordered (in the sense of Blackwell), then it is optimal to have the better informed vote earlier.[8] This provides an interesting contrast to the findings of Ottaviani and Sørensen (1998). They obtain the opposite optimal order in an environment in which information providers care not about the outcome but about appearing to be well informed. It is not difficult, however, to construct examples in which having the best-informed voter vote first is not optimal. Hence, it seems unlikely that general insights into this question can be obtained.

Appendix

Proof of Theorem 1

Suppose that s is an equilibrium for the one-period voting game. Note that player i is pivotal for several combinations of the votes of the other $n-1$ players. Let $P_i \subset \{Y, N\}^{n-1}$ denote the set of such combinations. For an n-tuple of signals x and a possible action (voting) profile of the other players $p \in P_i$, define $Y(p)$ to be the set of players voting Y in p and $N(p)$ to be the set of players voting N in p. Let $f(x|x_i)$ be the conditional probability that the vector of signals is x if i's signal is x_i. Let $f^s(p|x_i)$ be the conditional probability that the other players' action profile is p if i's signal is x_i, given that the strategy profile is s. Finally, let $f^s(x|x_i, p)$ be the conditional probability of x if i's signal is x_i and the realization of the other players' votes is p, given that the strategy profile is s. By the full support axiom and the informativeness of the strategy, every p in P_i is realized with positive probability, so the conditional probability is well defined:

$$f^s(x|x_i, p) = \frac{f(x|x_i)\prod_{j\in Y(p)} s^*(x_j)\prod_{j\in N(p)}[1 - s^*(x_j)]}{\sum_{x'\in X} f(x'|x_i)\prod_{j\in Y(p)} s^*(x'_j)\prod_{j\in N(p)}[1 - s^*(x'_j)]}. \quad \text{(A1)}$$

[8] The claim is proved by iteratively switching neighboring players who violate this order. We do not provide the details since the result is special and the argument is straightforward.

Now, s is an equilibrium if and only if the following conditions are true:

$$\sum_{p\in P_i} f^s(p|x_i) \sum_{x\in X} E_i(v_i|x)\, f^s(x|x_i, p) \begin{cases} \geq 0 & \text{if } s^*(x_i) = 1 \\ = 0 & \text{if } s^*(x_i) \in (0, 1) \\ \leq 0 & \text{if } s^*(x_i) = 0. \end{cases} \tag{A2}$$

Consider next a T-period game and the strategy profile as in the statement of the theorem, $s_i^T(h, x_i) = s^*(x_i)$. Consider player i, after history h, and let $P_i(h)$ be the subset of P_i consistent with h. If $P_i(h)$ is empty, then player i can have no effect on the outcome, and hence sequential rationality must hold. So suppose that $P_i(h)$ is not empty, and for $p \in P_i(h)$, let $f^{s^T}(p|x_i, h)$ be the conditional probability of p if i's signal is x_i and the realization of votes up to $t(i)$ is h, given that the strategy profile is s^T. By the full support axiom and because the strategy s^* is informative, this conditional probability is well defined. Finally, let $f^{s^T}(x|x_i, p, h)$, for $p \in P_i(h)$, denote the conditional probability of x given that the history is h, i's signal is x_i, and the profile of other actions is p, and given that the strategy profile is s^T. In the T-period game, s^T is sequentially rational if and only if the following conditions are true:

$$\sum_{p\in P_i(h)} f^{s^T}(p|x_i, h) \sum_{x\in X} E_i(v_i|x)\, f^{s^T}(x|x_i, p, h) \begin{cases} \geq 0 & \text{if } s^*(x_i) = 1 \\ = 0 & \text{if } s^*(x_i) \in (0, 1) \\ \leq 0 & \text{if } s^*(x_i) = 0. \end{cases} \tag{A3}$$

We now argue that conditions (A2) hold if and only if conditions (A3) are satisfied. The first step is to note that for every p, $p' \in P_i$, symmetry implies $\sum_{x\in X} E_i(v_i|x)\, f^s(x|x_i, p) = \sum_{x\in X} E_i(v_i|x)\, f^s(x|x_i, p')$. Therefore, the sign of the left-hand side of conditions (A2) is the same as the sign of $\sum_{x\in X} E_i(v_i|x)\, f^s(x|x_i, p)$ for any $p \in P_i$. The second step is to note that, for $p \in P_i(h)$, $f^{s^T}(x|x_i, p, h) = f^s(x|x_i, p)$ since h does not add information beyond p. Now the first step can be applied to conditions (A3), completing the proof. Q.E.D.

Proof of Theorem 2

Part 1. The proof of this claim follows from combining two arguments. The first argument is the one contained in the proof of theorem 1. The second is the straightforward claim that a pure-strategy Nash equilibrium of a simultaneous move game is a Nash equilibrium of a sequential version of the same game. (This in turn is essentially the same as the claim that a pure-strategy Nash equilibrium of a static game of incomplete information is also a Nash equilibrium of the sequential version of the same game if the strategies in the Nash equilibrium do not depend on the players' private information.) Q.E.D.

Part 2. Consider the case in which $s_i = Y$ for $i \geq k$. For $i < k$, let Z be the set of signals for which i votes Y. Since s is an equilibrium, $E(v_i; x_i, n_p - 2 - n + k, N - n_p, Z) \geq 0$ for x_i in Z and $E(v_i; x_i, n_p - 2 - n + k, N -$

n_p, $Z) \leq 0$ for x_i in the complement of Z, for any $i < k$. By monotonicity, $E(v_i; x_i, n_1, n_2, Z)$ is nondecreasing in n_1 and nonincreasing in n_2 for any x_i.

Let Θ be the set of histories that are realized with positive probability under the strategy profile s^T. If a history h is in Θ, sequential rationality after h follows by applying arguments analogous to the proof of theorem 1. Suppose then that h is not in Θ. Since $t(i)$ is nondecreasing in i, parts i and iii imply that only players $i \geq k$ are voting after h and that deviations in h can be attributed only to players $i \geq k$ voting N instead of Y. Let D denote the number of deviations in h and select off-equilibrium beliefs for which deviations are independent of signals. Then, sequential rationality for a player i after h is obtained if $E(v_i; x_i, n_p - 1 - n + k + D, N - n_p - D, Z) \geq 0$ for any x_i. Since s is an equilibrium and $i \geq k$, $E(v_i; x_i, n_p - 1 - n + k, N - n_p, Z) \geq 0$ for any x_i. The claim then follows since $E(v_i; x_i, n_1, n_2, Z)$ is nondecreasing in n_1 and nonincreasing in n_2 for any x_i. The proof for the case in which $s_i = N$ for $i \geq k$ is analogous. Q.E.D.

Proof of Theorem 3

We continue to use notation defined in the proof of theorem 1. Note that i is pivotal only if all other players voted Y. So we can simplify (A1) as follows:

$$f^{s^T}(x|x_i, (Y)^{n-1}) = \frac{f(x|x_i)\prod_{j\neq i} s_j^T(x_j, (Y)^{n_{t(i)}})}{\sum_{x'\in X} f(x'|x_i)\prod_{j\neq i} s_j^T(x'_j, (Y)^{n_{t(i)}})}.$$

This is well defined because, for each i, $s_i(x_i) > 0$ for some x_i. The equilibrium conditions then become

$$\sum_{x\in X} E_i(v_i|x)\, f^s(x|x_i, (Y)^{n-1}) \begin{cases} \geq 0 & \text{if } s_i(x_i) = 1 \\ = 0 & \text{if } s_i(x_i) \in (0, 1) \\ \leq 0 & \text{if } s_i(x_i) = 0. \end{cases}$$

Consider next the T-period game and a strategy profile s^T such that, for each i, $s_i^T((Y)^{n_{t(i)}}, x_i) > 0$ for some x_i. We can then define similarly

$$f^s(x|x_i, (Y)^{n-1}) = \frac{f(x|x_i)\prod_{j\neq i} s_j(x_j)}{\sum_{x'\in X} f(x'|x_i)\prod_{j\neq i} s_j(x'_j)}.$$

In the T-period game, s_i^T is sequentially rational if and only if the following conditions are true:

$$\sum_{x\in X} E_i(v_i|x)\, f^{s^T}(x|x_i, (Y)^{n-1}) \begin{cases} \geq 0 & \text{if } s_i^T(x_i, (Y)^{n_{t(i)}}) = 1 \\ = 0 & \text{if } s_i^T(x_i, (Y)^{n_{t(i)}}) \in (0, 1) \\ \leq 0 & \text{if } s_i^T(x_i, (Y)^{n_{t(i)}}) = 0. \end{cases}$$

If, for all $j \neq i$, $s_j^T(x_j, (Y)^{n_{t(j)}}) = s_j(x_j)$, then the conditions for s_i and for s_i^T to be sequentially rational coincide. Hence, since only histories of all Y votes are relevant, the sets of sequential equilibria coincide. Q.E.D.

To prove theorem 4, we first develop several lemmas for roll-call voting games; this restriction to n-period games is implicit in all the following lemmas. To simplify the notation, in what follows it is assumed that $t(i) = i$ so that the time and the player indexes are identical. It is helpful to think about finding optimal profiles in this common-value environment as a *single-person* decision problem: one decision maker deciding on a strategy profile for everyone that maximizes everyone's expected payoff. (The problem is a little subtle because the decision maker has imperfect recall: the vote in the tth period cannot depend on previous signals, only on the current signal and previous votes.) Recall that we say that a player's strategy is uninformative if her vote is the same regardless of her signal. The following notation will be useful: let $Y(h)$ denote the number of Y votes and $N(h)$ denote the number of N votes in history h.

Lemma A1. There exists an M-optimal *pure*-strategy profile.

Proof. For any given history, the expected value of the election conditional on a vector of signals is linear in the probability of a player's vote. So, given any M-optimal mixed-strategy profile, we can change this, player by player, to a pure-strategy profile. Q.E.D.

Lemma A2. There exists an M-optimal pure-strategy profile in which all the uninformative voting occurs at the end. That is, for any t and t', with $t' > t$, and any t-period history $h = (a_1, \ldots, a_{t-1}) \in \{Y, N\}^{t-1}$ and any t'-period continuation of that history $h' = (a_1, \ldots, a_{t-1}, \ldots, a_{t'-1}) \in \{Y, N\}^{t'-1}$, if $s_{I(t)}(h, G) = s_{I(t)}(h, B)$, then $s_{I(t')}(h', G) = s_{I(t')}(h', B)$.

Proof. Consider any pair of players voting one after the other in which after some history the first votes uninformatively but the second votes informatively. Then clearly, the strategies of these two players after that history can be switched without changing the payoff. So, by iteratively switching any such pair of players, we can move all the uninformative strategies to the end of the game. Q.E.D.

Lemma A3. There exists an M-optimal pure-strategy profile in which all the uninformative voting occurs at the end and such that (i) after any history h in which the number of Y votes is less than $n_p - 1$, the player votes Y on observing G, that is, $Y(h) < n_p - 1 \Rightarrow s_i(h, G) = 1$; and (ii) after any history h in which the number of N votes is less than $n - n_p$, the player votes N on observing B, that is, $N(h) < n - n_p \Rightarrow s_i(h, B) = 0$.

Proof. Since uninformative voting occurs at the end, if at a history h a player votes N on observing G, then all following strategies are uninformative and the outcome of the election is already determined at h. So consider an M-optimal pure-strategy profile with uninformative voting at the end that does not satisfy condition i of the theorem. Change the strategy of the first player, who violates condition i so that she votes Y after G, and if necessary, change a subsequent player's strategy to N after the new history, so that the outcome of the election is not changed. Similar arguments can be used for condition ii. Q.E.D.

Consider an i-tuple of signals $(x_1, \ldots, x_{i-1}, G)$ such that the number of

G's is n_p and the number of B's is less than or equal to $n - n_p$. Denote by Z' the set of all such vectors for any i. Given a strategy profile $(s_1, \ldots, s_n)$ as in lemmas A2 and A3, consider the history $(h_1, \ldots, h_i)$ generated by an i-tuple of signals $(x_1, \ldots, x_{i-1}, G)$ in Z'. Then, for $j \leq i - 1$, $h_j = Y$ if and only if $x_j = G$ since, at any time a Y vote is cast, the number of Y's is strictly less than $n_p - 1$ and, at any time an N vote is cast, the number of N's is strictly less than $n - n_p$. Moreover, the outcome of the election is determined by the action following the observation of G by voter i: Y passes the alternative whereas N is uninformative, and hence all subsequent voting is also uninformative. Consider now an i-tuple of signals $(x_1, \ldots, x_{i-1}, B)$ such that the number of G's is less than or equal to $n_p - 1$ and the number of B's is equal to $n - n_p + 1$. Denote by Z'' the set of all such vectors for any i. Again, for a strategy profile $(s_1, \ldots, s_n)$ as in lemmas A2 and A3, consider the history $(h_1, \ldots, h_i)$ generated by an i-tuple of signals $(x_1, \ldots, x_{i-1}, B)$ in Z''. As before, for $j \leq i - 1$, $h_j = Y$ if and only if $x_j = G$, and the outcome of the election is determined by the action following the observation of B by voter i: N fails the alternative whereas Y is uninformative, and so is all subsequent voting.

Hence, by the preceding lemmas, we need to specify the election outcome for an M-optimal strategy after the realization of signal vectors in $Z' \cup Z''$. For this purpose, let $Y^* = \{h\colon N(h) \geq n - n_p \text{ and } Y(h) \geq \gamma\}$.

LEMMA A4. If $e(n_p - 1, n - n_p + 1) \geq 0$, then the strategy profile $(s_1, \ldots, s_n)$ is M-optimal, where $s_i(h, G) = s_i(h, B) = 1$ for $h \in Y^* \cap \{Y, N\}^{i-1}$, and $s_i(h, G) = 1$ and $s_i(h, B) = 0$ for $h \notin Y^* \cap \{Y, N\}^{i-1}$.

Thus an M-optimal strategy profile in this case is to vote informatively after histories not in Y^* and to vote Y after histories in Y^*.

Proof. A strategy profile as in lemmas A2 and A3 induces a function d: $Z' \cup Z'' \rightarrow \{0, 1\}$, which specifies the outcome—zero for the status quo, one for the alternative—after the realization of a signal vector in $Z' \cup Z''$.

Let z be a vector of r signals in Z' or Z'', and let $f(z)$ denote the probability of such a signal: $f(z) = \sum_{\{x: x_1 = z_1, \ldots, x_r = z_r\}} f(x)$. Note that (i) for any vector of signals z in $Z' \cup Z''$ and an arbitrary vector of signals c, the vector (z, c) cannot be in $Z' \cup Z''$; and (ii) for any complete realization of signals $(x_1, \ldots, x_n)$, there is an i-tuple z in $Z' \cup Z''$ such that $x_j = z_j$ for $j \leq i$. Then, if $G(z)$ denotes the number of G's in z and $B(z)$ the number of B's in z, the expected value of the election is

$$\sum_{z \in Z'} d(z)\, f(z) e(n_p, B(z)) + \sum_{z \in Z''} d(z)\, f(z) e(G(z), n - n_p + 1).$$

The lemma is proved if the strategy profile specified maximizes this expression. The specification yields $d(z) = 1$ either if $z \in Z'$ or if $z \in Z''$ and $G(z) \geq \gamma$, and $d(z) = 0$ otherwise. In the case under consideration, where $e(n_p - 1, n - n_p + 1) \geq 0$, we know that $e(n_p, B(z)) > 0$ for all $z \in Z'$, so the first term in the expected value is maximized by setting $d(z) = 1$ for all $z \in Z'$, exactly as the strategy profile specified in the lemma does. The second term has both positive and negative elements in the summation; $e(G(z), n - n_p + 1)$ is positive exactly when $G(z) \geq \gamma$, which is when the profile specified yields $d(z) = 1$. Thus $d(z)$ is optimal. Q.E.D.

Now let $N^* = \{h: Y(h) \geq n_p - 1 \text{ and } N(h) \geq \beta\}$.

LEMMA A5. If $e(n_p, n - n_p) < 0$, then the strategy profile $(s_1, \ldots, s_n)$ is *M*-optimal, where $s_i(h, G) = s_i(h, B) = 0$ for $h \in N^* \cap \{Y, N\}^{i-1}$, and $s_i(h, G) = 1$ and $s_i(h, B) = 0$ for $h \notin N^* \cap \{Y, N\}^{i-1}$. If $e(n_p, n - n_p) \geq 0$ and $e(n_p - 1, n - n_p + 1) < 0$, then the strategy profile $(s_1, \ldots, s_n)$ is *M*-optimal, where $s_i(h, G) = 1$ and $s_i(h, B) = 0$ for all h.

Proof. The proof is exactly like that of the preceding lemma. Q.E.D.

Proof of Theorem 4

For any strategy profile of any *T*-period voting game, there is a realization-equivalent strategy profile of the roll-call game. Since the strategies in the statement of the theorem are history-independent, it is therefore sufficient to prove the theorem for roll-call games.

Consider the *M*-optimal strategy profile in lemma A4. Let z be the first $n - n_p + \gamma$ signals in some vector of signals $x \in X$. If the number of *G* signals in z is γ or greater, then the alternative passes regardless of future signals; if the number is less than γ, then the alternative fails regardless of future signals. This yields exactly the same outcome as the strategy profile given in the statement of the theorem, so we have proved part 1 of the theorem. Parts 2 and 3 follow similarly by using lemma A5. Q.E.D.

Proof of Theorem 5

It remains to prove that in the simultaneous voting game if the optimal equilibrium involves informative strategies, then the *M*-optimal equilibrium identified by the theorem is generically strict; and when it is strict, it is the unique strict equilibrium. The basic intuition is that in simultaneous voting, *M*-optimality and optimality coincide (step 1 below), and generically the strategies defined are strictly optimal; hence they are a strict equilibrium. Formally, the conclusion follows from the steps below. Let n_N denote the number of voters choosing *N* uninformatively (i.e., regardless of their signal), n_Y the number of voters choosing *Y* uninformatively, and n_i the number of informative voters.

1. Clearly a nonmonotonic strategy cannot be a strict best reply in the simultaneous voting game. If it is a strict best reply to vote *N* when observing *G*, then it must be a strict best reply to vote *N* when observing *B*. Therefore, all informative strategies are monotonic.

2. There is no strict equilibrium in which at least one player, say i, uninformatively votes *N* and at least one other, say j, uninformatively votes *Y*. When i is pivotal, she knows that $n_p - 1$ others voted *Y* and $n - n_p$ voted *N*. So when she observes *G* and is pivotal, she knows that (including herself) $n_p - n_Y$ players observed *G* and $n - n_p - n_N$ observed *B*; so strictly preferring *N* implies that $e(n_p - n_Y, n - n_p - n_N) < 0$. On the other hand, when j observes *B* and is pivotal, he knows that $n_p - n_Y - 1$ observed *G* and $n - n_p - n_N + 1$ observed *B*; so strictly preferring *Y* implies that $e(n_p - n_Y - 1, n - n_p - n_N + 1) > 0$, which yields a contradiction.

3. Thus any strict equilibrium has the form in which some players vote

informatively and others use the same uninformative strategy. Consider henceforth the case in which $e(n_p - 1, n - n_p + 1) \geq 0$; the other cases are argued similarly. Then it cannot be that the uninformative all vote N. For that to be a strict equilibrium, at least $n_p - 1$ must vote informatively. But then an uninformative voter is pivotal only if $n_p - 1$ good signals were observed, in which case voting Y is better than N since by assumption $e(n_p - 1, b) \geq e(n_p - 1, n - n_p + 1) \geq 0$, for all b such that $0 \leq b \leq n - n_p + 1$. So $n_N = 0$.

4. A pivotal informative voter will strictly prefer voting Y when she observes G if and only if $e(n_p - n_Y, n - n_p) > 0$, whereas she will strictly prefer voting N when she observes B if and only if $e(n_p - n_Y - 1, n - n_p + 1) < 0$. An uninformative voter strictly prefers voting Y when B is observed if $e(n_p - n_Y, n - n_p + 1) > 0$. But these last two inequalities imply that $n_p - n_Y = \gamma$. Q.E.D.

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