World Academy of Science, Engineering and Technology
International Journal of Cognitive and Language Sciences
Vol:11, No:11, 2017

Sfard's Commognitive Framework as a Method of Discourse Analysis in Mathematics

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***Abstract*—**This paper discusses Sfard's commognitive approach and provides an empirical study as an example to illustrate the theory as method. Traditionally, research in mathematics education focused on the acquisition of mathematical knowledge and the didactic process of knowledge transfer. Through attending to a distinctive form of language in mathematics, as well as mathematics as a discursive subject, alternative views of making meaning in mathematics have emerged; these views are therefore "critical," as in critical discourse analysis. The commognitive discourse analysis method has the potential to bring more clarity to our understanding of students' mathematical thinking and the process through which students are socialized into school mathematics.

***Keywords*—**Commognitive framework, discourse analysis, mathematical discourse, mathematics education.

I. Introduction

WHY and how do students use and process mathematical thinking? Most research in the field of mathematics education attempts to answer this question and dwells on student struggles associated with the process of mathematical thinking and of building mathematical concepts. Regarding student struggles in mathematics, the acquisition metaphor [1] is useful to describe the cognitive aspect of teaching and learning in which it is assumed that students acquire knowledge transferred from the teacher. This is in contrast to the participation metaphor [1] in which the focus shifts to the evolving relationships between the individual (the student) and others (the teacher and peers), and meta-discursive rules in the relationships. More importantly, the participation metaphor frames learning as a process of becoming a member of a community and attends to communication within the community as a function of meaning-making, understanding, and discourse including verbal and non-verbal activities.

We believe that the participation metaphor better explains student struggles and misunderstandings in the process of mathematical thinking and problem solving. In the acquisition framework student misunderstandings are the error to correct, but in the participation framework student misunderstandings are a natural part of participating in a community of subjective views; hence, the opportunity to resolve different understandings and build new and improved concepts through discourse.

Sfard's commognitive framework [2] conceptualizes mathematics as a discourse—a form of communication consisting mainly of its word use, visual mediators, routines, and narratives. Her theory affords researchers a theoretical basis from which to engage in discourse analysis of the mathematics classroom, especially how individuals (student-student and student-teacher) engage in discourse. As the theory encapsulates both cognition and communication, a discourse analysis based on the framework can explain the relationship between interpersonal communication and the cognitive process and how teachers and students move towards a meaningful discourse through participation. Relatively new, however, is the method of mathematical discourse analysis based on the commognitive approach. There have been few studies that illustrate a commognitive discourse analysis in mathematics. Sfard's framework needs more literature on the research methods with clear procedures and follow-up studies that validate the methods or propose alternative research designs.

In light of the need for the application of Sfard's commognitive framework, we develop our view on conducting a commognitive discourse analysis in the research of mathematics education, and more specifically, discuss how to collect and analyze data. This paper aims to contribute to the existing body of literature by developing a research design along with our views related to the assumptions, procedures, and challenges faced when implementing a commognitive discourse analysis.

II. Theoretical Background

A. Mathematical Discourse

Vygotsky [3] viewed that language serves as the instrument to develop thought, and Wittgenstein [4] argued that people use language not only to reflect the world in words but also to create meanings through language with logical structures. Building from these two theorists, it can be argued that learning concepts in mathematics is facilitated by language and the meanings people create in discourse. Sfard's commognitive approach is useful in theorizing the learning of mathematics as discursive activities. This approach frames learning as a unified body of cognition and communication. In that sense, the aforesaid language shall be the language of the objectified mathematics as well as the language of individuals participating in mathematical discourse. The meanings, through language with logical structures, then emerge as intersubjectivity towards mathematics produced in discourse. This view also explains the definitional change of discourse

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quite well in the field of mathematics education: from discourse as talk and text, human interaction through verbal and non-verbal language, the whole communicative event [5], to ultimately Gee's big-D-discourse [6], as the way of using language and other artifacts to identify oneself as a member of a social group where beliefs and actions are an essential part of social participation.

B. Commognitive Approach

In the commognitive approach, the classroom community, the individual, and the dialogic interactions constitute the discourse. First, students become aware of the conventions and norms in the mathematics classroom discourse and acquire mathematical practice including belief, value, autonomy, and disposition through a process of socialization into school mathematics. In this way, the communication in the math classroom has the reflective relationship between individual students and the discursive practices of a classroom community. Discerning mathematical discourse from others, the commognitive approach involves constructs such as word use, visual mediators, narratives, and routines. *Word use* refers to the way students use vocabulary in discourse – for example, ¼ is said as one fourth and words like triangle, function, or differentiation have mathematical meaning with specific properties. *Visual mediators* refer to mathematical objects including symbols, diagrams, and graphs such as ¼, x^2, dy/dx, $\ll$, which are used for mathematical communication. *Routines* refer to various metarules which regulate participants' actions in discourse. For example, students create algebraic equations from word problems; students simplify expressions whenever possible; students know when to apply a theorem and when to prove a theorem; or graphs of a function are assumed accurate although not drawn to scale. *Endorsed narratives* refer to spoken or written mathematical statements such as $2 + 2 = 4$, $(x^2)' = 2x$, or "the sum of the measures of the interior angles of a triangle equals 180º" to which participants can argue for truth or falsity through word use, mediators, and routines.

C. Commognitive Approach as Method

In principle, discourse analysis identifies the interplay between language use and the socialization process. Concerning the notion about interplay between the language use, meaning-making, and participation in the mathematics classroom community, a discourse analysis based on commognitive approach can afford findings which are not developed in other methods. First, the analysis shows the significant role of language use in not only cognition but also meta-cognition for mathematics learning by revealing different uses of the same word among students that impact on meta-discursive rules. Second, the analysis can look into the continuum of mathematical thinking. Related, participants' interactions in discourse offer authentic opportunities to connect theory and practice. Third, analysis enables the study of routines or, mathematical norms associated with specific mathematical objects, which are not readily accessible through an analysis of student cognition. Lastly, analysis affords the opportunity to consider a broader picture of thinking and learning since contexts and cultures are important elements of language and socialization.

III. Commognitive Discourse Analysis in Mathematics

We identify four important steps in a research method based on a commognitive approach: (1) preparing for interview, (2) collecting data, (3) transcribing data, and (4) analyzing data. For illustration, we provide sample data and related instruments from published studies in which commognitive discourse analysis was the primary analytical framework.

A. Step 1: Preparing for Interview

It is important to develop an instrument (e.g., tasks, questions) that enables participants to engage in discourse (e.g., interviews, discussions) and creates data with the potential to answer research questions. The instrument should involve *mathematics* with which the participants will have the opportunity to demonstrate word use, visual mediator, routines, and narratives (Table I). Discourse through interviews should be designed to articulate the process of student thinking. For example, a researcher-student(s) interview is appropriate when the interview collects data on power relations or the locus of knowledge in discourse. A student-student interview can be more appropriate when the interview focuses on transmission of evolving ideas, or lack thereof, and possibilities for new meaning-making through peer interactions. Related, prompts for student responses should be designed to best elicit a student's own thinking.

TABLE I
Example of Interview Tasks that Involve Mathematics [7]

	Questions/Tasks
1.	Make a sentence using the following words: "limitless," "endlessly," "of limit"
2.	A student argues that a sequence $a_1, a_2, a_3, \ldots$ goes to infinity. What would he mean?
3.	Decide which one has more numbers than others? Explain your reasoning. (1) odd numbers vs. even numbers (2) odd numbers vs. integers
4.	Which set has more elements than the other? Explain your reasoning. A = {1, 2, 3, 4, 5, . . . } B = {2, 4, 6, 8, 10, . . . }
5.	Given two infinite sets, A and B, a student argues the set A is larger than the set B. What would the student mean?

The participants' demographic data should be collected prior to the data collection stage so that the researcher can design the instrument and interview protocols in a way that engages participants in discourse so that they produce data to serve the study without bias. In that regard, wording of the questions should enable the participants to share and participate, rather than become confused or intimidated, or to feel forced; alternative versions of prompts should be prepared in case participants prefer different tasks or if they feel different interview questions would allow for better communication. The timing and environment (e.g. seating, room temperature, space) for interviews need to be part of the planning, and a decision regarding the format of the interview - structured, semi-structured, or unstructured - should be made carefully, depending on the nature of the questions and tasks, as well as the qualification of the interviewers.

We recommend providing scripted information for interviewees to include: context and purpose of research, overview of procedures, method(s) of recording, and common Q & A; this should also include ensuring compliance with all ethical standards (e.g., informed consent, voluntary participation and withdrawal, data use agreement) and institutional policies/regulations on research as stipulated by an IRB.

TABLE II
SELECTING INTERVIEWEES FROM REPRESENTATIVE GROUPS [8]

Algebraic thinking assessment		
High algebraic thinking High geometric thinking (Two interviewees)	Low algebraic thinking High geometric thinking (Two interviewees)	Geometric thinking assessment
High algebraic thinking Low geometric thinking (Two interviewees)	Low algebraic thinking Low geometric thinking (Two interviewees)	

When selecting interviewees, instead of random sampling, the researcher should consider attributes such as gender, nationality, ethnicity, age, socioeconomic status, or academic levels that qualify an interviewee as a representative sample of the participants who are of interest in the study. For example: a study investigating student discourse on algebraic thinking versus geometric thinking could group participants by ability based on the use of a math test so that four groups are formulated by student performance (Table II). In this case, with a number of students in each representative category, the researcher may randomly select students in each group and decide on the number of selected students in order to justify findings that are common to the selected participants, and eventually support the conclusion of the research.

B. Step 2: Collecting Data

It is important to share the protocol(s) for an interview with participants prior to the interview. For example, the researcher should make sure participants know that they are not being evaluated during the interview. Other norms during the interview include the participants should speak loudly enough so that audio is captured clearly, the participants should be comfortable to ask clarifying questions about tasks and prompts, and the interviewer should take the liberty to ask probing questions.

The essence of an interview is to elicit authentic student thinking and capture related commognitive processes. In this regard, open-ended questions (Table III) should be the primary mode of prompting student responses.

Videotaping interactions in the discourse is another important part of data collection. To capture classroom interactions, we recommend two cameras be used – one records the class from the back of the classroom so that the teacher's actions and the board are well captured, and another records the class from the front so that student interactions are captured. To capture interactions during an interview (Fig. 1), one camera can capture the interviewees, while another camera captures how participants complete the instrument. It helps to do frequent checks of the camera batteries and to use a microphone to capture clear audio. Another suggestion, run audio-taping along with video recording in case of camera malfunction.

TABLE III
EXAMPLES OF OPEN-ENDED QUESTIONS VS. CLOSED ONES [9], [10]

Open-ended questioning	Close-ended questioning
113 ME 2-2: The angle AOP is congruent to the angle COP (…) therefore the measure of angle POQ is sixty degrees. (…) 117 T: Yes, feel free to ask questions. 118 S2-5: Did you get the angle circle and the angle x separately? (…) 125 T: Why was this not possible? (…) 129 T: Why is this possible, can anyone explain it?	A95 T: So it is going to look like a concave up curve, right? A96 S: Yes. A97 T: Then let's verify that. Let me draw it. This is a graph of $y = x^2$. Isn't this a parabola with concave up? A98 S: Yes. A99 T: This is $y = 2x^2$. This one is also concave up. Some of you were talking about when plotting. How is the shape of $2x^2$ getting closer to the y-axis than the shape of x^2? In other words, the graph is narrower, isn't it? A100 S: Yes A101 T: Absolutely we can verify it this way.

Fig. 1 Videotaping participants engaging in discourse and completing an instrument

C. Step 3: Transcribing Data

Transcribed data should capture interactions among participants. The data should indicate the sequence of turns in the conversation, speaker, spoken language, and non-verbal language (gestures, facial expressions and other body languages). Ordering each turn in the conversation is important because the numbers can serve as a reference point in the discussion of data (Table IV) and the numbers can indicate the frequency of interactions in the discourse. Developing an efficient code for each speaker is an important part of coding data. Table IV discusses a discourse on conic sections. In the data, students are indicated by the letter A and B, H indicates hyperbola, and E indicates ellipse. In this way, HA and HB represents the speakers in a discourse on the problem solving of Hyperbola, and the discourse participants in the problem solving of Ellipse can be coded as EA, EB and EC.

Transcribed data also show nonverbal communication, including silence during the conversation (Table V). It is important to reflect the discourse "as is" and avoid presenting researcher interpretation. For example, when a speaker nods his or her head a few times, the data should indicate "head nods 3 times" as opposed to "showing agreement." Since it is common to analyze discursive patterns on the same question/task across participants, it is important to add an additional code to indicate a question or task number. After a transcription is completed, we recommend the researcher play a videotaped discourse against coded data and check for accuracy.

TABLE IV
ORDERING OF THE TURN IN DISCOURSE AS REFERENCE POINT [11]

… this problem is finding an equation that is tangent to $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and perpendicular to $y = x + 2$. Student HB couldn't solve the problem but began to ask the meaning of math terms and formulas in the intellectually safe classroom environment (see [a turn reference], [a turn reference]) …

TABLE V
AN ENGLISH SPEAKING PAIR COMPARES ODDS WITH EVENS [12]

Turn	Speaker	What is said	What is done
38	I	What about, you know, odd number and even number?	Showing the card "A: Odd numbers B: Even numbers"
39	E1	We do technically (…) I'd believe it'd be even because the numbers go on for, you know, infinity, and there are (…) they're infinite numbers (…) so every time you go up one, go down one, etcetera. It's either an odd number or even number and it's never ending. So, I'd say that it would be even because we know that there won't be one more odd number than one more even number.	
40	E2	Yeah, that's pretty much it, that's what I put too, I think.	Turing his head to E1
41	I	You first responded, you know (…)	Looking to E1 and smiling
42	E2	Oh, I put even numbers I think. Yeah.	Looking at his questionnaire
43	I	Right	
44	E2	I don't know what I think (…) I was just trying to figure out the answer so I put even numbers because, uh, this doesn't even make sense now that I look at it again. So, it was just, kind of, putting something down, but, I mean, I couldn't think of a way that you could have more even numbers or more odd numbers. So, I think there's the same too, I just thought we had to check a box, so I checked a box.	Shrugging his shoulders when saying "I couldn't"
45	I	But then first, you know, when you checked even numbers, what was your reasoning?	
46	E2	Uh, just kind of, um, zero is an even number. And so, you start with zero, and then, for some reason I was thinking, the numbers, it just seems like zero is before all the numbers and so there would be more even numbers, but that's not right (…) I don't think.	Writing a zero on paper

I = interviewer

D.Step 4: Analyzing Data

The researcher should have an analytical framework that is aligned with research purposes and questions. Sfard's commognitive framework is a useful framework for mathematical discourse analysis. Using a study [12] on college students' use and process of the mathematical words *infinity* and *limit* in two languages, we explain how Sfard's commognitive framework is applied for the analysis.

1. Word Use

Our research was interested in how students use and process mathematical words such as *infinity* and *limit*. More specifically, we inquired into how students use these words depending on contexts (e.g., arguments, illustrations) and how students use the synonyms and antonyms to express meanings. During the interview, we asked participants to engage in tasks such as making sentences, identifying synonyms, verbalizing definitions, and using the words in explaining solutions. Our analysis suggested that Korean students use these two words (*infinity* and *limit*) as *object* and English speaking students use these two words as *process*.

2. Visual Mediators

Along with words, visual mediators serve as a medium for meaning-making. Our research was interested in how students use and process mediators associated with the concepts *infinity* and *limit*. We categorized the modes in the case of symbolic mediators such as tables, graphs, and algebraic expressions for *infinity* and *limit* as syntactic, concrete, or objectified. For each mode, participants had a mathematical task in order to engage in discourse. We found that English speaking students used the shape of graphs and geometric mediators such as asymptotes and slopes to explain that "*x* goes to positive infinity." In contrast, Korean students resorted to algebraic expressions and seemed to make a quick transition from the written phrases to a different mode for finding limits.

3. Endorsed Narratives

It is important to be aware that endorsed narratives can appear in students' responses while not being articulated explicitly. So we recommend the researcher design a mathematical task that affords the opportunity to make mathematical statements. Our research was interested in how students make mathematical arguments related to *infinity* and *limit* and posed a question about the limit of an infinite sequence (Table VI). The limit of the sequence as a number (15.1 % and 45.2 % in category 4 and category 5, respectively, in Table VI) dominated as the idea of limit among Korean students.

4. Routines

Individual students demonstrate patterns in engaging with mathematics. Given the opportunity to identify the pattern, the researcher should consider when and how. Students may show different routines *when* a graph is provided in the problem or not, or *when* a table is provided in the problem or not. Students show different routines with regard to the *way* they solve the problems. In our study, we provided two tasks of limit finding where a table was provided and where a function as expression was provided. Table VII shows a limit finding task with a table and the distribution of the ways (i.e., *how*) participants engaged in problem solving. English speaking students used routines based on the processual use of limit, and Korean students employed routines on the basis of the structural use of limit.

TABLE VI
DISTRIBUTION OF ENDORSED NARRATIVES [12]

Endorsed narratives	English speaking students		Korean students	
	Interview	Survey	Interview	Survey
1) The sequence is increasing/ decreasing (without the limit)	30 %	29.5 %	0 %	2.4 %
2) The sequence is getting close to (or approaches) 0.1	30 %	29.5 %	30 %	27.8 %
3) The sequence approaches 0.1 but never reach it	30 %	8.3 %	5 %	2.4 %
4) The sequence converges to or becomes 0.1	0 %	3.0 %	15 %	15.1 %
5) The limit of the sequence is 0.1	10 %	12.9 %	50 %	45.2 %
6) Other	0 %	10.6 %	0 %	4.0 %
7) No answer	0 %	6.1 %	0 %	3.2 %

TABLE VII
DISTRIBUTION OF ROUTINES [12]

Routines	English speaking students		Korean students	
	Interview	Survey	Interview	Survey
1) Describing a process by looking at patterns of numbers in the sequence	20 %	27.3 %	0 %	1.6 %
2) Describing a process by characterizing the given function	5 %	6.1 %	0 %	0 %
3) Finding the limit by looking at patterns of numbers in the sequence	70 %	50.0 %	25 %	44.4 %
4) Rationalization	0 %	1.5 %	60 %	41.3 %
5) l'Hôspital's rule	0 %	0 %	15 %	7.1 %
6) Unidentified	5 %	10.6 %	0 %	2.4 %
7) No answer	0 %	4.5 %	0 %	3.2 %

IV. CONCLUSION

This paper provides an empirical example of Sfard's commognitive approach as an analytical framework in mathematical discourse analysis in the field of mathematics education. We view commognitive discourse analysis as a viable and generative methodology for studying how students participate in the learning of mathematics because of these two concluding remarks.

First, mathematical discourse involves the use of language, and other semiotic resources, so Sfard's framework affords a unique theoretical, as well as analytical lens, to better understand how our students engage in mathematical thinking and reasoning as discourse. Commognitive discourse analysis is useful in examining the role of language and non-language elements, and how these two interact and become a discourse. More specifically, word use and endorsed narratives in discourse analysis can account for language-dependent elements in learning, and visual mediators play an important role as tools for communication. With an analysis of routines, researchers can have access to student thinking that is not so much strictly related to language. We provided a sample data and analysis in this paper that investigated how those constructs are interdependent among students in two particular speech communities and how they are similar or dissimilar from each other.

Second, discourse analysis in mathematics should consider interactions among students and between students and the teacher. Also there are other elements to analyze including interpersonal dynamics in learning environments and the norms of the community, including the ways students are encouraged to engage in conversations and express their thinking. We argue that the essence of discourse analysis, based on the commognitive approach lies in the analysis of student discourse that considers the variety of communicative mediums, verbal and nonverbal languages, that explores the past and the present meaning through participation, and that clarifies the interplay between texts, contexts, and culture in discourse.

ACKNOWLEDGEMENT

This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2017S1A2A2040608).

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