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Boston – Delft

Full text available at: http://dx.doi.org/10.1561/2000000013

Foundations and Trends® in Signal Processing

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
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The preferred citation for this publication is W. A. Pearlman and A. Said, Set Partition Coding: Part I of Set Partition Coding and Image Wavelet Coding Systems, Foundations and Trends® in Signal Processing, vol 2, no 2, pp 95–180, 2008

ISBN: 978-1-60198-178-3
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Volume 2 Issue 2, 2008

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Foundations and Trends® in Signal Processing, 2008, Volume 2, 4 issues. ISSN paper version 1932-8346. ISSN online version 1932-8354. Also available as a combined paper and online subscription.

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Foundations and Trends® in
Signal Processing
Vol. 2, No. 2 (2008) 95–180
© 2008 W. A. Pearlman and A. Said
DOI: 10.1561/2000000013

Set Partition Coding: Part I of Set Partition Coding and Image Wavelet Coding Systems

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Abstract

The purpose of this two-part monograph is to present a tutorial on set partition coding, with emphasis and examples on image wavelet transform coding systems, and describe their use in modern image coding systems. Set partition coding is a procedure that recursively splits groups of integer data or transform elements guided by a sequence of threshold tests, producing groups of elements whose magnitudes are between two known thresholds, therefore, setting the maximum number of bits required for their binary representation. It produces groups of elements whose magnitudes are less than a certain known threshold. Therefore, the number of bits for representing an element in a particular group is no more than the base-2 logarithm of its threshold

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rounded up to the nearest integer. SPIHT (Set Partitioning in Hierarchical Trees) and SPECK (Set Partitioning Embedded blocK) are popular state-of-the-art image coders that use set partition coding as the primary entropy coding method. JPEG2000 and EZW (Embedded Zerotree Wavelet) use it in an auxiliary manner. Part I elucidates the fundamentals of set partition coding and explains the setting of thresholds and the block and tree modes of partitioning. Algorithms are presented for the techniques of AGP (Amplitude and Group Partitioning), SPIHT, SPECK, and EZW. Numerical examples are worked out in detail for the latter three techniques. Part II describes various wavelet image coding systems that use set partitioning primarily, such as SBHP (Subband Block Hierarchical Partitioning), SPIHT, and EZBC (Embedded Zero-Block Coder). The basic JPEG2000 coder is also described. The coding procedures and the specific methods are presented both logically and in algorithmic form, where possible. Besides the obvious objective of obtaining small file sizes, much emphasis is placed on achieving low computational complexity and desirable output bitstream attributes, such as embeddedness, scalability in resolution, and random access decodability.

This monograph is extracted and adapted from the forthcoming textbook entitled *Digital Signal Compression: Principles and Practice* by William A. Pearlman and Amir Said, Cambridge University Press, 2009.

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1

Introduction

The goal of this monograph is to elucidate the principles of a data compression method called *set partition coding,* mainly by showing its use in image compression. One of the most interesting aspects of this coding technique is that it is based on some very simple principles, but is also very effective, surpassing other coding methods that may seem theoretically much more sophisticated (and probably with a fraction of the computational complexity).

This technique is the primary entropy coding method in the well-known SPIHT (Set Partitioning In Hierarchical Trees) [18] and SPECK (Set Partitioning Embedded bloCK) [15] algorithms and is an auxiliary coding method in the JPEG2000 standard [11, 12]. It also has been successfully applied in the method of EZBC (Embedded Zero Block Coder) for video coding [9]. There are many books and articles explaining SPIHT and SPECK, and its many variations and modifications. These works state the algorithms and present worked-out examples, but none touches on the logic underlying these methods.

What had been missing is a proper explanation of why a method as simple as set partition coding can be so effective. A possible reason for this omission is that the usual approach for teaching data compression

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is to start from information theory principles, and some simple statistical models, like i.i.d. sources, and then possibly move to somewhat more complicated cases. However, rarely there is any real intention of realism, since the data sources defined by the main multimedia compression applications (audio, images, and video) are quite complex. They can be extremely difficult to model accurately, and we may not even know what are all the aspects involved (e.g., if a Stradivarius violin sounds like no other, how to preserve that information?).

A superficial interpretation of information theory can also give the impression that increasingly more complex statistical models are the only thing needed to improve compression. Actually what is needed is the development of *workable* models (explicit or implicit) that are flexible enough to allow more *accurate* modeling for *coding,* without necessarily providing descriptive information. An *ad hoc* scheme based on empirical knowledge of the source's statistical properties can be much better than methods that in theory could be used for data sources of any complexity, but in practice need to be very strongly constrained to be manageable.

This monograph is an attempt to remedy this situation and help the reader understand and separate the various principles that are at play in effective coding systems. We chose an approach that keeps the presentation intuitive and simple, without formal statistical models, but always related to real data sources. The risk, in this case, is that the methods we present may seem at first too simplistic and limited, even naïve. The reader should keep in mind that even the simplest versions are quite effective, and could not be so if they were not able to exploit fundamental statistic properties of the data sources.

Set partition coding uses recursions of partitioning or splitting of sets of samples guided by a threshold test. The details will be explained at length in Part I of this monograph. There are different tests and different ways that these sets can be split. One unique feature is the presentation of the steps of the algorithms in separate boxes, in addition to the explanations. Contrary to what is written in many places, these methods are not married to bit-plane coding and do not by themselves produce embedded bitstreams. Bit-plane coding is an option and often an unnecessary one. Similarly, some authors assume that

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these methods are meant to work only on wavelet transforms, without knowing that they had been successfully applied to other types of data.

Embedded coding within the framework of set partition coding will be carefully described. Integer data that tend to contain clusters of close value, such as those produced by certain mathematical transformations, are especially amenable to compression by most methods, including this method. Numerous examples of set partition coding of transformed data are presented. In particular, the SPIHT, SPECK and EZW (Embedded Zerotree Wavelet) [20] algorithms are explained thoroughly and compared in an actual example of coding a wavelet transform.

Part II describes current-day wavelet transform image coding systems. As before, steps of the algorithms are explained thoroughly and set apart. An image coding system consists of several stages of procedure: transformation, quantization, set partition or adaptive entropy coding or both, decoding including rate control, inverse transformation, de-quantization, and optional processing in certain stages. Wavelet transform systems can provide many desirable properties besides high efficiency, such as scalability in quality, scalability in resolution, and region-of-interest access to the coded bitstream. These properties are built into the JPEG2000 standard, so its coding will be fully described. Since JPEG2000 codes sub-blocks of subbands, other methods, such as SBHP (Subband Block Hierarchical Partitioning) [4] and EZBC [9], that code subbands or its subblocks independently are also described. The emphasis in this part is the use of the basic algorithms presented in the previous part in ways that achieve these desirable bitstream properties. In this vein, we describe a modification of the tree-based coding in SPIHT whose output bitstream can be decoded partially, corresponding to a designated region of interest and is simultaneously quality and resolution scalable.

Although not really necessary for understanding the coding methods presented here, we have included an appendix describing some mathematical transforms, including subband/wavelet transforms, used in coding systems. Some readers may feel more comfortable in reading this monograph with some introduction to this material.

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This monograph is intended to be a tutorial and not a survey. Although many coding methods are described, some others that fit into the framework of this monograph are not, and the bibliography is certainly not comprehensive. We hope that any omissions will be forgiven. Nonetheless, we believe this tutorial is unprecedented in its manner of presentation and serves its stated purpose. We hope the readers agree.

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