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SERIES REPRESENTATIONS OF THE REMAINDERS IN THE EXPANSIONS FOR CERTAIN TRIGONOMETRIC FUNCTIONS AND SOME RELATED INEQUALITIES, I

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Abstract. We present series representations of the remainders in the expansions for certain trigonometric and hyperbolic functions. From these results, we establish some inequalities for trigonometric and hyperbolic functions.

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