Severely Fading MIMO Channels: Models and Mutual Information

Seung Ho Choi and Peter Smith
Department of Electrical and Computer Engineering
University of Canterbury
Christchurch, New Zealand
Email: shc27@student.canterbury.ac.nz, p.smith@elec.canterbury.ac.nz

Ben Allen and Wasim Q. Malik
Department of Engineering Science
University of Oxford, Oxford, UK
Email: ben.allen@eng.ox.ac.uk, wasim.malik@eng.ox.ac.uk

Mansoor Shafi
Telecom New Zealand Ltd
PO Box 293, Wellington, New Zealand
Email: mansoor.shafi@telecom.co.nz

***Abstract*— In most wireless communications research, the channel models considered experience less severe fading than the classic Rayleigh fading case. In this work, however, we investigate MIMO channels where the fading is more severe. In these environments, we show that the coefficient of variation of the channel amplitudes is a good predictor of the link mutual information, for a variety of models. We propose a novel channel model for severely fading channels based on the complex multivariate t distribution. For this model, we are able to compute exact results for the ergodic mutual information and approximations to the outage probabilities for the mutual information. Applications of this work include wireless sensors, RF tagging, land-mobile, indoor-mobile, ground-penetrating radar, and ionospheric radio links. Finally, we point out that the methodology can also be extended to evaluate the mutual information of a cellular MIMO link and the performance of various MIMO receivers in a cellular scenario. In these cellular applications, the channel itself is not severely fading but the multivariate t distribution can be applied to model the effects of inter-cell interference.**

I. Introduction

Typical wireless communications channels experience considerable multipath propagation which causes signal fading at the receiver. Several amplitude probability distributions have been used in the literature to describe random fading. These include the Rician distribution for a range of line-of-sight and non-line-of-sight channels, the Nakagami distribution for wideband channels, and the Weibull distribution for propagation in non-homogeneous media. The Rayleigh distribution is commonly used to model the fading in strongly dispersive linear channels comprising homogeneous media, such as some urban wireless channels. Based on the Jakes model, this description assumes a large number of overlapping taps with uniformly distributed phases and angles-of-arrival. Results on the mutual information of Rayleigh-faded MIMO channels are widely available in the literature [1].

It is well known that the Nakagami-m distribution can model the amplitude for cases when the fading is more severe than Rayleigh. These scenarios arise in a variety of applications. For example, measurements reported in [2] were considerably more severe than Rayleigh for an RF tagging application. Other areas where severe fading models are useful include wireless sensors [3], land-mobile, indoor-mobile and ionospheric radio links [4]. Despite the range of applications where severe fading may be encountered, relatively little attention has been given to such channels, with the exception of the Nakagami model. For severely fading MIMO channels, there is even less information available in literature, partly due to the difficulty of performing statistical analysis of random Nakagami matrices.

In this paper, we investigate MIMO channels where the fading is more severe than Rayleigh fading. We propose a novel channel model for this scenario which is simple, flexible, lends to closed form analysis, and is a direct extension of the Rayleigh model to the severe fading region. Here, fading severity is defined in terms of the probability of deep fading events. It does not concern the level crossing rate, the average fade duration, or Doppler effects, and the propagation channel is assumed to be narrowband. We show that the coefficient of variation of the channel amplitudes yields a good prediction of the mutual information. We also propose using the complex multivariate t distribution to model the channel amplitudes for a variety of channels. We compute the associated ergodic mutual information and approximate outage probabilities. Finally, we show how the methodology is easily extended to evaluate the mutual information of cellular MIMO and the performance of various MIMO receivers. For these scenarios, the multivariate t distribution is used to model the inter-cell interference.

II. A Comparison of Severe Fading Channel Models

A. System Model

Consider a single user (n_T, n_R) MIMO system with n_T transmit antennas and n_R receive antennas. The system equation is given by [5], [6]

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{r}$ is the $n_R \times 1$ received signal vector, $\mathbf{s}$ is the complex $n_T \times 1$ transmitted signal vector, $\mathbf{n}$ is an $n_R \times 1$ additive white complex Gaussian noise vector with unit modulus variance, and $\mathbf{H}$ is an $n_R \times n_T$ complex channel matrix. In this section we consider the channel coefficients, $h_{rs} = (\mathbf{H})_{rs}$, to be of the form, $h_{rs} = A_{rs} \exp(j\theta_{rs})$, where θ_{rs} are independent and identically distributed (iid) uniform phase variables over $[0, 2\pi]$ and the A_{rs} are iid amplitudes drawn from a range of distribution types. As is standard practice, eg. [5], [6], we normalize the amplitude distributions so that the mean power is unity, $E(A_{rs}^2) = 1$. This allows a fair comparison across distribution types. Classical models for the amplitude probability distribution include Rayleigh, Ricean and Nakagami. We also consider a range of alternative distributions including lognormal, gamma, beta, uniform, truncated-t and truncated-Gaussian. Both the truncated distributions are truncated at zero to give positive amplitudes. The distributions are chosen to give a wide variation in the type of amplitude variable and to cover severe fading scenarios. Note that the selected models include short tailed (Gaussian), long tailed (lognormal), finite support (uniform) and infinite support (Nakagami) distributions. The Weibull distribution is another distribution that is worth consideration in a more comprehensive study, since it also covers the severe fading region.

Hence, there are considerable differences between the amplitude distributions considered in this work, covering a large variety of channel propagation conditions.

B. Coefficient of Variation and Mutual Information

The fading severity of a channel model can be measured by the coefficient of variation (CV), which is defined by [7]

$$CV = \frac{\sqrt{Var(A)}}{E(A)} = \sqrt{\frac{1}{E(A)^2} - 1} \tag{2}$$

for the case of amplitudes, A, normalized to give $E(A^2) = 1$. Another measure of fading is the amount of fading (AF), which is defined as $AF = CV^2 = Var(A)/[E(A)]^2$. Clearly, CV and AF are closely related, and we will use the CV for the analysis in the rest of this paper.

In this section, we study the ergodic mutual information of the system in (1) as a function of the CV. The mutual information (MI) is defined by [5], [6]

$$\mathcal{I} = \log_2 \left[\det \left(\mathbf{I}_{n_R} + \frac{\rho}{n_T} \mathbf{H}\mathbf{H}^\dagger \right) \right] \quad b/s/Hz \tag{3}$$

where ρ is the average SNR per receiver branch, and † denotes the complex conjugate transpose.

Note that the MI in (3) is often described as the capacity for the scenario when the transmitter has no channel information. We investigate the sensitivity of the ergodic MI to the type of amplitude distribution and whether the CV is a good measure of the effect of the fading severity on the ergodic MI.

C. Amplitude Distributions

The amplitude distributions considered here are all 2-parameter distributions. Distributions such as the uniform ($U[a, b]$) and Beta ($Beta[p, q]$) have 2 parameters in their standard form. Others, such as the t, have only 1 parameter in their standard form but can be scaled by a constant to give an extra parameter. The 2 parameters are necessary since one is fixed by the normalization $E(A^2) = 1$ and the remaining parameter is varied to give a range of CV values. This allows us to investigate the effect of CV on ergodic MI for a fixed type of normalized amplitude distribution. For example, if A is uniform over $[a, b]$, for $0 < a < b$, then $0 < a < 1$ is required to satisfy the normalization condition, which can be written as $E(A^2) = (a^2 + b^2 + ab)/3 = 1$. Solving this constraint for b gives, $b = -a/2 + \sqrt{12 - 3a^2}/2$. Now, we compute the mean amplitude as, $E(A) = (a + b)/2 = a/4 + \sqrt{12 - 3a^2}/4$. As a varies from 0 to 1, the mean varies from its minimum value ($E(A) = \sqrt{3}/2$) to the maximum value ($E(A) = 1$). Inserting these values in (2) gives a possible range for the CV values in the interval $[0, 1/\sqrt{3}]$. Hence, for uniform amplitudes we are only able to consider the variation in ergodic MI over $0 < \text{CV} < 1/\sqrt{3}$. Several of the other distributions also have restrictions on the range of CV values that can be achieved. A thorough description of all these distributions can be found in [8].

D. Results

For a range of amplitude distributions and a wide range of CV values the ergodic MI was simulated from (3) using 100,000 replicates. Results for a (4,2) MIMO system with a SNR of 18 dB and 0 dB are shown in Figs. 1 and 2, respectively. Note that the legends in these figures refer to ‘t’ and ‘Gaussian’, whereas truncated versions of these distributions were actually used. Figure 1 shows that at high SNR the CV is an excellent predictor of the ergodic MI which is not greatly affected by the precise amplitude distribution. Figure 2 shows that this conclusion begins to break down at low SNR where the ergodic MI is more sensitive to the type of distribution. Four of the distributions considered can only give CV values less than 1. Hence the MI curves for these distributions are overlaid by the others and are difficult to see. As a reference point, the CV for a Rayleigh channel is 0.526. Clearly, for small CV values, all the amplitude distributions give very similar results. The top curve, which diverges from the others, corresponds to the beta distribution. This achieves the higher CV values by becoming bimodal. Hence the amplitude distribution is very different to physical reality and this case is an extreme example. In Fig. 2, where the curves diverge, the short-tailed distributions (Nakagami and gamma) are similar and the long-tailed distributions (lognormal and truncated-t) are similar. Hence, at high SNR all the curves are similar, whereas at low SNR, distributions which are similar in nature show similar patterns.

These results motivate the development of a simple channel model which can cover the full range of severely fading channels, $\text{CV} > \text{CV}_{\text{Rayleigh}}$, and act as an approximation to other severely fading channels with the same CV. In the next section we propose the complex multivariate t model and show that it has the desired properties.

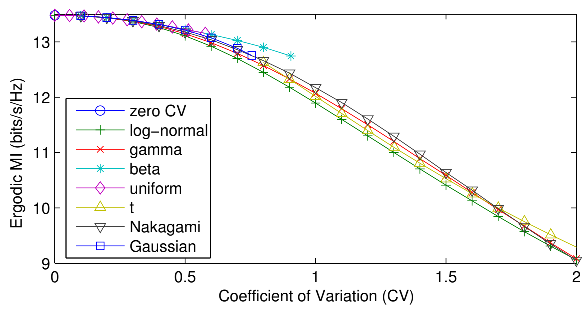

Fig. 1. Ergodic MI vs CV for a range of amplitude distributions in a (4,2) MIMO system with SNR=18dB

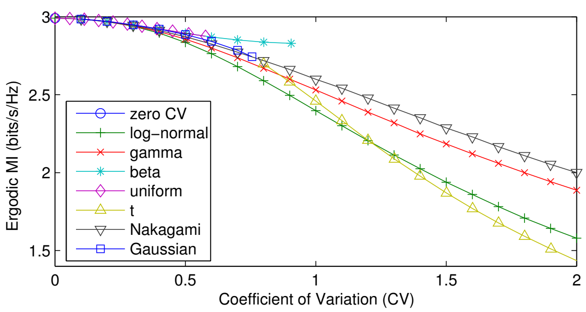

Fig. 2. Ergodic MI vs CV for a range of amplitude distributions in a (4,2) MIMO system with SNR=0dB

III. The Complex Multivariate T Channel Model

A. Introduction

Motivated by the desire to study severe fading in MIMO channels and the results in section III, we seek a model for the elements of the channel matrix which is simple, analytically tractable and covers the full range of cases in the region $\text{CV} > \text{CV}_{\text{Rayleigh}}$. Such a model is the complex multivariate t (CMT) distribution [9], [10]. The CMT is one of the family of elliptical multivariate distributions [9] and hence a wide body of knowedge is available. Also, the CV of the amplitudes resulting from the use of the CMT satisfies the requirement that $\text{CV}_{\text{CMT}} > \text{CV}_{\text{Rayleigh}}$. Furthermore, the marginal distributions for any channel matrix element are complex t distributed and this includes the baseline case of complex Gaussian entries as a special case. Hence the CMT model generalizes the classical complex Gaussian matrix to the severe fading scenario.

The simplest description of the CMT channel matrix, $\mathbf{H}_{\text{CMT}}$, is to express it as the ratio of an iid complex Gaussian matrix, $\mathbf{H}$ and the square root of an independent scaled χ^2 variable [9], [10]. This formulation is:

$$\mathbf{H}_{\text{CMT}} = \frac{\mathbf{H}}{\sqrt{\frac{S^2}{r-1}}} \tag{4}$$

where S^2 has a complex χ^2 distribution with r degrees of freedom and probability density function $f_{S^2}(x) = x^{r-1}\exp(-x)/\Gamma(r)$, where $r > 1$, $x > 0$ and $\Gamma(\cdot)$ is the

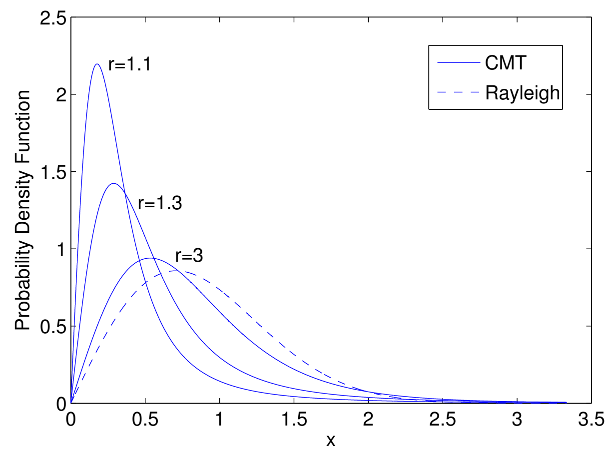

Fig. 3. A comparison of the amplitude distributions for the CMT model (r=1.1, 1.3 and 3) with the Rayleigh case.

gamma function. Note that the division of the iid matrix $\mathbf{H}$ by a single, common random variable results in dependent entries for $\mathbf{H}_{\text{CMT}}$. However, the independence of the $\mathbf{H}$ matrix preserves the uncorrelated nature of the entries. With this definition, the elements of $\mathbf{H}_{\text{CMT}}$ are uncorrelated, identically distributed, complex t variables with uniform phase over $[0, 2\pi]$. When $r \to \infty$, the model collapses to the baseline iid complex Gaussian matrix and the elements of $\mathbf{H}_{\text{CMT}}$ have amplitudes with CV equal to $\text{CV}_{\text{Rayleigh}}$. For finite r, the amplitudes are longer tailed than the Rayleigh case and give severe fading scenarios with a CV easily derived as

$$CV_{CMT} = \sqrt{\frac{\Gamma(r)^2}{\Gamma(r-1/2)^2\Gamma(3/2)^2(r-1)} - 1} \tag{5}$$

From (5) it is straightforward to show that $\text{CV}_{\text{CMT}} \to \text{CV}_{\text{Rayleigh}}$ as $r \to \infty$ and $\text{CV}_{\text{CMT}} \to \infty$ as $r \to 1$. The probability density functions for the amplitudes of the entries of the $\mathbf{H}_{\text{CMT}}$ matrix are shown in Fig. 3. Three densities are shown, corresponding to $r = 1.1$, $r = 1.3$ and $r = 3$. These are compared to the Rayleigh density and it can be seen that the severe fading cases put more of the probability close to zero, but balance this by a longer tail to keep the average power at unity. Note that the Nakagami distribution achieves larger CV values by the rather non-physical mechanism of placing a peak at the origin. Hence, the Nakagami distribution gives severe fading channels where amplitudes close to zero have the largest probability. This is in contrast to the CMT channel where the basic shape of the density is similar to the Rayleigh but pushed towards the origin.

Representations for $\mathbf{H}_{\text{CMT}}$, including the probability density function, can be obtained from the viewpoint of multivariate statistical theory [9], [10] but the representation in (4) is sufficient for our purposes and leads to a simple analytical method for computing both MI and system performance.

B. Mutual Information

Consider the link model (1) with the CMT channel matrix $\mathbf{H}_{\text{CMT}}$. We use the representation for the ergodic MI, $E(\mathcal{I})$, defined by [6]

$$E(\mathcal{I}) = E\left[m \log_2\left(1 + \frac{\rho}{n_T}\lambda_{CMT}\right)\right] \tag{6}$$

where λ_{CMT} is an arbitrary, non-zero eigenvalue of $\mathbf{H}_{CMT}\mathbf{H}_{CMT}^{\dagger}$ and $m = \min(n_R, n_T)$. Using (4), we can write (6) as

$$E(\mathcal{I}) = E\left[m \log_2\left(1 + \frac{\rho}{n_T}\frac{(r-1)\lambda}{S^2}\right)\right] \tag{7}$$

where λ is an arbitrary, non-zero eigenvalue of the usual Wishart matrix, $\mathbf{H}\mathbf{H}^{\dagger}$. The expectation in (7) is over both λ and S^2. Note that the effect of the CMT model is to leave the form of solution for an iid Rayleigh fading MIMO channel unchanged, but to replace the fixed SNR, ρ by a variable SNR, $\rho(r-1)/S^2$. This formulation allows us to derive the ergodic MI exactly, as shown below.

$$\begin{aligned} E(\mathcal{I}) &= E\left[m \log_2\left(1 + \delta\frac{\lambda}{S^2}\right)\right] \\ &= \frac{m}{\log 2}\int_0^\infty \log(1+\delta\, x)\, f_X(x)\, dx \end{aligned} \tag{8}$$

where δ is defined as $\rho(r-1)/n_T$ and $X = \lambda/S^2$ has density $f_X(x)$. Note that log, with no subscript, refers to the natural logarithm. We derive the density of X below.

Consider the pair of transformations $X = \lambda/S^2$, $Y = S^2$. Standard transformation theory for random variables gives:

$$f_{X,Y}(x,y) = f_{\lambda,S^2}(xy, y)\, J \tag{9}$$

where $J = \det\begin{bmatrix} y & x \\ 0 & 1\end{bmatrix}$. Hence

$$\begin{aligned} f_{X,Y}(x,y) &= f_\lambda(xy)\, f_{S^2}(y)\, y & (10)\\ &= \sum_{j=n-m}^{n+m-2} \frac{c_j}{\Gamma(r)} x^j y^{r+j} e^{-(1+x)y}\, dy & (11)\end{aligned}$$

where we have used the pdf for λ given in [11] and $n = \max(n_R, n_T)$. Hence, $f_X(x)$ is evaluated as

$$\begin{aligned} f_X(x) &= \sum_{j=n-m}^{n+m-2}\left(\frac{c_j}{\Gamma(r)}x^j\right)\int_0^\infty y^{r+j}e^{-(1+x)y}\, dy \\ &= \sum_{j=n-m}^{n+m-2}\frac{c_j x^j}{\Gamma(r)}\frac{\Gamma(r+j+1)}{(1+x)^{r+j+1}} \end{aligned} \tag{12}$$

Substituting (12) in (8), $E(\mathcal{I})$ is written as

$$E(\mathcal{I}) = \frac{m}{\log 2}\sum_{j=n-m}^{n+m-2}\frac{c_j\Gamma(r+j+1)}{\Gamma(r)}\int_0^\infty \frac{\log(1+\delta\, x)\, x^j}{(1+x)^{r+j+1}}\, dx \tag{13}$$

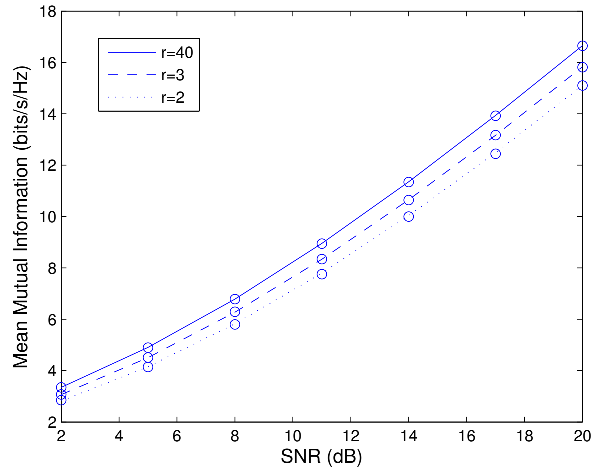

Fig. 4. Ergodic Mutual Information vs SNR for a (3,3) MIMO system with the CMT channel and $r = 2, 3, 40$. Lines represent analytical results and circles denote simulations.

where c_j is defined as [11]

$$c_j = \frac{1}{m}\sum_{l=\lceil \frac{i}{2}\rceil}^{m-1}\sum_{k=l}^{m-1}\frac{(-1)^{j+m-n}(2l)!\binom{2k-2l}{k-l}\binom{2n-2m+2l}{2l-j-m+n}}{2^{2k-j-m+n}l!(j+m-n)!(n-m+l)!} \tag{14}$$

and $i = j + m - n$ with $\lceil\ \rceil$ representing the ceiling function.

After evaluating the integral in (13), a closed-form expression for the MI of the CMT channel is derived as

$$\begin{aligned} E(\mathcal{I}) &= \frac{m}{\Gamma(r)\log 2}\sum_{j=n-m}^{n+m-2} c_j\Gamma(r+j+1)\times \\ &\quad \sum_{j}^{i=0}\binom{j}{i}(-1)^{j-i}\frac{1}{(r+j-i)^2}\times \\ &\quad {}_2F_1\left(1, r+j-i, r+j-i+1, -\frac{1}{\theta}\right) \end{aligned} \tag{15}$$

where ${}_2F_1$ is a Gauss hypergeometric function and θ is given by $\theta = \delta/(1-\delta)$. Note that when r is an integer, (15) can be further simplified since ${}_2F_1$ can be written in closed form. In Fig. 4, the ergodic MI is computed via (15) for a $(3, 3)$ MIMO system in a CMT channel with $r = 2, 3, 40$ and a range of SNR values. The analytical results are verified by simulation. The ergodic MI is shown to increase with r, i.e., as the CMT channel approaches a Rayleigh channel, which matches the results in Figs. 1 and 2.

C. Mutual Information Outage

For a fixed value of S^2, the MI is known to be well-approximated by a Gaussian random variable [12], [13]. Hence, we can approximate the MI outage of the CMT model by

$$P(\mathcal{I} < x) = \int_0^\infty \Phi\left(\frac{x - \mu_{\mathcal{I}}(y)}{\sigma_{\mathcal{I}}(y)}\right) f_{S^2}(y)dy \tag{16}$$

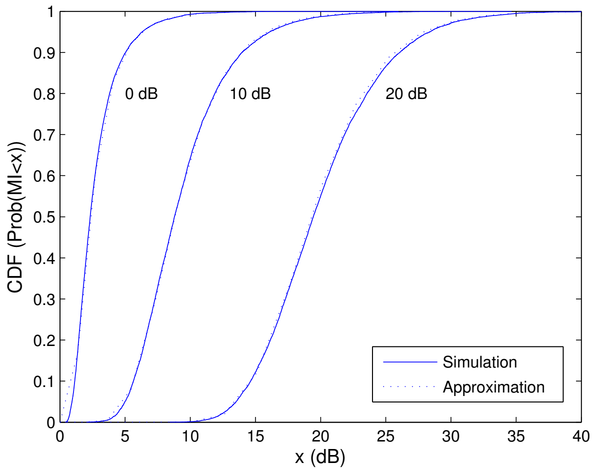

Fig. 5. CDF for the MI of a (4,4) MIMO system in a CMT channel with r=2 and SNR=0,10,20 dB.

where $\mu_{\mathcal{I}}(y)$ and $\sigma_{\mathcal{I}}(y)$ are the mean and standard deviation of the MI respectively, for a fixed SNR denoted $P=\rho(r-1)/y$. In (16), the function Φ is the cumulative distribution function of the standard Gaussian distribution. It is possible to use the exact mean and variance results for $\mathcal{I}$ found in [6], [12], but for simplicity we prefer to use the more compact asymptotic results [11], [14], obtained for the case where $n_T \to \infty$ and $n_T/n_R \to \beta$. The variance result is

$$\sigma^2_{\mathcal{I}}(y) = -\log\left(1-\frac{(1-\eta)^2}{\beta}\right) \tag{17}$$

where $\beta=\frac{n_T}{n_R}$, $\eta = 1-\frac{F(\gamma,\beta)}{4\gamma}$, $\gamma=\frac{Pn_R}{n_T}$ and

$$F(\gamma,\beta) = (\sqrt{\gamma(1+\sqrt{\beta})^2+1}-\sqrt{\gamma(1-\sqrt{\beta})^2+1})^2$$

The result for the mean is $\mu_{\mathcal{I}}(y)=mv$ where

$$\begin{aligned} v &= \beta\log\left(1+\gamma-\frac{F(\gamma,\beta))}{4}\right)+\log\left(1+\gamma\beta-\frac{F(\gamma,\beta)}{4}\right) \\ &- \frac{\log_e}{4\gamma}F(\gamma,\beta) \end{aligned} \tag{18}$$

In Fig. 5, the cumulative distribution function (CDF) of the MI is computed via (16) for a $(4,4)$ MIMO system in a CMT channel with $r=2$ and $SNR=0,10,20$dB. The analytical approximation is shown by simulation to be very accurate for all SNR values. Figures 4 and 5 demonstrate that the CMT channel leads to compact, closed form expressions for both the ergodic MI and outage probabilities. In addition, from (5), the CMT channel has a CV in the desired region, $\text{CV}_{\text{Rayleigh}} < \text{CV}_{\text{CMT}} < \infty$. Furthermore, as discussed in section IV, the CMT model can lead to a range of analytical results, including performance analysis, in addition to the work on MI described in section III.

IV. Applications to Cellular MIMO

A. Mutual Information

The approach taken in Section III is to average the well known results for an iid Rayleigh fading channel over a variable SNR. This is exactly the scenario considered in [15] where a cellular MIMO system is investigated and the system equation is written as

$$\mathbf{r}=\sqrt{\Gamma}\mathbf{H}\mathbf{s}+\mathbf{n}. \tag{19}$$

In (19), $\mathbf{r}$, $\mathbf{H}$, $\mathbf{s}$ and $\mathbf{n}$ are defined as before and Γ is the SINR resulting from noise and inter-cellular interference. Note the equivalence to the system equation for a CMT channel which can be written

$$\mathbf{r}=\mathbf{H}_{\text{CMT}}\mathbf{s}+\mathbf{n}=\sqrt{\tfrac{r-1}{S^2}}\mathbf{H}\mathbf{s}+\mathbf{n}. \tag{20}$$

The distribution of Γ was simulated in [15] for various cellular scenarios, including the effects of lognormal shadowing, distance attenuation, cell size, frequence reuse, sectorization and random user locations. The scenarios considered in [15] are primarily defined by the number of sectors per cell and the frequency reuse. We consider 3, 6 or 12 sectors per cell, denoted S3, S6 and S12, and a reuse factor of 1 and 1/3, denoted F1 and F3 respectively. With F1, the same frequency is used in all sectors, whereas under F3, each cell uses 3 disjoint frequency bands. Full details of the other parameters used, such as cell size, path loss exponent, etc., can be found in [15]. The 6 cellular scenarios considered can be abbreviated by S3F1, S3F3, S6F1, S6F3, S12F1 and S12F3.

For each of the 6 scenarios, we have replicated the results in [15] and have found that the distribution of Γ is well approximated by the scaling variable in the CMT model, namely a variable of the form, $(r-1)/S^2$, a scaled inverse χ^2 variable. For increased accuracy in fitting we allow the variable S^2 to have fractional degrees of freedom. Hence, we are able to quantify the behaviour of cellular MIMO systems in terms of ergodic MI and MI outage by a direct application of the results in section III. Sample results are given in Fig. 6, which shows the ergodic MI for the 6 scenarios as computed via (15) and also from the cellular simulations. Clearly, the analytical results are very accurate in all cases.

B. System Performance

The CMT model can be considered as a classic Rayleigh fading channel with variable SNR. This makes a wide range of results available. For example, consider a MIMO receiver with outage probability $P_{\text{out}}(\text{SNR})$ and bit error rate, $P_{\text{e}}(\text{SNR})$ in an iid Rayleigh channel. Both metrics are functions of the SNR and we obtain the corresponding results for the CMT channel by averaging the classical results over the inverse χ^2 scaling variable. Hence, we have:

$$P_{\text{out,CMT}}(\text{SNR}) = E\left(P_{\text{out}}(\text{SNR})\right) \tag{21}$$

and

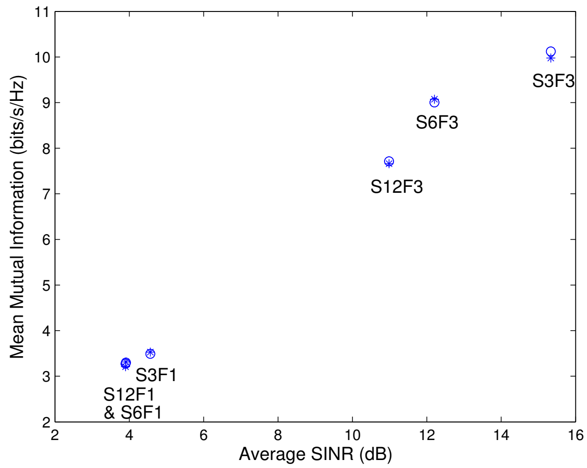

Fig. 6. Ergodic MI vs average SNR for a (2,2) MIMO system and 6 cellular scenarios. Stars represent the cellular simulations and circles denote the CMT approximation.

$$P_{\mathrm{e,CMT}}(\mathrm{SNR}) = E\left(P_{\mathrm{e}}(\mathrm{SNR})\right) \tag{22}$$

where SNR $= \rho(r-1)/S^2$, and the expectation is over the variable S^2. For reasons of space, results are not presented here, but a preliminary analysis shows that several systems can be handled in this way, e.g., linear MIMO receivers using zero-forcing or minimum-mean-squared-error (MMSE) combiners. In section IV, we have shown that the CMT model can be applied to cellular MIMO systems and allows a closed form evaluation of both MI and performance.

V. Conclusion

Fading severity is often measured by the CV of the amplitudes and we have shown that for a fixed CV, the ergodic MI of a MIMO channel is relatively insensitive to the exact type of amplitude distribution. This conclusion is especially valid when the fading is not too severe or at high SNR. In the severe fading region, i.e., $\mathrm{CV} > \mathrm{CV}_{\mathrm{Rayleigh}}$, we have proposed a novel model which is simple and analytically tractable, based on the complex multivariate t distribution. For this model, exact ergodic MI results are derived as well as approximations to the MI outage. The model has applications to wireless sensors, RF tagging, land-mobile, indoor-mobile and ionospheric radio links in addition to cellular MIMO systems where we have shown that the model fits simulated cellular scenarios very well. Furthermore, the model has obvious extensions to MIMO performance analysis through the simple approach of averaging well-known iid Rayleigh fading results over the inverse χ^2 scaling variable.

Acknowledgment

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References

[1] A. J. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, UK: Cambridge University Press, 2003

[2] D. Kim and M.A. Ingram, "Measurements of small-scale fading and path loss for long range RF tags," *IEEE Trans. Antennas Propagat.*, vol. 51, pp. 1740-1749, Aug. 2003.

[3] J. Galbreath and J. Frolik, "Channel allocation strategies for wireless sensors statically deployed in multipath environments," *Proc. IPSN,* pp. 334-341, 2006.

[4] H. Bai, H. Aerospace and M. Atiquizzaman, "Error modeling scheme for fading channels in wireless communications: A survey," *IEEE Commun. Surveys*, vol. 5, no.2, Fourth quarter, 2003.

[5] G.J. Foschini and M.J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol.6, no.3, pp. 311-335, Mar. 1998.

[6] I.E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, vol.10, no.6, pp. 586-595, Nov./Dec. 1999.

[7] R. U. Nabar, *Signaling for General MIMO Channels*, PhD Thesis, Stanford University, Feb. 2003.

[8] N. L. Johnson, S. Kotz and N. Balakrishnan *Continuous Univariate Distributions, Volume 1/2*, 2nd ed. New York, USA: John Wiley & Sons, 1994/1995.

[9] T. W. Anderson, *An Introduction to Multivariate Statistical Analysis*, 2nd ed. New York, USA: John Wiley & Sons, 1984.

[10] A. Dogandzic, A. Nehorai, and J. Wang, "Maximum likelihood estimation of compound-Gaussian clutter and target parameters," *Proc. Annual Workshop on Adaptive Sensor Array Processing*, Lincoln Laboratory, Lexington, MA, Mar. 2004.

[11] H. Shin and J. H. Lee, "On the capacity of MIMO wireless channels," *IEICE Trans. Commun.*, vol. E87-B, no. 3, pp. 671-677, Mar. 2004.

[12] P. J. Smith and M. Shafi, "An approximate capacity distribution for MIMO systems," *IEEE Trans. Commun.*, vol. 52, no. 6, pp. 887-890, 2004.

[13] Z. Wang, G. B. Giannakis, "Outage mutual information of space-time MIMO channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 4, pp. 657-662, 2004.

[14] A. M. Tulino, S. Verdu, "Asymptotic Outage Capacity of Multiantenna Channels," *IEEE Int. Conf. Acoustics, Speech Sig. Proc.*, Philadelphia, Mar. 2005.

[15] H. Huang and R. A. Valenzuela, "Fundamental Simulated Performance of Downlink Fixed Wireless Cellular Networks with Multiple Antennas," *IEEE PIMRC*, vol. 1, pp. 161- 165, Sept. 2005.