

Several Differentiation Formulas of Special Functions. Part VII

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Summary. In this article, we prove a series of differentiation identities [2] involving the arctan and arccot functions and specific combinations of special functions including trigonometric and exponential functions.

MML identifier: FDIFF_11, version: 7.10.01 4.111.1036

The papers [13], [15], [1], [10], [16], [5], [12], [3], [6], [9], [4], [11], [8], [14], and [7] provide the terminology and notation for this paper.

For simplicity, we adopt the following rules: x denotes a real number, n denotes an element of \mathbb{N} , Z denotes an open subset of \mathbb{R} , and f, g denote partial functions from \mathbb{R} to \mathbb{R} .

Next we state a number of propositions:

- (1) Suppose $Z \subseteq \text{dom}(\text{(the function arctan)} \cdot \text{(the function sin)})$ and for every x such that $x \in Z$ holds $-1 < \sin x < 1$. Then
 - (i) $\text{(the function arctan)} \cdot \text{(the function sin)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function arctan)} \cdot \text{(the function sin)})'_{|Z}(x) = \frac{\cos x}{1+(\sin x)^2}$.
- (2) Suppose $Z \subseteq \text{dom}(\text{(the function arccot)} \cdot \text{(the function sin)})$ and for every x such that $x \in Z$ holds $-1 < \sin x < 1$. Then
 - (i) $\text{(the function arccot)} \cdot \text{(the function sin)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{(the function arccot)} \cdot \text{(the function sin)})'_{|Z}(x) = -\frac{\cos x}{1+(\sin x)^2}$.
- (3) Suppose $Z \subseteq \text{dom}(\text{(the function arctan)} \cdot \text{(the function cos)})$ and for every x such that $x \in Z$ holds $-1 < \cos x < 1$. Then

- (i) (the function \arctan) \cdot (the function \cos) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \arctan) \cdot (the function \cos))' $_{|Z}(x) = -\frac{\sin x}{1+(\cos x)^2}$.
- (4) Suppose $Z \subseteq \text{dom}((\text{the function } \text{arccot}) \cdot (\text{the function } \cos))$ and for every x such that $x \in Z$ holds $-1 < \cos x < 1$. Then
- (i) (the function arccot) \cdot (the function \cos) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function arccot) \cdot (the function \cos))' $_{|Z}(x) = \frac{\sin x}{1+(\cos x)^2}$.
- (5) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \tan))$ and for every x such that $x \in Z$ holds $-1 < \tan x < 1$. Then
- (i) (the function \arctan) \cdot (the function \tan) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \arctan) \cdot (the function \tan))' $_{|Z}(x) = 1$.
- (6) Suppose $Z \subseteq \text{dom}((\text{the function } \text{arccot}) \cdot (\text{the function } \tan))$ and for every x such that $x \in Z$ holds $-1 < \tan x < 1$. Then
- (i) (the function arccot) \cdot (the function \tan) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function arccot) \cdot (the function \tan))' $_{|Z}(x) = -1$.
- (7) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \cot))$ and for every x such that $x \in Z$ holds $-1 < \cot x < 1$. Then
- (i) (the function \arctan) \cdot (the function \cot) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \arctan) \cdot (the function \cot))' $_{|Z}(x) = -1$.
- (8) Suppose $Z \subseteq \text{dom}((\text{the function } \text{arccot}) \cdot (\text{the function } \cot))$ and for every x such that $x \in Z$ holds $-1 < \cot x < 1$. Then
- (i) (the function arccot) \cdot (the function \cot) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function arccot) \cdot (the function \cot))' $_{|Z}(x) = 1$.
- (9) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \arctan x < 1$. Then
- (i) (the function \arctan) \cdot (the function \arctan) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \arctan) \cdot (the function \arctan))' $_{|Z}(x) = \frac{1}{(1+x^2) \cdot (1+(\arctan x)^2)}$.
- (10) Suppose $Z \subseteq \text{dom}((\text{the function } \text{arccot}) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \arctan x < 1$. Then
- (i) (the function arccot) \cdot (the function \arctan) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function arccot) \cdot (the function \arctan))' $_{|Z}(x) = -\frac{1}{(1+x^2) \cdot (1+(\arctan x)^2)}$.

- (11) Suppose $Z \subseteq \text{dom}(\text{(the function arctan)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \text{arccot } x < 1$. Then
- (the function arctan) · (the function arccot) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function arctan}) \cdot (\text{the function arccot}))'_{|Z}(x) = -\frac{1}{(1+x^2) \cdot (1+(\text{arccot } x)^2)}$.
- (12) Suppose $Z \subseteq \text{dom}(\text{(the function arccot)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \text{arccot } x < 1$. Then
- (the function arccot) · (the function arccot) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function arccot}) \cdot (\text{the function arccot}))'_{|Z}(x) = \frac{1}{(1+x^2) \cdot (1+(\text{arccot } x)^2)}$.
- (13) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (the function sin) · (the function arctan) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot (\text{the function arctan}))'_{|Z}(x) = \frac{\cos \arctan x}{1+x^2}$.
- (14) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (the function sin) · (the function arccot) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot (\text{the function arccot}))'_{|Z}(x) = -\frac{\cos \arccot x}{1+x^2}$.
- (15) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (the function cos) · (the function arctan) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function cos}) \cdot (\text{the function arctan}))'_{|Z}(x) = -\frac{\sin \arctan x}{1+x^2}$.
- (16) Suppose $Z \subseteq \text{dom}(\text{(the function cos)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (the function cos) · (the function arccot) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function cos}) \cdot (\text{the function arccot}))'_{|Z}(x) = \frac{\sin \arccot x}{1+x^2}$.
- (17) Suppose $Z \subseteq \text{dom}(\text{(the function tan)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (the function tan) · (the function arctan) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function tan}) \cdot (\text{the function arctan}))'_{|Z}(x) = \frac{1}{(\cos \arctan x)^2 \cdot (1+x^2)}$.
- (18) Suppose $Z \subseteq \text{dom}(\text{(the function tan)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (the function tan) · (the function arccot) is differentiable on Z , and
 - for every x such that $x \in Z$ holds $((\text{the function tan}) \cdot (\text{the function arccot}))'_{|Z}(x) = -\frac{1}{(\cos \arccot x)^2 \cdot (1+x^2)}$.

- (19) Suppose $Z \subseteq \text{dom}(\text{(the function cot)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function cot)} \cdot \text{(the function arctan)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cot)} \cdot \text{(the function arctan)}\big|_Z(x) = -\frac{1}{(\sin \arctan x)^2 \cdot (1+x^2)}$.
- (20) Suppose $Z \subseteq \text{dom}(\text{(the function cot)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function cot)} \cdot \text{(the function arccot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cot)} \cdot \text{(the function arccot)}\big|_Z(x) = \frac{1}{(\sin \text{arccot } x)^2 \cdot (1+x^2)}$.
- (21) Suppose $Z \subseteq \text{dom}(\text{(the function sec)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function sec)} \cdot \text{(the function arctan)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function sec)} \cdot \text{(the function arctan)}\big|_Z(x) = \frac{\sin \arctan x}{(\cos \arctan x)^2 \cdot (1+x^2)}$.
- (22) Suppose $Z \subseteq \text{dom}(\text{(the function sec)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function sec)} \cdot \text{(the function arccot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function sec)} \cdot \text{(the function arccot)}\big|_Z(x) = -\frac{\sin \text{arccot } x}{(\cos \text{arccot } x)^2 \cdot (1+x^2)}$.
- (23) Suppose $Z \subseteq \text{dom}(\text{(the function cosec)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function cosec)} \cdot \text{(the function arctan)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cosec)} \cdot \text{(the function arctan)}\big|_Z(x) = -\frac{\cos \arctan x}{(\sin \arctan x)^2 \cdot (1+x^2)}$.
- (24) Suppose $Z \subseteq \text{dom}(\text{(the function cosec)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function cosec)} \cdot \text{(the function arccot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function cosec)} \cdot \text{(the function arccot)}\big|_Z(x) = \frac{\cos \text{arccot } x}{(\sin \text{arccot } x)^2 \cdot (1+x^2)}$.
- (25) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \cdot \text{(the function arctan)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function sin)} \cdot \text{(the function arctan)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function sin)} \cdot \text{(the function arctan)}\big|_Z(x) = \cos x \cdot \arctan x + \frac{\sin x}{1+x^2}$.
- (26) Suppose $Z \subseteq \text{dom}(\text{(the function sin)} \cdot \text{(the function arccot)})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{(the function sin)} \cdot \text{(the function arccot)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $\text{(the function sin)} \cdot \text{(the function arccot)}\big|_Z(x) = \cos x \cdot \text{arccot } x - \frac{\sin x}{1+x^2}$.

- (27) Suppose $Z \subseteq \text{dom}(\text{the function } \cos)$ (the function \arctan) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \cos) (the function \arctan) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \cos) (the function \arctan))' $\upharpoonright_Z(x) = -\sin x \cdot \arctan x + \frac{\cos x}{1+x^2}$.
- (28) Suppose $Z \subseteq \text{dom}(\text{the function } \cos)$ (the function arccot) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \cos) (the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \cos) (the function arccot))' $\upharpoonright_Z(x) = -\sin x \cdot \text{arccot } x - \frac{\cos x}{1+x^2}$.
- (29) Suppose $Z \subseteq \text{dom}(\text{the function } \tan)$ (the function \arctan) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \tan) (the function \arctan) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \tan) (the function \arctan))' $\upharpoonright_Z(x) = \frac{\arctan x}{(\cos x)^2} + \frac{\tan x}{1+x^2}$.
- (30) Suppose $Z \subseteq \text{dom}(\text{the function } \tan)$ (the function arccot) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \tan) (the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \tan) (the function arccot))' $\upharpoonright_Z(x) = \frac{\text{arccot } x}{(\cos x)^2} - \frac{\tan x}{1+x^2}$.
- (31) Suppose $Z \subseteq \text{dom}(\text{the function } \cot)$ (the function \arctan) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \cot) (the function \arctan) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \cot) (the function \arctan))' $\upharpoonright_Z(x) = -\frac{\arctan x}{(\sin x)^2} + \frac{\cot x}{1+x^2}$.
- (32) Suppose $Z \subseteq \text{dom}(\text{the function } \cot)$ (the function arccot) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \cot) (the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \cot) (the function arccot))' $\upharpoonright_Z(x) = -\frac{\text{arccot } x}{(\sin x)^2} - \frac{\cot x}{1+x^2}$.
- (33) Suppose $Z \subseteq \text{dom}(\text{the function } \sec)$ (the function \arctan) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \sec) (the function \arctan) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \sec) (the function \arctan))' $\upharpoonright_Z(x) = \frac{\sin x \cdot \arctan x}{(\cos x)^2} + \frac{1}{\cos x \cdot (1+x^2)}$.
- (34) Suppose $Z \subseteq \text{dom}(\text{the function } \sec)$ (the function arccot) and $Z \subseteq]-1, 1[$. Then
- (i) (the function \sec) (the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function \sec) (the function arccot))' $\upharpoonright_Z(x) = \frac{\sin x \cdot \text{arccot } x}{(\cos x)^2} - \frac{1}{\cos x \cdot (1+x^2)}$.

- (35) Suppose $Z \subseteq \text{dom}(\text{the function cosec})$ (the function arctan) and $Z \subseteq]-1, 1[$. Then
- (i) (the function cosec) (the function arctan) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function cosec) (the function arctan))' $_{|Z}(x) = -\frac{\cos x \cdot \arctan x}{(\sin x)^2} + \frac{1}{\sin x \cdot (1+x^2)}$.
- (36) Suppose $Z \subseteq \text{dom}(\text{the function cosec})$ (the function arccot) and $Z \subseteq]-1, 1[$. Then
- (i) (the function cosec) (the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function cosec) (the function arccot))' $_{|Z}(x) = -\frac{\cos x \cdot \text{arccot } x}{(\sin x)^2} - \frac{1}{\sin x \cdot (1+x^2)}$.
- (37) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function arctan)+(the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function arctan)+(the function arccot))' $_{|Z}(x) = 0$.
- (38) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function arctan)-(the function arccot) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function arctan)-(the function arccot))' $_{|Z}(x) = \frac{2}{1+x^2}$.
- (39) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function sin) ((the function arctan)+(the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function sin) ((the function arctan)+(the function arccot)))' $_{|Z}(x) = \cos x \cdot (\arctan x + \text{arccot } x)$.
- (40) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function sin) ((the function arctan)-(the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function sin) ((the function arctan)-(the function arccot)))' $_{|Z}(x) = \cos x \cdot (\arctan x - \text{arccot } x) + \frac{2 \cdot \sin x}{1+x^2}$.
- (41) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function cos) ((the function arctan)+(the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function cos) ((the function arctan)+(the function arccot)))' $_{|Z}(x) = -\sin x \cdot (\arctan x + \text{arccot } x)$.
- (42) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function cos) ((the function arctan)-(the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function cos) ((the function arctan)-(the function arccot)))' $_{|Z}(x) = -\sin x \cdot (\arctan x - \text{arccot } x) + \frac{2 \cdot \cos x}{1+x^2}$.
- (43) Suppose $Z \subseteq \text{dom}(\text{the function tan})$ and $Z \subseteq]-1, 1[$. Then

- (i) (the function \tan) ((the function \arctan)+(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \tan) ((the function \arctan)+(the function arccot)))' $\Big|_Z(x) = \frac{\arctan x + \operatorname{arccot} x}{(\cos x)^2}$.
- (44) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \tan)$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \tan) ((the function \arctan)-(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \tan) ((the function \arctan)-(the function arccot)))' $\Big|_Z(x) = \frac{\arctan x - \operatorname{arccot} x}{(\cos x)^2} + \frac{2 \cdot \tan x}{1+x^2}$.
- (45) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \cot)$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \cot) ((the function \arctan)+(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \cot) ((the function \arctan)+(the function arccot)))' $\Big|_Z(x) = -\frac{\arctan x + \operatorname{arccot} x}{(\sin x)^2}$.
- (46) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \cot)$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \cot) ((the function \arctan)-(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \cot) ((the function \arctan)-(the function arccot)))' $\Big|_Z(x) = -\frac{\arctan x - \operatorname{arccot} x}{(\sin x)^2} + \frac{2 \cdot \cot x}{1+x^2}$.
- (47) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \sec)$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \sec) ((the function \arctan)+(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \sec) ((the function \arctan)+(the function arccot)))' $\Big|_Z(x) = \frac{(\arctan x + \operatorname{arccot} x) \cdot \sin x}{(\cos x)^2}$.
- (48) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \sec)$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \sec) ((the function \arctan)-(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \sec) ((the function \arctan)-(the function arccot)))' $\Big|_Z(x) = \frac{(\arctan x - \operatorname{arccot} x) \cdot \sin x}{(\cos x)^2} + \frac{2 \cdot \sec x}{1+x^2}$.
- (49) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \operatorname{cosec})$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function cosec) ((the function \arctan)+(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function cosec) ((the function \arctan)+(the function arccot)))' $\Big|_Z(x) = -\frac{(\arctan x + \operatorname{arccot} x) \cdot \cos x}{(\sin x)^2}$.
- (50) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \operatorname{cosec})$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function cosec) ((the function \arctan)-(the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function cosec) ((the function \arctan)-(the function arccot)))' $\Big|_Z(x) = -\frac{(\arctan x - \operatorname{arccot} x) \cdot \cos x}{(\sin x)^2} + \frac{2 \cdot \operatorname{cosec} x}{1+x^2}$.
- (51) Suppose $Z \subseteq]-1, 1[$. Then

- (i) (the function \exp) \cdot ((the function \arctan) + (the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \exp) \cdot ((the function \arctan) + (the function arccot))) $'_{|Z}(x) = \exp x \cdot (\arctan x + \operatorname{arccot} x)$.
- (52) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function \exp) \cdot ((the function \arctan) - (the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \exp) \cdot ((the function \arctan) - (the function arccot))) $'_{|Z}(x) = \exp x \cdot (\arctan x - \operatorname{arccot} x) + \frac{2 \cdot \exp x}{1+x^2}$.
- (53) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $\frac{(\text{the function } \arctan) + (\text{the function } \operatorname{arccot})}{\text{the function } \exp}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $\left(\frac{(\text{the function } \arctan) + (\text{the function } \operatorname{arccot})}{\text{the function } \exp}\right)'_{|Z}(x) = -\frac{\arctan x + \operatorname{arccot} x}{\exp x}$.
- (54) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $\frac{(\text{the function } \arctan) - (\text{the function } \operatorname{arccot})}{\text{the function } \exp}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $\left(\frac{(\text{the function } \arctan) - (\text{the function } \operatorname{arccot})}{\text{the function } \exp}\right)'_{|Z}(x) = \frac{(\frac{2}{1+x^2} - \arctan x) + \operatorname{arccot} x}{\exp x}$.
- (55) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \exp) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \exp) \cdot ((the function \arctan) + (the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \exp) \cdot ((the function \arctan) + (the function arccot))) $'_{|Z}(x) = 0$.
- (56) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \exp) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \exp) \cdot ((the function \arctan) - (the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \exp) \cdot ((the function \arctan) - (the function arccot))) $'_{|Z}(x) = \frac{2 \cdot \exp(\arctan x - \operatorname{arccot} x)}{1+x^2}$.
- (57) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \sin) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \sin) \cdot ((the function \arctan) + (the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function \sin) \cdot ((the function \arctan) + (the function arccot))) $'_{|Z}(x) = 0$.
- (58) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \sin) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function \sin) \cdot ((the function \arctan) - (the function arccot)) is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))'_{|Z}(x) = \frac{2 \cdot \cos(\arctan x - \operatorname{arccot} x)}{1+x^2}$.
- (59) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \cos) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))'_{|Z}(x) = 0$.
- (60) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \cos) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))'_{|Z}(x) = -\frac{2 \cdot \sin(\arctan x - \operatorname{arccot} x)}{1+x^2}$.
- (61) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \arctan) (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) (\text{the function } \operatorname{arccot}))'_{|Z}(x) = \frac{\operatorname{arccot} x - \arctan x}{1+x^2}$.
- (62) Suppose that
- (i) $Z \subseteq \operatorname{dom}(((\text{the function } \arctan) \cdot \frac{1}{f}) ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))$, and
- (ii) for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$.
- Then
- (iii) $((\text{the function } \arctan) \cdot \frac{1}{f}) ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})$ is differentiable on Z , and
- (iv) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot \frac{1}{f}) ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})'_{|Z}(x) = \frac{\arctan(\frac{1}{x}) - \operatorname{arccot}(\frac{1}{x})}{1+x^2}$.
- (63) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z ((\text{the function } \arctan) \cdot \frac{1}{f}))$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $\operatorname{id}_Z ((\text{the function } \arctan) \cdot \frac{1}{f})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\operatorname{id}_Z ((\text{the function } \arctan) \cdot \frac{1}{f}))'_{|Z}(x) = \arctan(\frac{1}{x}) - \frac{x}{1+x^2}$.
- (64) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $\operatorname{id}_Z ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\operatorname{id}_Z ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))'_{|Z}(x) = \operatorname{arccot}(\frac{1}{x}) + \frac{x}{1+x^2}$.
- (65) Suppose $Z \subseteq \operatorname{dom}(g((\text{the function } \arctan) \cdot \frac{1}{f}))$ and $g = \square^2$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $g((\text{the function } \arctan) \cdot \frac{1}{f})$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $(g((\text{the function } \arctan) \cdot \frac{1}{f}))'_{|Z}(x) = 2 \cdot x \cdot \arctan(\frac{1}{x}) - \frac{x^2}{1+x^2}$.
- (66) Suppose $Z \subseteq \text{dom}(g((\text{the function } \text{arccot}) \cdot \frac{1}{f}))$ and $g = \square^2$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $g((\text{the function } \text{arccot}) \cdot \frac{1}{f})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(g((\text{the function } \text{arccot}) \cdot \frac{1}{f}))'_{|Z}(x) = 2 \cdot x \cdot \text{arccot}(\frac{1}{x}) + \frac{x^2}{1+x^2}$.
- (67) Suppose $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds (the function \arctan)(x) $\neq 0$. Then
- (i) $\frac{1}{\text{the function } \arctan}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function } \arctan})'_{|Z}(x) = -\frac{1}{(\arctan x)^2 \cdot (1+x^2)}$.
- (68) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $\frac{1}{\text{the function } \text{arccot}}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function } \text{arccot}})'_{|Z}(x) = \frac{1}{(\text{arccot } x)^2 \cdot (1+x^2)}$.
- One can prove the following propositions:
- (69) Suppose $Z \subseteq \text{dom}(\frac{1}{n(\text{the function } \arctan)^n})$ and $Z \subseteq]-1, 1[$ and $n > 0$ and for every x such that $x \in Z$ holds $\arctan x \neq 0$. Then
- (i) $\frac{1}{n(\text{the function } \arctan)^n}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\frac{1}{n(\text{the function } \arctan)^n})'_{|Z}(x) = -\frac{1}{((\arctan x)^{n+1}) \cdot (1+x^2)}$.
- (70) Suppose $Z \subseteq \text{dom}(\frac{1}{n(\text{the function } \text{arccot})^n})$ and $Z \subseteq]-1, 1[$ and $n > 0$. Then
- (i) $\frac{1}{n(\text{the function } \text{arccot})^n}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\frac{1}{n(\text{the function } \text{arccot})^n})'_{|Z}(x) = \frac{1}{((\text{arccot } x)^{n+1}) \cdot (1+x^2)}$.
- (71) Suppose $Z \subseteq \text{dom}(2(\text{the function } \arctan)^{\frac{1}{2}})$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $\arctan x > 0$. Then
- (i) $2(\text{the function } \arctan)^{\frac{1}{2}}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(2(\text{the function } \arctan)^{\frac{1}{2}})'_{|Z}(x) = \frac{(\arctan x)^{-\frac{1}{2}}}{1+x^2}$.
- (72) Suppose $Z \subseteq \text{dom}(2(\text{the function } \text{arccot})^{\frac{1}{2}})$ and $Z \subseteq]-1, 1[$. Then
- (i) $2(\text{the function } \text{arccot})^{\frac{1}{2}}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(2(\text{the function } \text{arccot})^{\frac{1}{2}})'_{|Z}(x) = -\frac{(\text{arccot } x)^{-\frac{1}{2}}}{1+x^2}$.

- (73) Suppose $Z \subseteq \text{dom}(\frac{2}{3}(\text{the function } \arctan)^{\frac{3}{2}})$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $\arctan x > 0$. Then
- (i) $\frac{2}{3}(\text{the function } \arctan)^{\frac{3}{2}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{2}{3}(\text{the function } \arctan)^{\frac{3}{2}})'_Z(x) = \frac{(\arctan x)^{\frac{1}{2}}}{1+x^2}$.
- (74) Suppose $Z \subseteq \text{dom}(\frac{2}{3}(\text{the function } \text{arccot})^{\frac{3}{2}})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\frac{2}{3}(\text{the function } \text{arccot})^{\frac{3}{2}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{2}{3}(\text{the function } \text{arccot})^{\frac{3}{2}})'_Z(x) = -\frac{(\text{arccot } x)^{\frac{1}{2}}}{1+x^2}$.

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Received September 23, 2008
