Article

Shadowed Type-2 Fuzzy Sets in Dynamic Parameter Adaption in Cuckoo Search and Flower Pollination Algorithms for Optimal Design of Fuzzy Fault-Tolerant Controllers

Himanshukumar R. Patel *[ID] and Vipul A. Shah

Department of Instrumentation and Control Engineering, Faculty of Technology, Dharmsinh Desai University, Nadiad 387001, Gujarat, India
* Correspondence: himanshupatel.ic@ddu.ac.in or himanshupatelp32@gmail.com; Tel.: +91-7698-308-667

Abstract: In recent, various metaheuristic algorithms have shown significant results in control engineering problems; moreover, fuzzy sets (FSs) and theories were frequently used for dynamic parameter adaption in metaheuristic algorithms. The primary reason for this is that fuzzy inference system (FISs) can be designed using human knowledge, allowing for intelligent dynamic adaptations of metaheuristic parameters. To accomplish these tasks, we proposed shadowed type-2 fuzzy inference systems (ST2FISs) for two metaheuristic algorithms, namely cuckoo search (CS) and flower pollination (FP). Furthermore, with the advent of shadowed type-2 fuzzy logic, the abilities of uncertainty handling offer an appealing improved performance for dynamic parameter adaptation in metaheuristic methods; moreover, the use of ST2FISs has been shown in recent works to provide better results than type-1 fuzzy inference systems (T1FISs). As a result, ST2FISs are proposed for adjusting the Lèvy flight (P) and switching probability (P') parameters in the original cuckoo search (CS) and flower pollination (FP) algorithms, respectively. Our approach investigated trapezoidal types of membership functions (MFs), such as ST2FSs. The proposed method was used to optimize the precursors and implications of a two-tank non-interacting conical frustum tank level (TTNCFTL) process using an interval type-2 fuzzy controller (IT2FLC). To ensure that the implementation is efficient compared with the original CS and FP algorithms, simulation results were obtained without and then with uncertainty in the main actuator (CV_1) and system component (leak) at the bottom of frustum tank two of the TTNCFLT process. In addition, the statistical z-test and non-parametric Friedman test are performed to analyze and deliver the findings for the best metaheuristic algorithm. The reported findings highlight the benefits of employing this approach over traditional general type-2 fuzzy inference systems since we get superior performance in the majority of cases while using minimal computational resources.

Keywords: cuckoo search algorithm; shadowed type-2 fuzzy logic systems; flower pollination algorithm; fault-tolerant controller; conical frustum tank; fuzzy logic control

1. Introduction

Optimization is a branch of study that uses mathematical modeling in a variety of subjects, including science, engineering, economics, and others [1–4]. In general, the goal is to find an amicable alternative to an objective function defined across a search space [5]. There are two types of optimization algorithms: deterministic and stochastic. Deterministic techniques usually struggle to solve optimization problems since they only provide a hypothetical guarantee of finding a local minimum for the objective function [6–8]. On the other hand, stochastic techniques are frequently faster at discovering a global optimum [3,7,8]. Moreover, with the exception of deterministic approaches, they are easily adaptable to black-box formulations and severely ill-behaved functions, whereas deterministic methods usually depend heavily on at least some theoretical assumptions about the problem formulation and its analytical properties (such as Lipschitz continuity) [9].

Fuzzy controllers are now optimized using metaheuristics, and these controllers must be optimized because they frequently fail to achieve the optimal performance required for practical systems. These controllers are known as type-1 fuzzy logic controllers (T1FLCs) since they use the literal definition of fuzzy sets (FSs) [10]. Lotfi Zadeh described fuzzy logic for the very first time in ref. [11], in which he tried to introduce the concepts of fuzzy sets and fuzzy logic. In this scenario, the members of a set are assigned a numerical value as metrics of the uncertainty in the so-called membership functions (MFs), which define the linguistic variables of a fuzzy set's membership functions. In addition to the original fuzzy logic (type-1), type-2 fuzzy logic was later developed with the objective of dealing with more difficult issues, that is, problems with a higher level of uncertainty, than type-1 fuzzy logic can solve [12–14].

Zadeh also invented type-2 fuzzy sets, which are an extension of traditional fuzzy sets (type-1). A type-2 fuzzy collection's membership degrees are also fuzzy. In this context, a type-1 fuzzy set is a subset of a type-2 fuzzy set because its secondary membership function is a single-element subset [15].

Since type-2 fuzzy logic systems are made up of type-1 fuzzy logic systems, their capacity to interact with uncertainty is enhanced. In refs. [16–23], the authors describe the use of type-2 fuzzy systems to solve a wide variety of control problems with excellent results. The hypothetical developments of interval type-2 fuzzy logic controllers (IT2FLCs), as well as what needs to be done in this field, are presented in Ref. [24]. The dichotomy between a type-1 and a type-2 fuzzy controller, on the other hand, is properly explained in ref. [25,26]. Furthermore, the limitations of type-1 fuzzy controllers (T1FLCs) in dealing with situations with escalating levels of uncertainty were addressed, as were the advantages of interval type-2 fuzzy controllers (IT2FLCs).

Metaheuristics have previously been used to optimize both type-1 and type-2 fuzzy systems; for example, the optimization of type-1 fuzzy controllers is discussed in refs. [27,28], which use the firefly method to optimize the fuzzy controllers of autonomous mobile robots. In Ref. [29], the flower pollination algorithm (FPA) and genetic algorithm (GA) were also used to optimize a fuzzy controller for an autonomous robot by following a trajectory, and the dynamic adaptation of its most critical parameters for the galactic swarm optimization (GSO) algorithm's operation is described. In Ref. [29], the GSO algorithm was also used to optimize the coupled-water-tank fuzzy controller.

Some studies optimized fuzzy controllers using alternative metaheuristics. The genetic algorithm, for example, is used in Ref. [30] to transform the architecture of a type-2 fuzzy controller in real-world autonomous mobile robots. Other authors, as described in Ref. [31], have used fuzzy controllers to control autonomous robots following a trajectory. There are even more fuzzy controller applications because of their performance and efficiency, as demonstrated by refs. [32–34], which also compare type-1 and type-2 fuzzy controllers, and Ref. [35], which uses two fuzzy controllers to control the liquid-level process in a tank. Furthermore, the GA and FP algorithms have been used in the optimization of the water tank fuzzy controller in Ref. [36]. Furthermore, in recent years, fuzzy conformable fractional differential equations with highly generalized differentiability have been used in Refs. [37–41], which suggest a solution for the non-linear system. In [42] author uses the Grasshopper Optimization Algorithm (GOA) for the optimization of the interval type-2 fuzzy logic system and fuzzy system applied to the Australian national electricity market data for the forecasting of electricity loads and prices.

There are numerous examples where the different metaheuristic algorithms were used to optimize the non-linear equations or systems widely used in the engineering and science fields, and such examples are briefly discussed in Refs. [43–46]. In Ref. [45], an improved cuckoo search (ICS) algorithm was applied to cyber-physical systems (CPS), and 34 common non-linear equations that fit the nature of cyber security models are adopted to show the efficiency of the ICS algorithm. The proportional–integral–derivative (PID)-based controller parameters tuned with four different metaheuristics (i.e., particle swarm optimization (PSO), genetic algorithm (GA), ant colony optimization (ACO), and cuckoo search (CS)

algorithms) for non-linear six-degrees-of-freedom (DOF) medium-scale rotorcraft systems found that the GA gives the optimum PID parameters to fit the fitness function [47]. In Ref. [48], the author proposed a new probability-based stochastic fractal search (SFS) optimization algorithm for a photo voltaic (PV) system to optimize the conventional maximum power point tracking (MPPT) method; in addition, the author compared the proposed method results with results of other existing optimization algorithms. The wind speed prediction for a wind turbine within a short time span used a feedforward (FF) neural network (NN), and its optimization was proposed by using the improved flower pollination algorithm in Ref. [49].

In [50] Linguistic Pythagorean Fuzzy Numbers (LPFNs) are used for developing decision-making approaches, in [51] author use q-rung orthopair hesitant fuzzy stochastic method based on regret theory to capture the psychological behavior of decision makers (DMs) in decision making. Recently, reinforcement learning (RL) has been very popular to optimize fuzzy controllers or fuzzy rule base. In [52], authors proposed a fuzzy reinforcement learning (FRL)-based controller that generates a stable control action by Lyapunov constraining fuzzy linguistic rules for benchmark Inverted Pendulum (IP) and Rotational/Translational Proof-Mass Actuator (RTAC) control problems (with and without disturbances). In [53], authors propose a Lyapunov theory-based linguistic RL framework for stable tracking control of robotic manipulators and employ Lyapunov theory to constrain fuzzy rule consequents for ensuring the stability of the designed controller. Furthermore, Kumar and Sharma presented fuzzy Q-learning-based control, Lyapunov Markov game-based controller, and Linguistic Lyapunov RL controller for the non-linear system, such as a robotic manipulator (i.e., two-link arm manipulator, SCARA robot arm), and authors found that deterministic Fuzzy Q-Learning is not an efficient approach, especially in dealing with highly coupled non-linear systems, such as robotic manipulators and, hence, in [54,55] authors proposed the solutions for non-linear control problems (i.e., two-link robotic arm/manipulator, SCARA arm, Inverted Pendulum) and suggested metaheuristic algorithm based fuzzy Q-learning based control approach to give stable controller. As a result, we show the fuzzy logic system that was used to implement the metaheuristic algorithm's dynamic parameter adaptation and obtain an optimal fuzzy fault-tolerant controller for non-linear level control systems with uncertainties.

The main objective of this study is to provide an optimization technique that employs a fuzzy metaheuristic algorithm to achieve optimal performance in the control of a coupled frustum tank level control system with actuator and system component uncertainty. Because it has been demonstrated that using parameter adjustment in fuzzy metaheuristic algorithms for the optimization of non-linear control problems produces competitive results, we propose in this work that we use fuzzy logic to perform the dynamic adjustment of algorithm parameters and measure their performance in the optimization of the fuzzy controller of the coupled frustum tank level control.

The primary contributions are the proposed optimization methodology and identifying the optimal parameters for the optimization of an interval type-2 fuzzy controller. In this scenario, the control behavior of a non-linear level control system is studied, which attempts to follow a specific reference level with as little margin of error as possible, estimated using a predetermined metric. There is also a comparison of the optimization efficiency of two metaheuristic algorithms, the Cuckoo Search (CS) and Flower Pollination (FP) algorithms. Because CS and FP have previously been demonstrated to have high performance when they are applied to optimization problems in different engineering applications.

The following is the structure of this paper: the basics of the generalized and shadowed fuzzy sets are described in Section 2. The motivation and equations required by the CS and FP algorithms to execute their search and optimization are detailed in Section 3. The case study non-linear level control process with the mathematical model is described in Section 4. The proposed method for optimizing the interval type-2 fuzzy controller of the non-linear coupled frustum tank level process is provided in Section 5. The results of the

trials developed with the proposed approach for the frustum level control plant are shown in Section 6. Statistical discussion of the acquired results is offered in Section 7. Finally, conclusions and future work are described in Section 8.

2. General Type-2 Fuzzy Sets and Shadowed Sets

A 3D membership function $(\mu_{\tilde{A}}(x, u))$ represents a general type-2 fuzzy sets GT2 FS, which is described by a primary and secondary membership function and is represented in (1) [56,57].

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall u \subseteq [0, 1]\} \tag{1}$$

The set (2), which is constrained by a lower and upper membership function described in (3) and (4), respectively, defines the Footprint of Uncertainty (FOU) of $\tilde{A}$.

$$FOU(\tilde{A}) = \{(x, u) \in \{X \times [0, 1]\} \mid \mu_{\tilde{A}}(x, u) > 0\} \tag{2}$$

$$\bar{\mu}_{\tilde{A}}(x) = sup\{u \mid u \in [0, 1], \mu_{\tilde{A}}(x, u) > 0\} \tag{3}$$

$$\underline{\mu}_{\tilde{A}}(x) = inf\{u \mid u \in [0, 1], \mu_{\tilde{A}}(x, u) > 0\} \tag{4}$$

The primary membership function can be defined in (5), where $u = J_x \in \left[\bar{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x)\right]$. The secondary membership function of $\tilde{A}$ is indicated by (6), which is a vertical slice of $\mu_{\tilde{A}}(x, u)$, wherein the second membership function is a T1 FS on a specific value of x; hence, for every value of $x \in X$, there exists an embedded T1 membership function.

$$J_x = \{u \in [0, 1] \mid \mu_{\tilde{A}}(x, u) > 0\} \tag{5}$$

$$\mu_{\tilde{A}} : X \times [0, 1] \to [0, 1] \forall x \in X \tag{6}$$

General Type-2 Fuzzy Sets with Shadowed Sets

Nowadays, traditional type fuzzy sets have evolved to GT2 FS that allows not only a vagueness model but also an uncertainty modeling approach to be used. The mathematical expression of the GT2 FS is denoted in Equation (7):

$$\tilde{A} = \left\{\left((x, u), \mu_{\tilde{A}(x)}\right) \mid \forall x \in X, \forall u \in J_x^u \subseteq [0, 1]\right\} \tag{7}$$

There are several modeling options for the GT2 FS that have practical applications, including the vertical slices or z-slices portrayal [58], the geometric approximation [58], and the horizontal slices or α-planes representation [59]. Pedrycz [60] provided a developed approach for the application of a shadowing set as an approximation of a fuzzy set μ_A. We construct a shadow set from a fuzzy set described in $0 \le \beta \le \alpha$ for a pair of threshold (α, β) using $0 \le \beta \le \alpha$ presented in Equation (10). These α-planes are expressed by Equation (8) and can be computed as an IT2 FIS [59]. The GT2 FS is then modeled using the union of every α-plane, as shown in Equation (9):

$$\tilde{A}_\alpha = \{((x, u), \alpha) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1]\} \tag{8}$$

$$\tilde{A} = \bigcup \tilde{A}_\alpha \tag{9}$$

$$S_{\mu_A}(x) = \begin{cases} 1, & if \, \mu_A \ge \alpha \\ 0, & if \, \mu_A \le \alpha \\ [0, 1], & if \, \alpha \le \mu_A(x) \ge \beta) \end{cases} \tag{10}$$

A shadowed set can be interpreted in three different ways, according to the three-way decisions. They are the positive region, characterized by the membership degree of 1, the negative zone, defined by the membership degree of 0, and the shadow zone [25]. We used

a shadowed set to mimic a fuzzy set, as illustrated in Figure 1. Each of the three regions represents a change in the membership function. As a result, Pedrycz [60] proposed that the ideal pair of thresholds must meet this theory.

$$EA_{(\alpha,\beta)}(\mu_A) + RA_{(\alpha,\beta)}(\mu_A) = SA_{(\alpha,\beta)}(\mu_A) \tag{11}$$

EA is the elevation area, *RA* is the reduction area, and *SA* is the shadowing area.
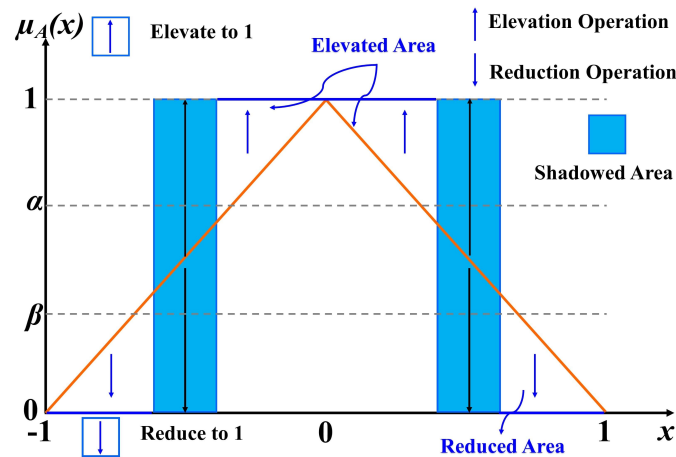

Figure 1. Characterization of shadowed set [25].

A shadowed set *S* in universe *U* is defined as a mapping from *U* to the set $0, [0,1], 1$. That is, $S : U \rightarrow 0, [0,1], 1$. The elements with a membership degree of 1 shape the core and the elements with a membership degree between 0 and 1 represent the shadow of *S*. For the application of a shadowed set as an approximation of a fuzzy set, a constructive method has been proposed by Pedrycz [60–62]. For a pair of threshold values (α, β) with $0 \leq \beta \leq \alpha$, we can design a shadowed set from a fuzzy set μ_A, which is defined in (10).

According to the three-way decision, a shadowed set can be interpreted by three regions: the positive region defined by the membership degree of 1, the negative region defined by the membership degree of 0 and the shadow region. We considered a shadowed set to approximate a fuzzy set that is illustrated in Figure 2. The three regions represent a change in the membership function. Based on this theory, Pedrycz [60–62] suggested that the optimal pair of the threshold values must satisfy the following expression (Equation (11)).

Linda and Manica proposed the use of shadowed sets in the secondary axis of the GT2 FIS in 2013 [60,63]. In this way, the optimized values of α and β are found and then these values are used as α-planes. Hence, the computational cost is reduced, and thereafter, the implementation of GT2 FIS for dynamic parameter adaption in metaheuristic algorithms is allowed.

The basic idea is to optimize a function $V(\alpha, \beta)$ that is defined in Equation (12) [25]. We take the optimized value for the $\alpha = 0.73$ and $\beta = 0.27$, which gives optimal value of trapezoidal membership function parameters.

The optimized values of the shadowed GT2 FS parameters $\alpha = 0.73$ and $\beta = 0.27$ are taken. Using GT2 FIS we proposed two fuzzy metaheuristic algorithms (cuckoo Search and flower pollination). We optimized type-2 fuzzy controller for two-tank non-interacting conical frustum tank level system, and found that the ST2 FS can outperform the ability of IT2 FS to model and system uncertainty with a better precision. Nevertheless, the computational cost of the GT2 FS is reduced by using ST2 FSs representation [64].

$$V(\alpha, \beta) = \left| \int_{x \in A_r} \mu_A(x)dx + \int_{x \in A_e} \mu_A(x)dx - \int_{x \in S} dx \right| \tag{12}$$

The ST2 FIS is based on the idea of Shadowed Fuzzy Sets and is an approximation of General Type-2 Fuzzy Inference Systems (GT2 FIS). The fundamental rationale for utilising ST2 FIS instead of GT2 FIS is that the computational cost of GT2 FIS is too high for this

application [65]. Hence, the ST2 FIS is the best choice: maintain the good results and have lesser computational burden. Recently, the authors have used ST2 FIS for different control applications and obtained significant results [66,67]. In [66] the authors proposed uncertainty handling capability for dynamic parameter adaption in Harmony Search (HS) and Differential Evolution (DE) algorithms and tested on DC motor position control system with noise in controller. The diagnosis problem in Artificial Neural Networks (ANNs), Support Vector Machines (SVMs), Fuzzy Inference Systems (FISs), Decision Threes (DTs), and hybrids of one or more these algorithms can be taken care by special approximation of the Type-2 Fuzzy Inference System, i.e., Shadowed Type-2 Fuzzy Inference System (ST2 FIS), as demonstrated in [67]. The other interesting and complex real-world problem like traffic management was addressed in [68] and real time traffic delay optimization (waiting time of traffic at a big junction/crossing) was done using ST2 FIS. The capability of the differential evolution algorithm with the utilization of shadowed and general type-2 fuzzy systems was presented in [69]. Comparative results were produced between shadowed and general type-2 fuzzy systems to one of the prime parameter adaption dynamically. It can be concluded that the ST2 FIS is better option with lesser computational cost, as compared to GT2 FIS.

3. Background of Optimization Algorithm and Mathematical Formulation

In a review of the literature, some studies focused on optimising membership functions with GSO, FA, FP, and GA were discovered, but none of these works used the experiments configurations in the same domain of engineering fields as are made available here. Since the combination of these metaheuristics is unusual in the literature, we will investigate the performance results of these methods for this type of optimization problem. In addition, correlative statistical results with analysis will be presented to assess the efficacy of the proposed IT2 FLC optimization method. Furthermore, the fault-tolerant control application for industrial processes using the proposed methodology is presented for the first time and produce promising results. This section presents the applicable theories and concepts for this research.

3.1. Cuckoo Search (CS) Algorithm

Cuckoo Search (CS), developed by Yang and Deb [70], is one of the most recent nature-inspired algorithms. The concept of CS is based on the brood parasitism of some cuckoo species. Furthermore, rather than simple isotropic random walks, this algorithm is enhanced by the so-called L∂vy flights [70]. Recent research indicates that CS has the potential to be far more efficient than Particle Swarm Optimization (PSO) and GA [71,72].

Cuckoo birds leave their eggs in the nests of other host birds (usually other species). They have incredible abilities, such as trying to pick nests with newly laid eggs and removing existing eggs to increase the hatchability of their own eggs. A few host birds can counteract the cuckoos' rapacious behavior by throwing out the identified alien eggs or constructing a new nest in a different place. The cuckoo breeding analogy was used to develop the CS algorithm, even though biological ecosystems are complex and most basic computer algorithms cannot precisely model them. Natural systems must be streamlined in order to be properly implemented in computer algorithms. Yang and Deb [72] simplified the cuckoo reproduction process into three idealized rules.

1. An egg, kept in a nest, represents a solution. An artificial cuckoo can lay only one egg at a time [70].
2. The cuckoo bird searches for the most suitable nest to lay its eggs in to maximize the survival rate of its eggs (solution). Because of an exclusivist selection strategy, only high-quality eggs (best solutions around the optimal value) that are more similar to the host bird's eggs have a chance to develop (next generation) and become mature cuckoos [70].
3. The population (number of host nests) remains constant. The host bird can find the alien egg with a probability of $p_a \in [0, 1]$ (worse solutions away from the optimal value), and these eggs are thrown away or the nest is neglected and a new nest is

established in a different location. Otherwise, the egg matures and lives to the next generation. Lèvy flights around the best current solutions aid in the selection of new eggs (solutions) laid by a cuckoo [70,72].

Using these three principles, the main steps of the Cuckoo Search (CS) can be represented as the pseudo-code provided in Algorithm 1.

Algorithm 1 Pseudo code for CS Algorithm [70,72]

Objective function $f(x) = x = (x_1, \cdots, x_d)^T$
Generate initial population of n host nests $x_i = (i = 1, 2, \cdots, n)$
while ($t <$ Max. Generation) or (Stop criterion) **do**
　　Get a cuckoo randomly by Lèvy flights
　　Evaluate the quality/fitness F_i
　　Choose a nest among n (say, j) randomly
　　if $F_i > F_j$ **then**
　　　　Replace j by the new solution
　　end if
　　A fraction (p_a) of worse nests are abandoned and new ones are built
　　Keep the best solutions (or nests with quality solutions)
　　Rank the solutions and find the current best
end while
Post-process results and visualization

When generating new solutions $x^{(t+1)}$ for, say, a cuckoo i, a Lèvy flight is performed

$$x_t^{t+1} = x_t^{(t)} + \alpha \oplus L\grave{e}vy(\lambda), \tag{13}$$

where $\alpha > 0$ signifies the step size that is connected to the measurements of the problem. In most cases, $\alpha = 1$ is sufficient. The above equation is the stochastic equation for the random walk. A random walk is a Markov chain in which the next status/position is solely determined by the current location (the first term in the above equation) and then changing and evolving (the second term) [71]. The term $\oplus$ relates to multiplications of entries one by one. This entry-wise product is similar to those used in PSO, but the random walk through Lèvy flight traverses the search space more efficiently in the long run due to its significantly longer step size. Although the random step length is drawn from an Lèvy distribution, the Lèvy flight produces a random walk [72].

$$L\grave{e}vy \sim u = t^{-\lambda}, (1 < \lambda \le 3), \tag{14}$$

It has an infinite mean and correspondingly infinite variance. The steps form a random walk process with a long tail and a power-law step-length distribution. Lèvy walk around the best solution acquired generates new solutions, speeding up the local search. A high proportion of the new solutions are generated using far-field randomization, with locations being sufficiently distant from the current best solution to prevent the system from being stuck in a local optimum [70].

3.2. Flower Pollination

The flower pollination algorithm (FPA) is a ground-breaking heuristic algorithm based on flower pollination behavior. In nature, pollination procedures for flowers are classified into two types: cross-pollination and self-pollination [73–75]. Cross-pollination takes place when certain birds act as global pollinators, transferring pollen between flowers on distant plants. In self-pollination, pollen is transported by wind and only takes place between neighborhood blooms on the same plant. As a result of transforming cross-pollination and self-pollination into global and local pollination operators, respectively, the FPA is formed.

Because of its simple concepts, few parameters, and ease of use, the FPA has received attention [74–78].

In nature, flower pollination promotes 'the survival of the fittest' and 'the optimum breeding of flowering plants'. Pollination in blooming plants can take two forms: biotic and abiotic [78,79]. Biological pollination is suitable for approximately 90% of flowering plants. Pollen is transported by pollinators, such as bees, birds, insects, or animals. Abiotic pollination, such as wind and water dispersal, accounts for approximately 10% of the remaining pollination. Pollination of plants can be accomplished through self-pollination or cross-pollination, as illustrated in Figure 2 [73,74]. The fertilization of one flower with pollen from another flower on the same plant (autogamy) is referred to as self-pollination (Geitonogamy). This takes place when a flower contains both male and female gametes. Self-pollination frequently occurs over short distances when pollinators are not present. It can be regarded as local pollination. Cross-pollination, also referred to as allogamy, occurs when pollen grains from one plant enter into the bloom of another. The stimulation of biotic or abiotic pollinators initiates the process. Biological cross-pollination can occur over long distances with biotic pollinators. Pollination is observed on a global scale. Bees and birds, as biotic pollinators, exhibit Lèvy flying behavior [77,78], with a leap or fly distance steps following an Lèvy distribution. Algorithm 2 presents the pseudo-code that can be used to synthesize Yang's FPA algorithm [78,79].


Algorithm 2 Pseudo code for FP Algorithm [73,74]

　　Initialize an objective as minimization
　　Define the population for n flower
　　Find current best solution f^* in the initial population
　　Describe the switch probability $p \in [0,1]$
　　while ($t <$ Max. Iteration) **do**
　　　　for $i = 1 : n$ **do**
　　　　　　if rand $< p$ **then**
　　　　　　　　Define step size P which follows Lèvy distribution
　　　　　　　　Use Equation (15) to perform global pollination
　　　　　　else
　　　　　　　　Define $\in$ for uniform distribution $[0,1]$
　　　　　　　　Randomly select a j and k among all the solution
　　　　　　　　Perform local pollination by Equation (16)
　　　　　　end if
　　　　　　Calculate new solution
　　　　　　If the calculated solution is better, then update the population
　　　　end for
　　　　Get the optimal solution f^*
　　end while
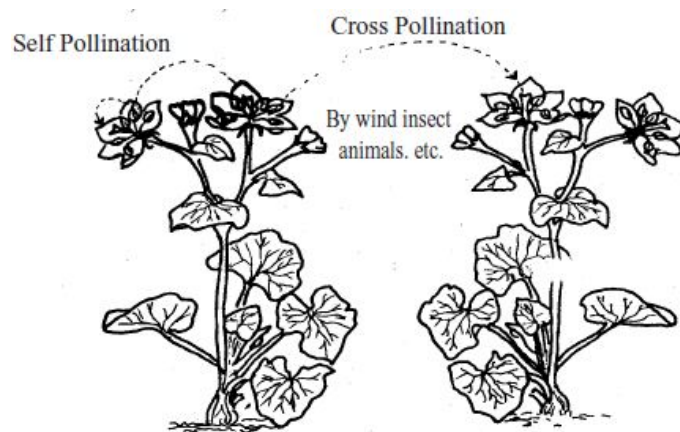
Figure 2. Flower pollination in nature [73,74].

3.3. Mathematical Modeling of FPA

Pollinators follow the rules based on the above characteristics of the pollination process [77]: when pollinators migrate pollen by conducting Lèvy flying with the process of global pollination, it is biotic and cross-pollination. The two types of local pollination are presumed to be abiotic and self-pollination. Pollinator consistency occurs when the identical characteristics of two flowers are proportionate to the likelihood of breeding. The transition probability $p \in [0, 1]$ is used to control the process of local and global pollination. The fundamental process of pollination is accelerated due to physical proximity and other factors, such as wind. The FPA optimization approach was developed based on the research on the aforementioned pollination process characteristics. As a result, the four principles listed above have been transformed into mathematical modeling equations. In the first step of global pollination, pollinators, such as flying insects, transport pollen gametes over long distances. Pollination and breeding of the fittest solution are defined as f^* in this procedure. Equation (15) can be used to show rule 1 and flower reliability [77].

$$y_i^{t+1} = y_i^t + P(y_i^t - f^*) \tag{15}$$

The initial value of y_i^t is selected as random value. y_i^t represents the pollen, i in Equation (15) or vector of solution y_i at generation t. The pattern f^* displays the current optimal solution after the current number of iterations. The pollination's durability is expressed by the element P, which is essentially the step size.

Because flying insects can travel long distances while transporting pollen, we can define their flight characteristics in terms of Lèvy flight. In other words, $0 > P$ is embellished from a Lèvy distribution [77].

$$P \sim \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \frac{1}{s^{1+\lambda}}, \ (s >> s_0 > 0). \tag{16}$$

where s represents the step size. Equation (16) expresses the classic gamma function as $\Gamma(\lambda)$, and this type of distribution is applicable for large steps $0 > s$. The accepted value of λ is 1.5. Local pollination is clarified by Equation (17), and rule 2 plus flower reliability can be mathematically modeled as [78,79]:

$$y_i^{t+1} = y_i^t + \in \left(y_j^t - y_k^t\right) \tag{17}$$

where pollen of different flowers on same plant shown by y_j^t and y_k^t. This essentially mimics the flower constancy in a limited neighborhood. If y_j^t and y_k^t belong to the same category and the same population, this becomes a local random walk if we express $\in$ from a uniform distribution in the range $[0, 1]$ [77].

Flower pollination can occur on all scales, both large and small. Pollen from nearby flower patches is more likely to fertilize neighborhood flower patches than pollen from distant flower patches. As a result, we can use the fourth rule (switch probability) p to transit from common global pollination to local pollination. The pseudo-code for the algorithm is as given in Algorithm 2.

We can find the most convenient parameter range by starting with $p = 0.5$ and performing a parametric analysis. $p = 0.7$ to 0.8 can be used to improve response in the majority of applications. The pseudo-code for the FP Algorithm [77–79] is shown below.

Estimate the new optimal solution from Algorithms 1 and 2 by minimizing the Mean Square Error (MSE) between the prior and current estimations. The MSE equation is shown in Equation (18).

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (\bar{Y}_i - Y_i)^2 \tag{18}$$

4. Two-Tank Conical Frustum Non-Interacting Level System with Mathematical Model

We employed different non-linear uncertain level control situations to test the efficacy of the proposed control method. A chemical process, petrochemical, refinery, food processing, dyes and paints, cement, and other industries have all employed level control systems [8,25,29,80–85]. Using the same non-linear uncertain system, the performance of numerous fault-control methods may also be evaluated [8,25,29,84,86–89]. As a consequence, we choose the two-tank conical frustum non-interacting level control (TTCFNLC) technique under actuator uncertainties.

The (TTCFNLC) process prototype model [84] is utilized as a benchmark problem in a variety of research disciplines, as shown in Figure 3.
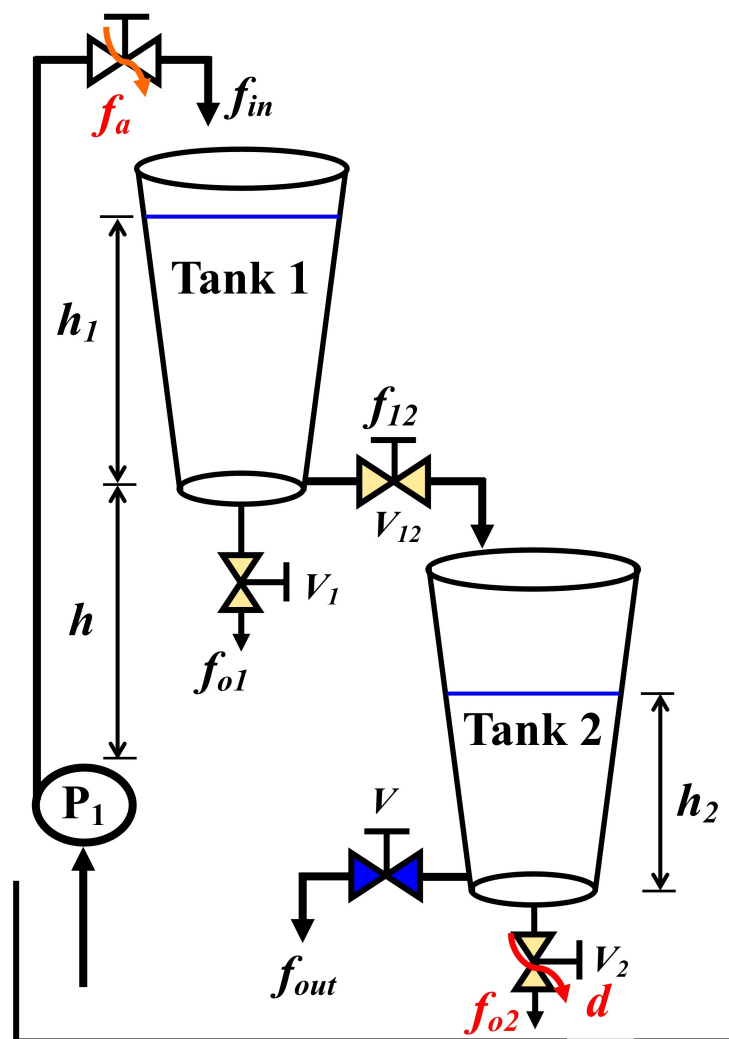

Figure 3. Prototype structure of frustum two-tank level control system [84].

The TTCFNLC process is a non-interacting single input single output (SISO) process in which the measured variable is the tank liquid level (h_2) and the manipulated variable is the inflow flow (f_1). Because the radius of the tank (r) changes, it is expressed as a ratio of the Frustum Tank's maximum radius (R) to its maximum height (H).

Modeling of Coupled Frustum Tank Level Control Process

We begin by looking into the single frustum tank system (FTS) illustrated in Figure 4 to derive the mathematical model for the TTCFNLC process.
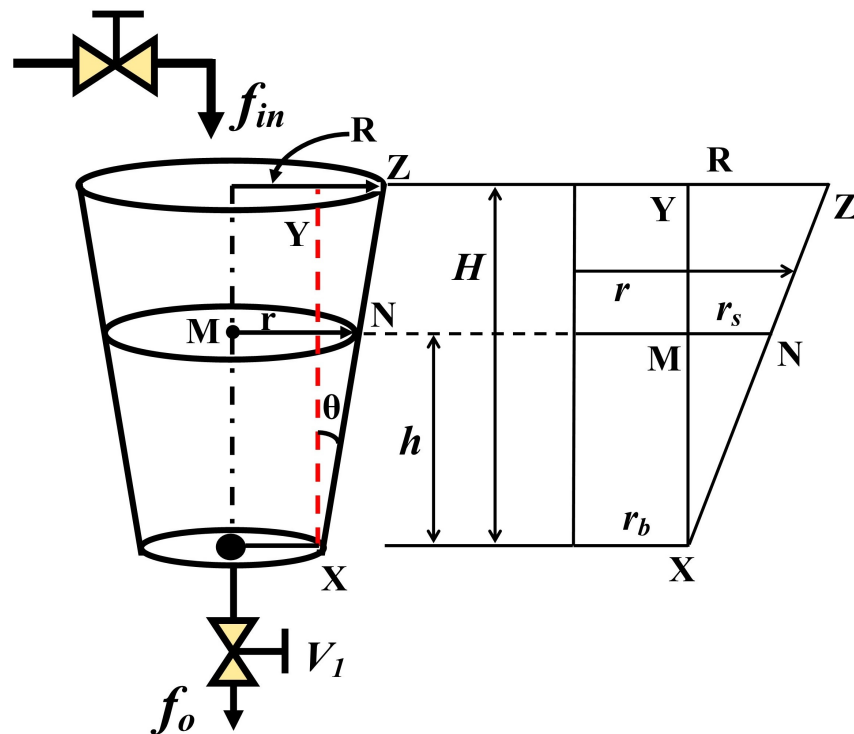
Figure 4. Variables of the frustum single tank for the non-linear model [90].

As per the mass balance equation, the mathematical model of the FTS is [84,91,92]:

Rate of accumulation = inflow − outflow

$$\frac{d(Vol)}{dt} = f_{in} - f_o \tag{19}$$

The liquid in the conical frustum tank has a volume *Vol*. As the tank's surface area changes, the volume of liquid changes as well. The volume of a conical frustum tank is calculated using Equation (20),

$$Vol = \frac{\pi}{3}\left(r_b^2 + r^2 + r_b r\right) \tag{20}$$

The bottom radius of the tank is r_b, and the top radius of the liquid is (r). The variable top radius of the liquid level is calculated using the trigonometric law.

The top radius of liquid level is $r = r_b + r_s$,

$$r = r_b + \frac{(R - r_b)}{H}h \tag{21}$$

The mathematical model of TTCFNLC's frustum tank 1 without an uncertainty can be written using Equation (22) and the non-interaction condition, according to [8,84].

$$\frac{dh_1}{dt} = \frac{f_{in1} - f_{o1} - f_{12}}{\frac{\pi}{3}\left[3r_{b1}^2 + 6r_{b1}\left(\frac{R-r_{b1}}{H_1}\right)h_1 + 3\left(\frac{R-r_{b1}}{H_1}\right)^2 h_1^2\right]} \tag{22}$$

The rate of accumulation equation for tank 2 in TTCFNLC process is represented by (23),

$$\frac{dh_2}{dt} = \frac{f_{12} - f_{o2} - f_{out}}{\frac{\pi}{3}\left[3r_{b2}^2 + 6r_{b2}\left(\frac{R-r_{b2}}{H_2}\right)h_2 + 3\left(\frac{R-r_{b2}}{H_2}\right)^2 h_2^2\right]} \tag{23}$$

We formulate the faulty model of the same system from healthy (without fault or uncertainty) mathematical model of the TTCFNLC, as shown by Equations (24) and (25). In TTCFNLC, an actuator defect in the main control valve is taken into account, which causes the manipulated variable inlet flow rate (f_{in}) of the conical frustum tank 1 to be disrupted. The system component (leak) fault in conical frustum tank 2 is the second fault considered in the TTCFNLC. Uncertain process disturbances (d) are also taken into account by the valve (V_1), which manipulates (f_{o2}).

In this study, only an actuator fault is considered during the simulation and produce the results.

$$\frac{dh_1}{dt} = \frac{(\alpha \times f_{in1}) - f_{o1} - f_{12}}{\frac{\pi}{3}\left[3r_{b1}^2 + 6r_{b1}\left(\frac{R - r_{b1}}{H_1}\right)h_1 + 3\left(\frac{R - r_{b1}}{H_1}\right)^2 h_1^2\right]} \tag{24}$$

The rate of accumulation equation for tank 2 in TTCFNLC process with uncertainty is represented by (25),

$$\frac{dh_2}{dt} = \frac{f_{12} - (d \times f_{o2}) - f_{out}}{\frac{\pi}{3}\left[3r_{b2}^2 + 6r_{b2}\left(\frac{R - r_{b2}}{H_2}\right)h_2 + 3\left(\frac{R - r_{b2}}{H_2}\right)^2 h_2^2\right]} \tag{25}$$

where (α) signifies a faulty actuator (loss of efficacy) in the primary actuator that controls the controlled variable input flow rate (f_{in}). Equations (24) and (25) show a faulty system model with uncertainties (actuator (f_a) fault and process disturbances (d)):

$$\begin{aligned} f_{in} &= k_p V & f_{out} &= \beta_2 a \sqrt{2gh_2} \\ f_{o1} &= \beta_1 a_1 \sqrt{2gh_1} & f_{o2} &= \beta_2 a_2 \sqrt{2gh_2} \\ f_{12} &= \beta_{12} a_{12} \sqrt{2gh_1} \end{aligned} \tag{26}$$

The bottom radius $r_{b1} = r_{b2} = 18$ cm and the top radius $R_1 = R_2 = 24$ cm are the same because the two frustum tanks are comparable. The frustum conical tank has two heights: H_1 and H_2 ($H_1 = 90$ cm and $H_2 = 90$ cm). Tank 1, 2's liquid levels are indicated by the h_1 and h_2. The valve coefficients in both tanks are the same ($\beta_1 = \beta_2 = 0.33$), and the interaction pipe valve coefficient is $\beta_{12} = 0.2$. The pump 1 gains are the $k_p = (25 \text{ cm}^3/\text{v.s})$ [84].

The incipient nature of the actuator fault (loss of effectiveness in the main control valve) that gradually limits the inlet flow f_{in} rate and the abrupt process disturbance d (occur in f_{o2}) (instant close the valve V_2) that causes the rate of aggregation in tank 2 to increase are both taken into account during the simulation process. Furthermore, the system component (leak) abrupt fault is considered in the conical frustum tank 2 in TTCFNLC process.

Two input variables, the error and derivative of the error, are granulated into three interval type-2 membership functions, two trapezoidal for the edges and one trapezoidal for the centre with a triangle lower membership function (LMF), labelled as "low", "moderate", and "high", respectively, to construct the fuzzy controller of the TTCFNLC. The *CV* action output variables are divided into three triangle membership functions labelled "low", "moderate", and "high". The inputs and outputs, making up the interval type-2 fuzzy controller of the coupled frustum tank process, granulated with triangle and trapezoidal interval type-2 membership functions, are depicted below. The type-2 membership functions are shown in Figure 5 for input and output of the IT2FLC.
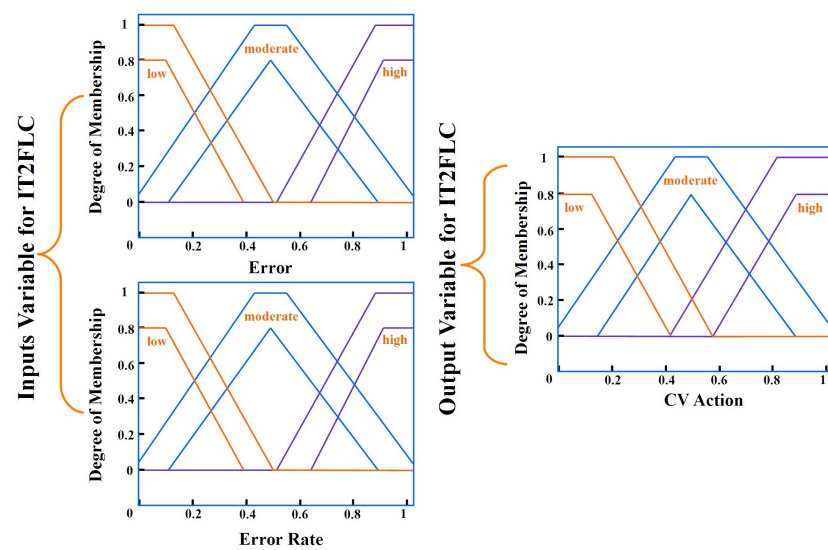
Figure 5. Fuzzy inference system using IT2FIS for TTCFNLC system.

5. Proposal and Methods

The work's main contributions are the proposed fuzzy base metaheuristic optimization algorithm methodology and its application for determining the best membership function parameters in the design of an interval type-2 fuzzy controller, which governs the behavior of the TTCFNLC system. The fuzzy controller attempts to follow a given set point with the smallest (negligible) margin of error, as determined by a defined control performance criterion. The main difference between using dynamic parameter adjustment in metaheuristic algorithms and using fixed parameters in metaheuristic algorithms is that the parameters chosen for dynamic adjustment are adjusted by the iterations' progress, resulting in better results.

Using a metaheuristic algorithm, the methodology optimizes the membership function parameters of the interval type-2 fuzzy logic level controller (IT2FLLC) for set-point tracking (regulatory control) with negligible steady-state error. In this scenario, we measure the interval type-2 fuzzy controller's performance and produce a result based on the established performance metric. We then continue with the optimization until a stopping criterion or a previously established number of iterations is met. Figure 6 depicts a simplified version of the process.
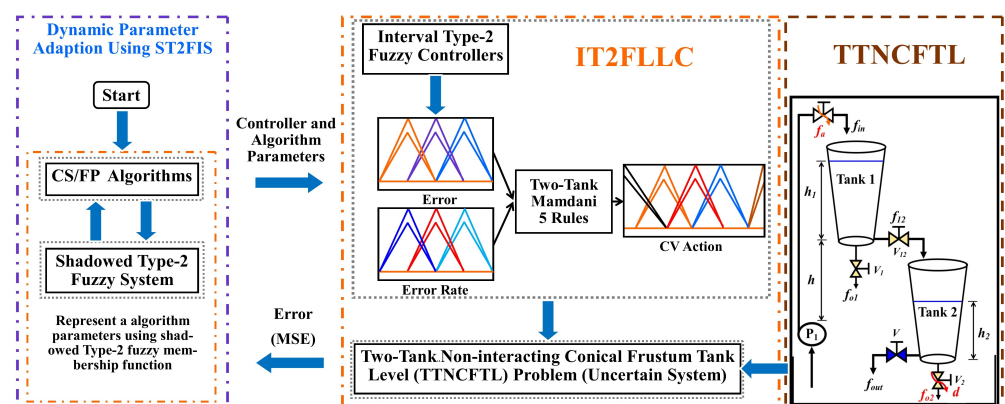

Figure 6. Proposed optimization method using metaheuristic algorithm.

The fuzzy system used for both algorithms has one input and one output. In the case of the CS algorithm, the input parameter is the "Simulation Iterations (SI)" and for the output, the "L*è*vy flight (P)" parameter is used, and for the FP algorithm, the input parameter is the "Simulation Iterations (SI)" and the output parameter is "Switching Probability (P')".

In Equation (27), the "Simulation Iterations (SI)" refers to "Iterations (I)" for the FCS method and generations for the FFP method. The current simulation represents the current iterations or generations and the maximum of simulations represents the maximum number of iterations and generations.

Equation (27) is used to calculate the input of the shadowed type-2 fuzzy system according to the method:

$$Simulation\ Iteration(SI) = \frac{Current\ Simulation}{Maximum\ of\ Simulations} \tag{27}$$

After finding the input of shadowed type-2 fuzzy inference system "Iteration (I)", we found the outputs "Lèvy flight (P)" and "Switching Probability (P')" for the fuzzy CS and fuzzy FP algorithm, respectively. As a result, after proposing ST2FIS for fuzzy CS and fuzzy FP algorithms, the optimized IT2FLS for a frustum two-tank level control system was realized. The IT2FLC optimized for nonlinear systems was tested both with and without uncertainties.

To optimize the type-2 fuzzy controller, two fuzzy metaheuristic algorithms are used, i.e., the CS and FP algorithms. In the first scenario, we use a shadowed type-2 fuzzy inference system to optimize the fuzzy CS variant. The parameter adjustment fuzzy system of the CS algorithm takes "Iteration (I)" as the input variable and the "Lèvy flight (P)" parameters as the output variables. Each variable, as shown in Figure 7, is composed of three trapezoidal membership functions labelled "low," "moderate," and "high". The fuzzy FP using ST2FIS that adjusts the FP's parameters employ the "Iteration (I)" variable as the input variable and the "Lèvy flight (P)" parameter as the output variable, and these variables are made up of three trapezoidal membership functions labeled as "low," "moderate," and "high", as shown in Figure 8.
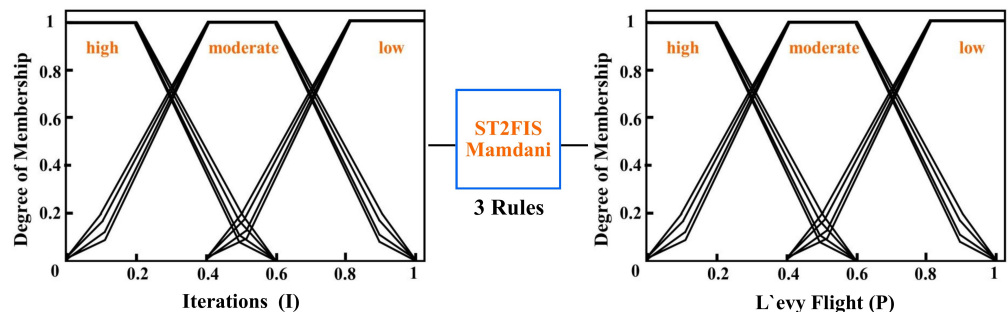

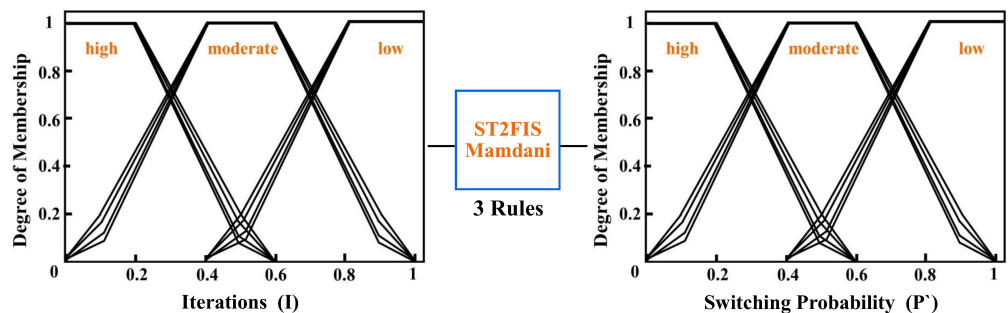**Figure 7.** Dynamic parameter adaption of fuzzy CS algorithm using ST2FIS.

Figure 8. Dynamic parameter adaption of fuzzy FP algorithm using ST2FIS.

The idea behind the use of fuzzy system rules is that algorithms can explore in early iterations and exploit in later iterations.

Fuzzy rules for Fuzzy CS using ST2FIS:

1.   If iteration is low, then Lèvy flight (P) is low;

2. If iteration is moderate, then Lèvy flight (P) is moderate;
3. If iteration is high, then Lèvy flight (P) is high.

Fuzzy rules for Fuzzy FP using ST2FIS:

1. If iteration is low, then switching Probability (P') is low;
2. If iteration is moderate, then switching Probability (P') is moderate;
3. If iteration is high, then switching Probability (P') is high.

The rules are based on human expertise and previous experimentation as presented in [25,29], while the ST2FCS and ST2FFP methods are used in a decreasing fashion.

In our proposal, each individual or feasible solution is defined by the required parameters for automatically generating the type-2 fuzzy controller rather than manually generating the fuzzy controller, with the goal of obtaining better results than those obtained with the basic fuzzy controller [93,94]. The settings that comprise the fuzzy controller are unique to each individual. In this case, the controller has two input variables built by two trapezoidal membership functions, each with eight parameters, and a triangular with six parameters, for a total of 22 parameters per input variable. The output variables are made up of three triangle membership functions, each with 6 parameters, resulting in a total of 18 parameters per output variable. When all the parameters for the input and output variables are collected, the fuzzy controller is built.

Individual structure in the CS and FP algorithms contributes to the formation of interval type-2 fuzzy controllers, which are formed by a triangle and trapezoidal membership functions. The following equations and illustrations employing interval type-2 fuzzy logic are shown.

The representation of the interval type-2 triangular membership functions (itritype2) of the fuzzy controller consists of 6 parameters a_{n1}, b_{n1}, c_{n1}, a_{n2}, b_{n2}, and c_{n2}, mathematically presented by Equation (28),

$$\mu(x) = itritype2(x, [a_{n1}, b_{n1}, c_{n1}, a_{n2}, b_{n2}, c_{n2}]), \tag{28}$$

where the membership function parameters value $a_{n1} < a_{n2}$, $b_{n1} < b_{n2}$, $c_{n1} < c_{n2}$.

The representation of the interval type-2 trapezoidal membership functions (*itrapatype2*) of the fuzzy controller consists of 8 parameters a_{n1}, b_{n1}, c_{n1}, d_{n1}, a_{n2}, b_{n2}, c_{n2}, d_{n2}, where $a_{n1} < a_{n2}$, $b_{n1} < b_{n2}$, $c_{n1} < c_{n2}$ and $d_{n1} < d_{n2}$ and the MATLAB syntax for the same illustrated in Equation (29),

$$\mu(x) = itrapatype2(x, [a_{n1}, b_{n1}, c_{n1}, d_{n1}, a_{n2}, b_{n2}, c_{n2}, d_{n2},]), \tag{29}$$

where the membership function parameters value $a_{n1} < a_{n2}$, $b_{n1} < b_{n2}$, $c_{n1} < c_{n2}$, $d_{n1} < d_{n2}$.

In Figure 9, we can find the graphical description of the interval type-2 triangular and trapezoidal membership functions, respectively.
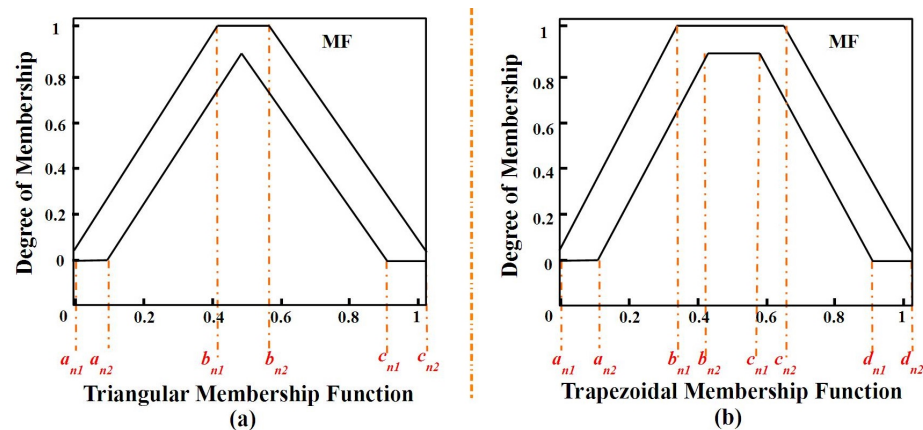

Figure 9. Interval type-2 membership function. (**a**) triangular membership function; (**b**) trapezoidal membership function.

Table 1 shows the vector size during the optimization process using the fuzzy CS and FP algorithms for a given non-linear control problem.

Table 1. Size of vector in the optimization by fuzzy CS and FP algorithm for control problem.

Control Problem	Input		Output		Total Size of Vector
	Total	**Type of MFs**	**Total**	**Type of MFs**	
TTCFNLC Process	3	2—IT2 Trapezoidal and 1—IT2 Triangular in each Input	3	3—IT2 Triangular in each output	80

6. Simulation Results

In non-linear plants, a series of performance indices are used to evaluate the efficiency of the CS algorithm in control, including the Integral Square Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE), Integral Time Squared Error (ITSE), Mean Square Error (MSE), and Root Mean Square Error (RMSE). Equations (30)–(34), and (18) show the respective mathematical representations.

$$ISE = \int_0^\infty e^2(t)dt \tag{30}$$

$$IAE = \int_0^\infty |e(t)|dt \tag{31}$$

$$ITAE = \int_0^\infty |e(t)t|dt \tag{32}$$

$$ITSE = \int_0^\infty e^2 t(t)dt \tag{33}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(x_t - \hat{x}_t)^2} \tag{34}$$

The MSE (see Equation (18)) is the fitness function of the CS and FP algorithms, which means the best individual in each execution is the particle with the lowest MSE in the entire population.

The simulations designed to evaluate the proposed approach are described in Section 5 to optimize the type-2 fuzzy controller for the non-linear uncertain level control system.

In non-linear plant, a total of 30 simulations were performed without fault, with a sudden actuator fault of 50%, at time $t = 4$ s and $M = 60\%$ at time $t = 6$ s. Additionally, abrupt system component fault in tank 2 bottom was also considered and simulation results were produced with leak fault of 50%, at time $t = 4$ s and $M = 60\%$ at time $t = 6$ s. Table 2 displays the findings for each model, as well as the average result for the minimum values to be found by the CS and FP algorithms of the four performance indexes and the control problem with and without actuator and system component (leak) fault.

Table 2. Results of average performance index of minimums values found by the CS and FP algorithm using ST2FIS.

Algorithm	Performance Index	Simulation Scenarios	
		Without Fault	**With Actuator Fault 1**
CS	ITAE	1.481×10^2	2.612×10^2
	ITSE	3.418×10^2	3.933×10^2
	IAE	0.891×10^1	0.989×10^1
	ISE	1.793×10^2	3.156×10^2
FP	ITAE	3.213×10^3	4.773×10^3
	ITSE	7.422×10^3	8.1014×10^3
	IAE	1.971×10^2	2.262×10^2
	ISE	3.9135×10^3	4.828×10^4

Comparative Analysis and Result Discussion

This section compares the results obtained when faults are included in the model. A visualization of the many measures utilized to assess the performance of the CS and FP algorithms is also examined.

Figures 10 and 11 depict the optimal distribution of the MFs for the IT2FIS discovered by the fuzzy FP and fuzzy CS algorithm for the frustum water tank level control problem using ST2FIS. Figure 10 displays the optimal MFs distribution found by the fuzzy CS using ST2FIS for the proposed frustum water tank level control problem, while Figure 11 depicts the best MFs vector distribution found by the fuzzy FP using ST2FIS for proposed frustum water tank level control problem. From the critical observation from Figures 10 and 11, we found the two inputs (Error and Error Rate) *x*-axis distributions of the IT2FLC is slightly changed for both the optimal MFs distribution found by fuzzy CS and fuzzy FP algorithms.
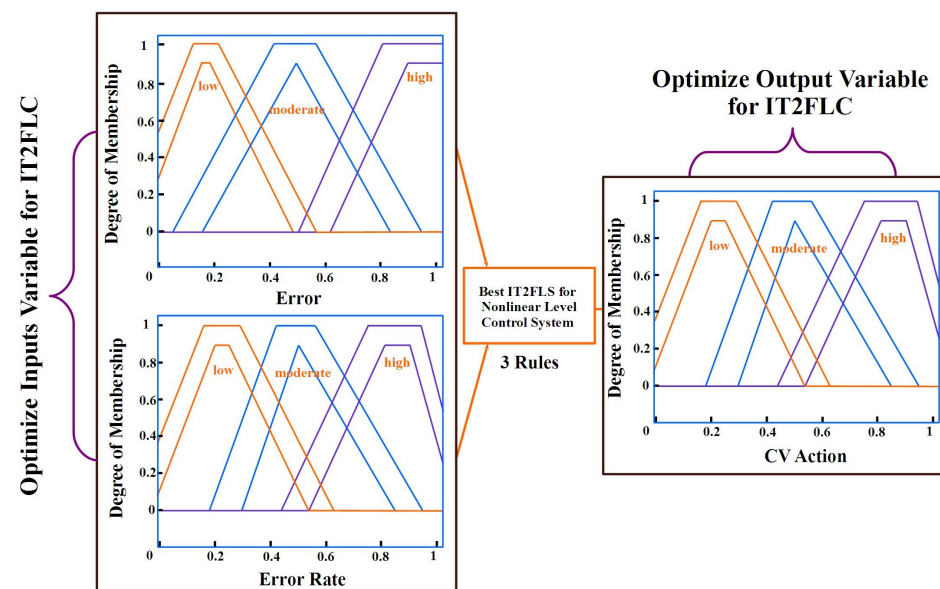
Figure 10. Best/Optimize IT2FLS that FP algorithm using ST2FIS finds for the frustum two-tank fuzzy Controller.
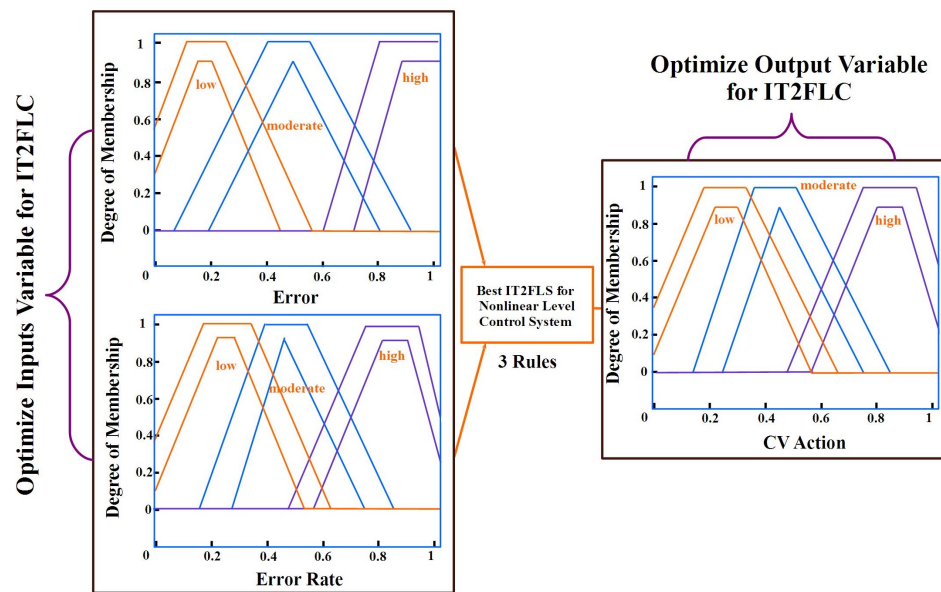
Figure 11. Best/Optimize IT2FLS that CS algorithm using ST2FIS finds for the frustum two-tank fuzzy Controller.

Finally, in Figures 12–14, the results achieved using the fuzzy CS and fuzzy FP approaches with ST2FIS are shown, with disparities between the desired set point and the real (process/measured value) created by the TTCFNLC with the best-optimized interval type-2 fuzzy logic controller from FCS using ST2FIS and FFP using ST2FIS. The observing results, clearly show that fuzzy CS using ST2FIS outperforms the other algorithms for the TTCFNLC system without fault and in two different faulty conditions.
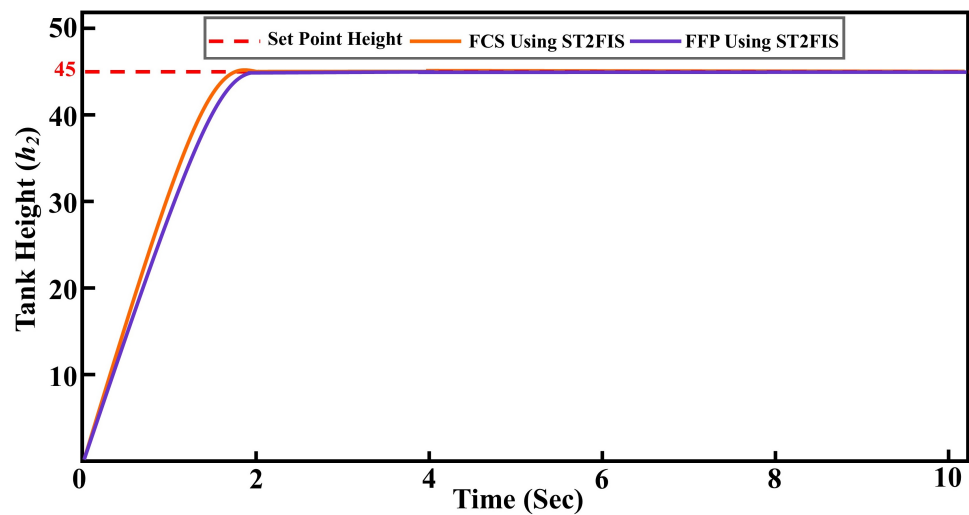

Figure 12. Comparative results of TTCFNLC process subject to without fault using fuzzy CS and FP algorithms.
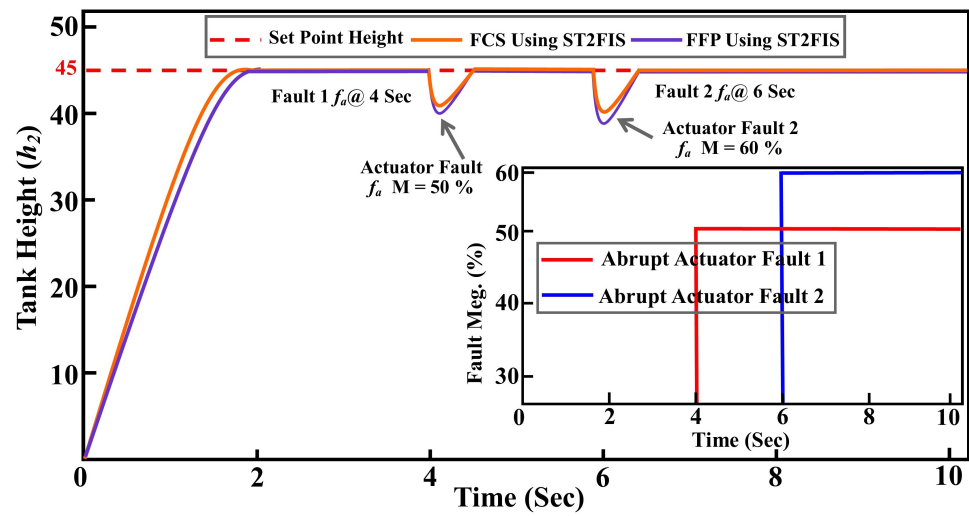
Figure 13. Comparative results of TTCFNLC process subject to actuator fault using fuzzy CS and FP algorithms.
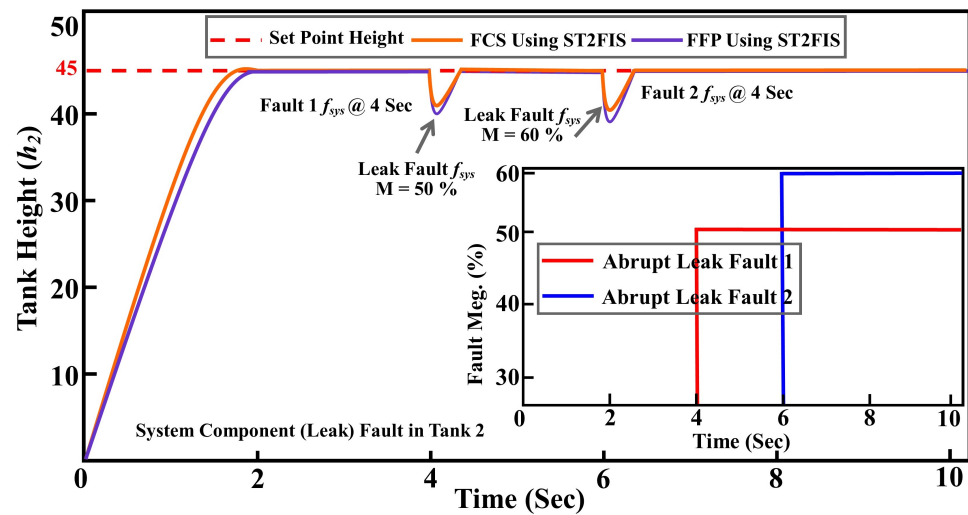

Figure 14. Comparative results of TTCFNLC process subject to system component (leak) fault using fuzzy CS and FP algorithms.

The MSE error comparison of the TTCFNLC process utilizing the CS and FP algorithm using ST2FIS is shown in Table 3. The fault considered during the simulations is abrupt in nature.

Table 3 shows the MSE error results for the two-tank conical frustum level control system, the best result of the fitness function MSE to find by fuzzy CS and fuzzy FP algorithm using shadowed type-2 fuzzy inference system with the metrics of "Best", "Worst", "Average", "Standard Deviation" (SD), without fault (not applied uncertainty), with actuator fault (in main actuator CV (f_a) fault), is presented. Additionally, the result in Table 3 shows the stabilization under the one uncertainty added in the two-tank level control system. For example, the best MSE for FCS using ST2FIS with actuator fault is 1.2027×10^{-2}, while in the case of FFP the best MSE is 0.2923. Similarly, the average MSE for FCS using ST2FIS with actuator fault is 9.0912×10^{-2}, while in the case of FFP the average MSE is 1.4091×10^{-1}.

Table 3. Results of metrics of Mean Square Error (MSE) values to found by CS and FP algorithm using ST2FIS.

Algorithm	Performance Index	Simulation Scenarios	
		Without Fault	With Actuator Fault 1
CS	BEST	1.1725×10^{-4}	1.2027×10^{-2}
	WORST	0.1671	0.2315
	AVERAGE	2.958×10^{-2}	9.0912×10^{-2}
	STANDARD DEVIATION	4.4234×10^{-2}	4.74513×10^{-2}
FP	BEST	2.3412×10^{-3}	0.2923
	WORST	0.2161	0.3019
	AVERAGE	7.3812×10^{-2}	1.4091×10^{-1}
	STANDARD DEVIATION	5.6124×10^{-2}	5.2341×10^{-2}

The RMSE error comparison of the TTCFNLC process utilizing the fuzzy CS and fuzzy FP algorithm using ST2FIS which is shown in Table 4. In Table 4, the RMSE is presented for both CS and FP algorithms using ST2FIS for the TTCFNLC system subject to actuator fault and without actuator fault. The RMSE metrics distributed with same mathematical functions, such as "BSET", "WORST", "AVERAGE", and "STANDARD DEVIATION". The SD value for FCS using ST2FIS gives 8.5209×10^{-2}, while FFP using ST2FIS gives 10.02813×10^{-2}, subject to actuator fault in TTCFNLC system. The average value for FCS is 2.9125×10^{-1} and for FFP it is 3.8342×10^{-1} for TTCFNLC system under actuator fault. As can be seen from RMSE and MSE error values, FCS using ST2FIS outperforms the FFP using ST2FIS for the TTCFNLC system subject to a fault and without fault.

Table 4. Results of metrics of Root Mean Square Error (RMSE) values to found by CS and FP algorithm using ST2FIS.

Algorithm	Performance Index	Simulation Scenarios	
		Without Fault	With Actuator Fault
CS	BEST	1.0913×10^{-2}	1.4230×10^{-1}
	WORST	3.8561×10^{-1}	4.4671×10^{-1}
	AVERAGE	1.3123×10^{-1}	2.9125×10^{-1}
	STANDARD DEVIATION	1.1901×10^{-1}	8.5209×10^{-2}
FP	BEST	4.7816×10^{-2}	2.6091×10^{-1}
	WORST	4.2891×10^{-1}	4.9891×10^{-1}
	AVERAGE	2.6543×10^{-1}	3.8342×10^{-1}
	STANDARD DEVIATION	1.2671×10^{-1}	10.02813×10^{-2}

Table 5 illustrates the fault recovery time analysis (t_{fr}) for TTCFNLC system subject to abrupt actuator and system component (leak). The comparative results clearly show that the fault recovery time for FCS using ST2FIS is superior as compared to FFP using ST2FIS for both uncertainties.

Table 5. Comparative results of fault recovery time (t_{fr}) for TTCFNLC system under two uncertainties found by fuzzy CS and FP algorithm using ST2FIS.

| Type of Uncertainties	Fault Magnitude	Nature of Uncertainties	Time of Occurrence in Second	Metaheuristic Algorithms	
				Fuzzy CS	**Fuzzy FP**
				(t_{fr}) in Second	
Actuator Fault 1	50%	Abrupt	4	0.47	0.54
Actuator Fault 2	60%		6	0.52	0.57
Leak Fault 1	50%	Abrupt	4	0.43	0.48
Leak Fault 2	60%		6	0.49	0.56

Simulation Results with Noise and Intermittent Fault

The proposed method's and type-2 fuzzy controller's robustness was tested in the presence of noise in the controller and faulty conditions in the proposed nonlinear level control system. In the second phase of simulation, we take two uncertainties: one is type-2 fuzzy controller noise, and the other is two types of intermittent fault in the level control TTCFNLC system. The noise applied to this type-2 fuzzy controller is 0.5 (Gaussian random number), and the two faults in the TTCFNLC system considered are actuator and system component (leak) faults with intermittent nature.

The simulation results are carried out in three different uncertainty conditions:

(1) Interval type-2 fuzzy controller with random noise with magnitude $M = 0.5$ (Gaussian random number);

(2) Interval type-2 fuzzy controller with random noise and TTCFNLC system with intermittent actuator fault in main actuator CV with magnitude of $M_1 = 50\%$ and $M_2 = 60\%$ at time $t = 4$ s and $t = 6$ s respectively;

(3) Interval type-2 fuzzy controller with random noise and TTCFNLC system with intermittent system component (leak) fault in bottom of the frustum tank 2 (additional flow rate f_{o2}) with magnitude of $M_1 = 50\%$ and $M_2 = 60\%$ at time $t = 4$ s and $t = 6$ s, respectively.

Figures 15–17 illustrate the best regulatory control responses obtained with the two fuzzy metaheuristic algorithms. One algorithm is fuzzy CS using ST2FIS and the other is fuzzy FP using ST2FIS methods with three different uncertainties, one is noise in interval type-2 fuzzy controller, the second is noise in interval type-2 fuzzy controller with actuator fault in TTCFNLC process and the third is noise in interval type-2 fuzzy controller with leak fault in TTCFNLC process, respectively.
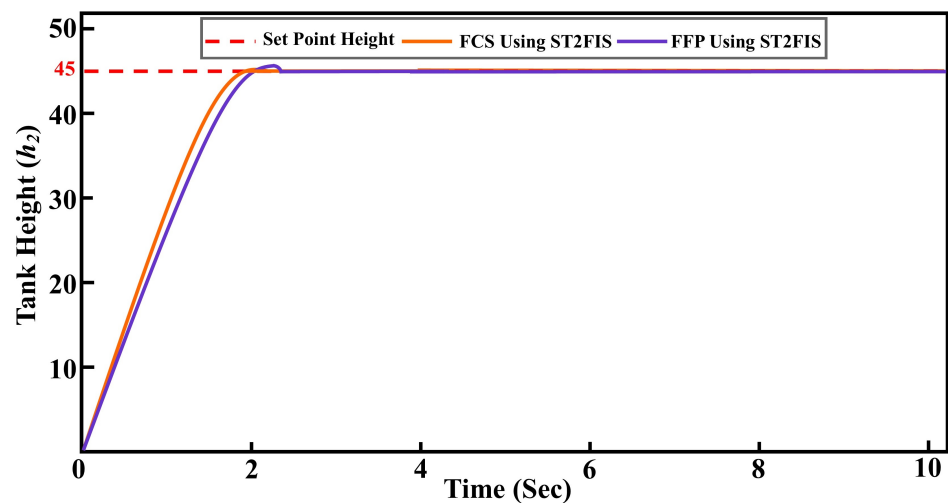
Figure 15. Comparative results of TTCFNLC process subject to random noise in fuzzy controller using fuzzy CS and FP algorithms.
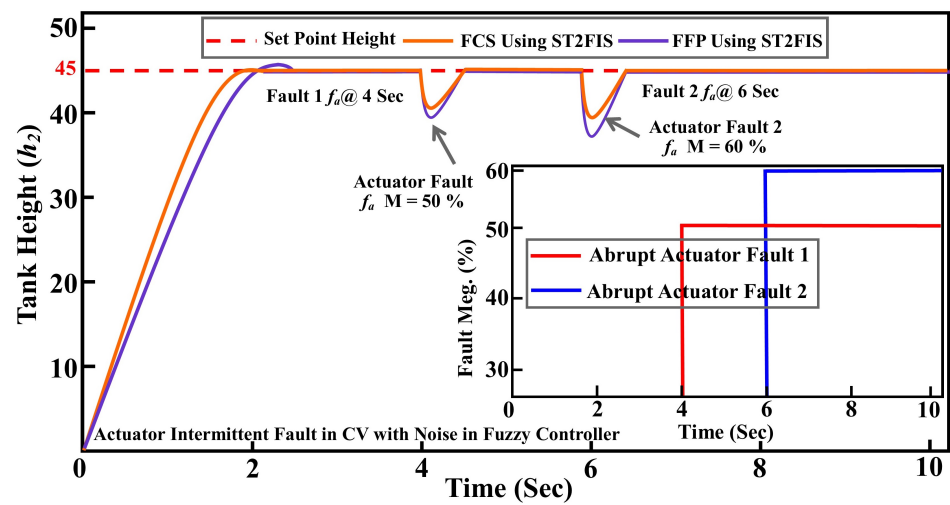
Figure 16. Comparative results of TTCFNLC process subject to random noise in the fuzzy controller and actuator fault in TTCFNLC process using fuzzy CS and FP algorithms.
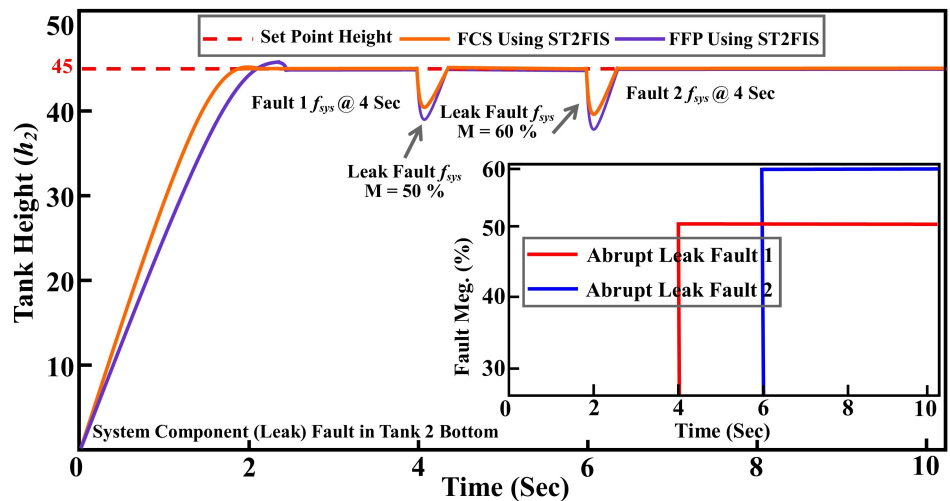

Figure 17. Comparative results of TTCFNLC process subject to random noise in the fuzzy controller and system component (leak) fault in TTCFNLC process using fuzzy CS and FP algorithms.

Tables 6 and 7 show the MSE and RMSE error results (with four performance indexes "BEST", "WORST", "AVERAGE", and STANDARD DEVIATION") obtained by optimizing the parameters of the TTCFNLC type-2 fuzzy controller in 30 simulations, with the FCS using ST2FIS and FFP using ST2FIS methods, respectively. The noise applied to this controller is 0.5 (Gaussian random number), and the two faults magnitude is $M_1 = 50\%$ and $M_2 = 60\%$ at time $t = 4$ s and $t = 6$ s, respectively.

Table 8 illustrates the fault recovery time analysis (t_{fr}) for the TTCFNLC system subject to the abrupt actuator fault, system component (leak) fault, and noise in the interval type-2 fuzzy controller for TTCFNLC, the comparative results clearly show that the fault recovery time for FCS using ST2FIS is superior as compared to FFP using ST2FIS for all the three types of uncertainties.

Finally, the MSE and RMSE error results obtained from the proposed method for the TTCFNLC system subject to noise in type-2 fuzzy controller are presented in Figures 18 and 19 in the radar plot, respectively. In order to validate the performance of the proposed fuzzy metaheuristic algorithms, we use the proposed optimization algorithms for optimizing the interval type-2 MFs parameters of the TTCFNLC fuzzy controller system with, two different uncertainties (actuator and leak faults) introduced into the system simultaneously with

random Gaussian noise into the type-2 fuzzy controller, and subsequently the MSE and RMSE error results for the different cases are presented in Figures 20 and 21, respectively, where one can note the proposed fuzzy CS using ST2FIS outperforms the other fuzzy FP algorithm using ST2FIS for proposed non-linear level control system subject to uncertainties, with the optimized type-2 fuzzy logic controller.

Table 6. Results of metrics of Mean Square Error (MSE) values to found by fuzzy CS and fuzzy FP algorithm using ST2FIS.

Algorithm	Performance Index	Simulation Scenarios		
		With Noise	With Noise and Actuator Fault 1 and 2	With Noise and Leak Fault 1 and 2
CS	BEST	7.16×10^{-2}	9.78×10^{-2}	8.37×10^{-2}
	WORST	0.2667	0.2767	0.2709
	AVERAGE	0.1415	0.1607	0.1447
	SD	5.24×10^{-2}	5.35×10^{-2}	5.52×10^{-2}
FP	BEST	9.73×10^{-2}	1.034×10^{-1}	9.87×10^{-2}
	WORST	0.3093	0.3477	0.3118
	AVERAGE	0.2154	0.2390	0.2179
	SD	6.67×10^{-2}	8.36×10^{-2}	6.89×10^{-2}

Table 7. Results of metrics of Root Mean Square Error (RMSE) values to found by fuzzy CS and fuzzy FP algorithm using ST2FIS.

Algorithm	Performance Index	Simulation Scenarios		
		With Noise	With Noise and Actuator Fault 1 and 2	With Noise and Leak Fault 1 and 2
CS	BEST	0.2675	0.3127	0.2893
	WORST	0.4934	0.5260	0.5204
	AVERAGE	0.37	0.3957	0.3737
	SD	6.17×10^{-2}	6.51×10^{-2}	6.23×10^{-2}
FP	BEST	0.3142	0.3215	0.3138
	WORST	0.5561	0.5896	0.5574
	AVERAGE	0.4583	0.4812	0.4610
	SD	7.39×10^{-2}	8.82×10^{-2}	7.44×10^{-2}

Table 8. Comparative results of fault recovery time (t_{fr}) for TTCFNLC system under three uncertainties found by fuzzy CS and FP algorithm using ST2FIS.

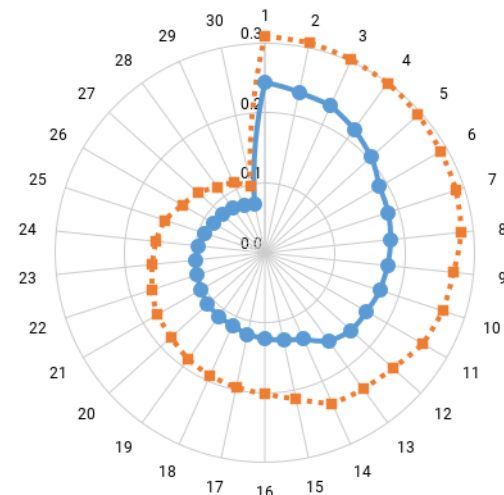| Type of Uncertainties	Fault Magnitude	Nature of Uncertainties	Time of Occurrence in Second	Metaheuristic Algorithms	
				Fuzzy CS	Fuzzy FP
				(t_{fr}) in Second	
Actuator Fault 1	50 %	Abrupt	4	0.49	0.64
Actuator Fault 2	60 %		6	0.53	0.61
Leak Fault 1	50 %	Abrupt	4	0.44	0.53
Leak Fault 2	60 %		6	0.51	0.59

Figure 18. MSE error for TTCFNLC process subject to random noise in fuzzy controller using fuzzy CS and FP algorithms for 30 simulations.
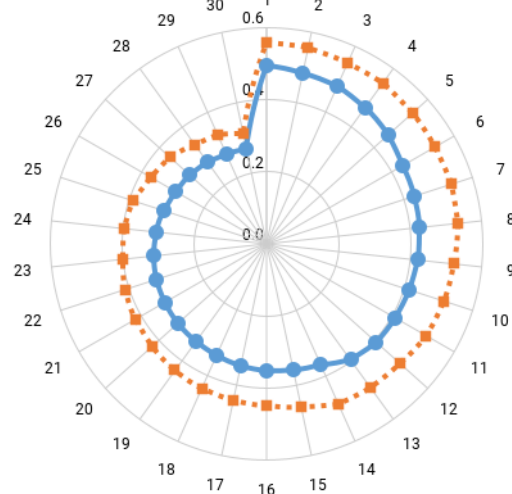

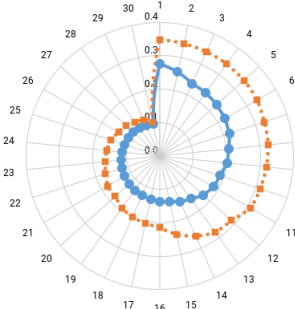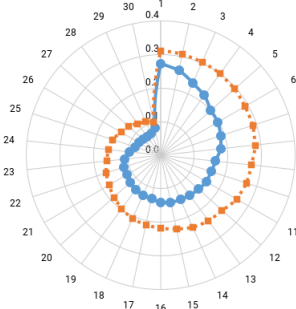**Figure 19.** RMSE error for TTCFNLC process subject to random noise in fuzzy controller using fuzzy CS and FP algorithms for 30 simulations.

Figure 20. MSE error comparison for 30 simulations. (**a**) TTCFNLC with actuator fault and noise in the fuzzy controller. (**b**) TTCFNLC with leak fault and noise in the fuzzy controller.
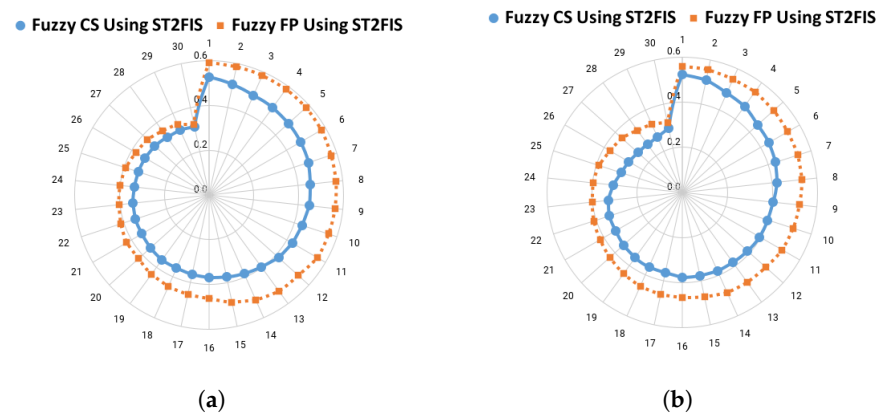
Figure 21. RMSE error comparison for 30 simulations. (**a**) TTCFNLC with actuator fault and noise in the fuzzy controller. (**b**) TTCFNLC with leak fault and noise in the fuzzy controller.

7. Statistical Results and Analysis

A statistical comparison is made to find evidence that the fuzzy metaheuristic algorithms are used to test the proposed optimization methodology for optimizing the interval type-2 membership function parameters of the TTCFNLC fuzzy controller subject to uncertainties. Table 5 depicts the values used in the statistical test to evaluate whether the proposed strategy generated better results in the optimization of membership function parameters, generating a data vector, and assisting in the improvement of the interval type-2 fuzzy controller for the coupled frustum tank system performance under faulty condition.

The null hypothesis (H_0) says that the average MSE error for the CS algorithm and the variants (μ_1) are greater than or equal to the average MSE error for the FP algorithm and the variants (μ_2), as shown in Table 9. According to the alternative hypothesis (H_a), the average MSE error for the CS algorithm and the variations (μ_1) is less than the average MSE error for FP algorithm and the variants (μ_2).

Table 10 displays the averages, standard deviations, and z values for each of the comparisons. The CS method and the variants receive significant evidence against the FP algorithm and the variants. As a result, H_0 is rejected and H_a is accepted with a 95% level of significance. The rejection zone for values is less than -1.645.

Table 9. Statistical z-test parameters.

Parameter	Value
H_0	$\mu_1 \geq \mu_2$
H_a	$\mu_1 < \mu_2$ (Claim)
Level of significance	95 %
A	0.05
Critical value	-1.645

The statistical z-test, based on Equation (35), was used and the settings for this test were α of 0.05 and a 95% level of confidence. The main goal is to demonstrate that combining FTC and interval Type-2 approaches yields a better result than utilizing the shadowed Type-2 fuzzy system, for any values less than -1.645:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{(\sigma_1 - \sigma_2)} \tag{35}$$

As previously discussed, metaheuristic algorithms produce good results in optimization issues. In this study, we optimized an interval type-2 fuzzy controller that regulates the TTCFNLC system, and we then described the experimental results. The experiment is carried out with ST2FIS optimization approaches for interval type-2 fuzzy controllers in order to compare the methods and determine the best. The CS optimization approach offers the best outcomes according to the hypothesis test, as shown in Table 10. This is because the flower pollination algorithm gives a bigger difference error between the desired (set point) and measured level value of the TTCFNLC process.

In Table 10, statistical *z*-test results are presented for the proposed fuzzy CS and fuzzy FP algorithm method for an uncertain non-linear TTCFNLC system. The *z*-values obtained in Table 10 demonstrate that the proposed approach with the FCS using ST2FIS outperforms the FPA using ST2FIS.

Table 10. Results of the Z-test for the CS and the FP algorithm.

CS		FP		*z*-Value
Average	**Std**	**Average**	**Std**	
FCS ST2FIS		**FPA ST2FIS**		
3.7981×10^{-1}	3.3761×10^{-1}	7.9023×10^{-1}	4.5021×10^{-1}	$-5.081 \times 10^{+01}$

In addition, we performed the non-parametric Friedman test to increase our confidence in the results' validity. The Friedman test [95] is a non-parametric test used to assess multiple algorithms in order to determine the best.

In the first case, when we used ST2FIS to compare the fuzzy CS and FP approaches, we discovered that the CS has a significant advantage with a *p* value of 0.000061 (see Table 11). In the second case, we compared the fuzzy CS and FP algorithms using ST2FIS for the TTCFNLC system with actuator fault and found that the CS has a significant advantage with a *p* value of 0.0000589 (see Table 12). We conclude that the cuckoo search (and the fuzzy variants) outperforms the fuzzy flower pollination method.

Table 11. Friedman test in comparing FCS Using ST2FIS vs. FPA Using ST2FIS.

FCS Using IT2FIS vs. FPA Using IT2FIS	
Test Statistic	***p*-Value**
16.13333	
$Q = \frac{12n}{k(K+1)} \left[\sum_j R_j^2 - \frac{k(K+1)^2}{3} \right]$	0.000061
Q = 0.066666 × 4292 − 270	
Q = 16.13333	

Table 12. Friedman test in comparing FCS Using ST2FIS vs. FPA Using ST2FIS for TTCFNLC with abrupt actuator fault 1.

FCS Using T1FIS vs. FPA Using T1FIS	
Test Statistic	***p*-Value**
16.13333	
$Q = \frac{12n}{k(K+1)} \left[\sum_j R_j^2 - \frac{k(K+1)^2}{3} \right]$	0.0000589
Q = 0.066666 × 4292 − 270	
Q = 16.13333	

8. Conclusions

This paper presents an efficient implementation of the fuzzy CS and fuzzy FP algorithms using ST2FIS for the designing of optimized interval type-2 membership functions (IT2MFs) of interval type-2 fuzzy logic controller (IT2FLC) for a given non-linear control problem with uncertainties. When uncertainty is introduced, the proposed structure for determining the best distribution of interval type-2 MFs in a given control problem demonstrates that the CS method can control a stabilization in a nonlinear control problem; coupled frustum water tank controller. The exploration and exploitation abilities of the CS algorithm are an important feature that is investigated and reflected in the positive results. Many such metrics are developed to assess the effectiveness of the CS and FP algorithms, including ITAE, ITSE, ISE, IAE, MSE, and RMSE.

The proposed implementation shows the CS algorithm is a useful tool for determining the best design in IT2MFs for interval type-2 fuzzy logic controller and fuzzy controller stabilization. The simulations have very few errors, as shown in Tables 2 and 4. Figures 18–21 show that the CS algorithm converges quickly and produces very few errors after only a few iterations, owing to the algorithm's exploitability. It can also be utilized for various fuzzy controllers, such as without fault simulations and with actuator fault in a non-interacting two-tank conical frustum level system. The best optimize IT2FLC system using fuzzy FP and fuzzy CS algorithms with ST2FIS for TTCFNLC system are presented in Figures 10 and 11 respectively. In addition, the fault recovery time analysis is presented for both the metaheuristic algorithm using dynamic parameter adaption techniques using ST2FIS.

In the future, different types of fuzzy systems for dynamic parameter adjustment in metaheuristic algorithms will be investigated and applied to various control challenges with higher levels of uncertainties. Other metaheuristic algorithms can also be used and tested for the non-linear control problems with fault-tolerant control capability with fault recovery time (t_{fr}) analysis.

Different types of fuzzy systems for dynamic parameter adjustment in metaheuristic algorithms will be investigated and applied to various control applications in future research. In addition, it would be interesting to monitor and assess the shadowing type-2 fuzzy logic systems' executing time and performance when greater noise levels are incorporated into the control process.

It is also proposed to use generalized type-2 fuzzy systems to perform dynamic parameter tweaking and to extend the controller to a generalized type-2 fuzzy controller [25,29,96]. Furthermore, a study of the controllers could be conducted by introducing various levels of disturbance to the plants to assess the efficacy of the proposed method in various scenarios.

Abbreviations

The following abbreviations are used in this manuscript:

ANNs	Artificial Neural Networks
ACO	Ant Colony Optimisation
CS	Cuckoo Search
CV	Control Valve
CPS	Cyber-Physical Systems
DE	Differential Evolution
DMs	Decision Makers
DOF	Degrees of Freedom
DTs	Decision Threes
EA	Elevation Area
FF	Feedforward
FP	Flower Pollination
FSs	Fuzzy Sets
FLC	Fuzzy Logic Controller
FISs	Fuzzy Inference Systems
FTS	Frustum Tank System
GA	Genetic Algorithm
GT2FS	General Type-2 Fuzzy Sets
GT2FIS	General Type-2 Fuzzy Inference System
GOA	Grasshopper Optimization Algorithm
GSO	Galactic Swarm Optimization
LFM	Lower Membership Function
LPFNs	Linguistic Pythagorean fuzzy numbers
MFs	Membership Functions
MSE	Mean Square Error
MPPT	Maximum Power Point Tracking
NN	Neural Network
PID	Proportional Integral Derivative
PSO	Particle swarm optimisation
PV	Photo Voltaic
RTAC	Rotational/Translational Proof-Mass Actuator
RA	Reduction Area
RL	Reinforcement Learning
RMSE	Root Mean Square Error
SA	Shadowing Area
SISO	Single Input Single Output
ST2FIS	Shadowed Type-2 Fuzzy Inference System
SFC	Stochastic Fractal Search
SVMs	Support Vector Machines
TTCFNLCS	Two-Tank Conical Frustum Non-Interacting Level System
T1FLCs	Type-1 Fuzzy Logic Controllers
t_{fr}	Fault recovery time
HS	Harmony Search
IP	Inverted Pendulum
IAE	Integral Absolute Error
ICS	Improved Cuckoo Search
ISE	Integral Square Error
ITAE	Integral Time Absolute Error
ITSE	Integral Time Absolute Error
IT2FLCs	Interval Type-2 Fuzzy Logic Controllers
IT2FIS	Interval Type-2 Fuzzy Inference System

References

1. Patel, H.R.; Shah, V.A. Fuzzy Logic Based Metaheuristic Algorithm for Optimization of Type-1 Fuzzy Controller: Fault-Tolerant Control for Nonlinear System with Actuator Fault. *IFAC-PapersOnLine* **2022**, *55*, 715–721. [CrossRef]
2. Patel, H.R.; Shah, V.A. Comparative Analysis Between Two Fuzzy Variants of Harmonic Search Algorithm: Fuzzy Fault Tolerant Control Application. *IFAC-PapersOnLine* **2022**, *55*, 507–512. [CrossRef]
3. Patel, H.R.; Raval, S.K.; Shah, V.A. A novel design of optimal intelligent fuzzy TID controller employing GA for nonlinear level control problem subject to actuator and system component fault. *Int. J. Intell. Comput. Cybern.* **2021**, *14*, 17–32. [CrossRef]
4. Georgieva, A.; Jordanov, I. Global optimization based on novel heuristics, low-discrepancy sequences and genetic algorithms. *Eur. J. Oper. Res.* **2009**, *196*, 413–422. [CrossRef]
5. Ji, Y.; Zhang, K.-C.; Qu, S.-J. A deterministic global optimization algorithm. *Appl. Math. Comput.* **2007**, *185*, 382–387. [CrossRef]
6. Patel, H., Shah, V. An Optimized Intelligent Fuzzy Fractional Order TID Controller for Uncertain Level Control Process with Actuator and System Component Uncertainty. In *Fuzzy Information Processing 2020*; Advances in Intelligent Systems and Computing; Bede, B., Ceberio, M., De Cock, M., Kreinovich, V., Eds.; Springer: Cham, Switzerland, 2020; Volume 1337, pp. 183–195. [CrossRef]
7. Verma, M.; Ghritlahre, H.K.; Chaurasiya, P.K.; Ahmed, S.; Bajpai, S. Optimization of wind power plant sizing and placement by the application of multi-objective genetic algorithm (GA) in Madhya Pradesh, India. *Sustain. Comput. Inform. Syst.* **2021**, *32*, 100606. [CrossRef]
8. Patel, H.R.; Shah, V.A. A Fractional and Integer Order PID Controller for Nonlinear System: Two Non-interacting Conical Tank Process Case Study. In *Advances in Control Systems and its Infrastructure*; Lecture Notes in Electrical Engineering; Mehta, A., Rawat, A., Chauhan, P., Eds.; Springer: Singapore, 2020; Volume 604, pp. 37–55. [CrossRef]
9. Luo, J.; Chen, H.; Zhang, Q.; Xu, Y.; Huang, H.; Zhao, X. An improved grasshopper optimization algorithm with application to financial stress prediction. *Appl. Math. Model.* **2018**, *64*, 654–668. [CrossRef]
10. Zadeh, L.A. Fuzzy sets. *Inf. Control.* **1965**, *8*, 338–353. [CrossRef]
11. Zadeh, L.A. Fuzzy logic. *Computer* **1988**, *21*, 83–93. [CrossRef]
12. Zadeh, L.A. Fuzzy logic computing with words. *IEEE Trans. Fuzzy Syst.* **1996**, *4*, 103–111. [CrossRef]
13. Patel, H.R.; Shah, V.A. Fuzzy logic based passive fault tolerant control strategy for a single-tank system with system fault and process disturbances. In Proceedings of the 2018 5th International Conference on Electrical and Electronic Engineering (ICEEE), Istanbul, Turkey, 3–5 May 2018; pp. 257–262. [CrossRef]
14. Patel, H.R.; Shah, V.A. A Framework for Fault-tolerant Control for an Interacting and Non-interacting Level Control System using AI. In Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics ICINCO, Porto, Portugal 29–31 July 2018; Volume 1, pp. 180–190. [CrossRef]
15. Liang, Q.; Mendel, J.M. Interval type-2 fuzzy logic systems: Theory and design. *IEEE Trans. Fuzzy Syst.* **2000**, *8*, 535–550. [CrossRef]
16. Du, Z.; Kao, Y.; Zhao, X. An input delay approach to interval type-2 fuzzy exponential stabilization for nonlinear unreliable networked sampled-data control systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2021**, *51*, 3488–3497. [CrossRef]
17. Patel, H.R.; Shah, V.A. Stable fuzzy controllers via LMI approach for non-linear systems described by type-2 T–S fuzzy model. *Int. J. Intell. Comput. Cybern.* **2021**, *14*, 509–531.
18. Patel, H.R., Shah, V.A. Passive Fault-Tolerant Control Based on Interval Type-2 Fuzzy Controller for Coupled Tank System. In *Proceedings of International Conference on Artificial Intelligence, Smart Grid and Smart City Applications. AISGSC 2019*; Kumar, L., Jayashree, L., Manimegalai, R., Eds.; Springer: Cham, Switzerland, 2020; pp. 145–154. [CrossRef]
19. Patel, H.R.; Shah, V.A. Fault Tolerant Controller Using Interval Type-2 TSK Logic Control Systems: Application to Three Interconnected Conical Tank System. In *Fuzzy Techniques: Theory and Applications*; IFSA/NAFIPS 2019; Advances in Intelligent Systems and Computing; Kearfott, R., Batyrshin, I., Reformat, M., Ceberio, M., Kreinovich, V., Eds.; Springer: Cham, Switzerland, 2019; Volume 1000, pp. 466–482.
20. Patel, H.R.; Shah, V.A. Stable Fault Tolerant Controller Design for Takagi–Sugeno Fuzzy Model-Based Control Systems via Linear Matrix Inequalities: Three Conical Tank Case Study. *Energies* **2019**, *12*, 2221. [CrossRef]
21. Patel, H.R.; Shah, V.A. Fault Tolerant Control Design Based on Takagi-Sugeno Fuzzy Logic: Application to a Three-Tank System. In *Proceedings of the 11th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2019)*; Springer: Cham, Switzerland, 2019; pp. 256–266. [CrossRef]
22. Patel, H.R.; Shah, V.A. Design of Interval Type 2 Fuzzy Fault-Tolerant Controller for a Non-Minimum Phase System: Application to quadruple conical tank system. In *Proceedings of the 11th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2019)*, Springer: Cham, Switzerland, 2019; pp. 358–369. [CrossRef]
23. Patel, H.R.; Shah, V.A. Fault Tolerant Control Using Interval Type-2 Takagi-Sugeno Fuzzy Controller for Nonlinear System. In *Intelligent Systems Design and Applications*; Advances in Intelligent Systems and Computing; Abraham, A., Cherukuri, A., Melin, P., Gandhi, N., Eds.; Springer: Cham, Switzerland, 2018; Volume 941, pp. 150–164.
24. Mendel, J.M. Advances in type-2 fuzzy sets and systems. *Inf. Sci.* **2007**, *177*, 84–110. [CrossRef]
25. Patel, H.R.; Shah, V.A. General Type-2 Fuzzy Logic Systems Using Shadowed Sets: A New Paradigm Towards Fault-Tolerant Control. In Proceedings of the 2021 Australian & New Zealand Control Conference (ANZCC), Gold Coast, Australia, 25–26 November 2021; pp. 116–121.

26. Patel, H.R.; Shah, V.A. Actuator and system component fault tolerant control using interval type-2 Takagi-Sugeno fuzzy controller for hybrid nonlinear process *Int. J. Hybrid Intell. Syst.*, **2019**, *15*, 143–153. [CrossRef]

27. Lagunes, M.L.; Castillo, O.; Soria, J. Methodology for the optimization of a fuzzy controller using a bio-inspired algorithm. In Proceedings of the North American Fuzzy Information Processing Society Annual Conference, Cancun, Mexico, 16 October 2017; Springer: Cham, Switzerland, 2018; Volume 648, pp. 131–137.

28. Lagunes, M.L.; Castillo, O.; Soria, J. Optimization of membership function parameters for fuzzy controllers of an autonomous mobile robot using the firefly algorithm. In *Fuzzy Logic Augmentation of Neural and Optimization Algorithms: Theoretical Aspects and Real Applications*; Studies in Computational Intelligence; Springer: Berlin, Germany, 2018; Volume 749, pp. 199–206.

29. Patel, H.R. Fuzzy Based Metaheuristic Algorithm for Optimization of Fuzzy Controller: Fault Tolerant Control Application. *Int. J. Intell. Comput. Cybern.* **2022**, *15*, 599–624. [CrossRef]

30. Wagner, C.; Hagras, H. A genetic algorithm based architecture for evolving type-2 fuzzy logic controllers for real world autonomous mobile robots. In Proceedings of the 2007 IEEE International Fuzzy Systems Conference, London, UK, 23–26 July 2007; pp. 1–6.

31. Astudillo, L.; Castillo, O.; Melin, P.; Alanis, A.; Soria, J.; Aguilar, L.T. Intelligent control of an autonomous mobile robot using type-2 fuzzy logic. *Eng. Lett.* **2006**, *13*, 199–206.

32. Wu, D.; Tan, W.W. Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers. *Eng. Appl. Artif. Intell.* **2006**, *19*, 829–841. [CrossRef]

33. Wu, D. On the fundamental differences between interval type-2 and type-1 fuzzy logic controllers. *IEEE Trans. Fuzzy Syst.* **2006**, *20*, 832–848. [CrossRef]

34. Wu, D.; Tan, W.W. A type-2 fuzzy logic controller for the liquid-level process. In Proceedings of the 2004 IEEE International Conference on Fuzzy Systems (IEEE Cat. No. 04CH37542), Budapest, Hungary, 25–29 July 2004; pp. 953–958.

35. Yang, X.-S. *Nature-Inspired Metaheuristic Algorithms*; Luniver Press: Beckington, UK, 2010.

36. Patel, H.R.; Shah, V.A. Comparative Study of Interval Type-2 and Type-1 Fuzzy Genetic and Flower Pollination Algorithms in Optimization of Fuzzy Fractional Order $PI^\lambda D^\mu$ Controllers. In *Intelligent System and Computing*; Yi, Y., Ed.; IntechOpen: Rijeka, Croatia, 2020. [CrossRef]

37. Arqub, O.A.; Al-Smadi. M. Fuzzy conformable fractional differential equations: Novel extended approach and new numerical solutions. *Soft Comput.* **2020**, *24*, 12501–12522. [CrossRef]

38. Abu Arqub, O.; AL-Smadi, M.; Momani, S. Numerical solutions of fuzzy differential equations using reproducing kernel Hilbert space method. *Soft Comput.* **2016**, *20*, 3283–3302.

39. Abu Arqub, O. Adaptation of reproducing kernel algorithm for solving fuzzy Fredholm–Volterra in-tegrodifferential equations. *Neural Comput. Appl.* **2017**, *28*, 1591–1610. [CrossRef]

40. Abu Arqub, O.; Singh, J.; Maayah, B.; Alhodaly, M.S. Reproducing kernel approach for numerical solutions of fuzzy fractional initial value problems under the Mittag–Leffler kernel differential operator. *Math. Methods Appl. Sci.* **2021**. [CrossRef]

41. Abu Arqub, O.; Singh, J.; Maayah, B.; Alhodaly, M.S. Adaptation of kernel functions-based approach with Atangana–Baleanu–Caputo distributed order derivative for solutions of fuzzy fractional Volterra and Fredholm integrodifferential equations. *Math. Methods Appl. Sci.* **2021**. [CrossRef]

42. Saima H.; Mojtaba Ahmadieh K.; Nazar Kalaf H.; Samir Brahim B.; Usman Amjad and Wali Khan M. Optimization of Interval Type-2 Fuzzy Logic System using Grasshopper Optimization Algorithm for Electricity Load and Price Forecasting. *Comput. Mater. Contin.* **2022**, *71*, 3513–3531. [CrossRef]

43. Abdel-Basset, M.; Mohamed, R.; Saber, S.; Askar, S.S.; Abouhawwash, M. Modified Flower Pollination Algorithm for Global Optimization. *Mathematics* **2021**, *9*, 1661.

44. Templos-Santos, J.L.; Aguilar-Mejia, O.; Peralta-Sanchez, E.; Sosa-Cortez, R. Parameter Tuning of PI Control for Speed Regulation of a PMSM Using Bio-Inspired Algorithms. *Algorithms* **2019**, *12*, 54. [CrossRef]

45. Abdel-Basset, M.; Mohamed, R.; Mohammad, N.; Sallam, K.; Moustafa, N. An Adaptive Cuckoo Search-Based Optimization Model for Addressing Cyber-Physical Security Problems. *Mathematics* **2021**, *9*, 1140. [CrossRef]

46. Ali, E.M.; Abdelsalam, A.K.; Youssef, K.H.; Hossam-Eldin, A.A. An Enhanced Cuckoo Search Algorithm Fitting for Photovoltaic Systems' Global Maximum Power Point Tracking under Partial Shading Conditions. *Energies* **2021**, *14*, 7210. [CrossRef]

47. Mpanza, L.J.; Pedro, J.O. Optimised Tuning of a PID-Based Flight Controller for a Medium-Scale Rotorcraft. *Algorithms* **2021**, *14*, 178 . [CrossRef]

48. Rezk, H.; Fathy, A. Stochastic Fractal Search Optimization Algorithm Based Global MPPT for Triple-Junction Photovoltaic Solar System. *Energies* **2000**, *13*, 4971. [CrossRef]

49. Ren, Y.; Li, H.; Lin, H.-C. Optimization of Feedforward Neural Networks Using an Improved Flower Pollination Algorithm for Short-Term Wind Speed Prediction. *Energies* **2019**, *12*, 4126. [CrossRef]

50. Khan, M.S.A.; Jana, C.; Khan, M.T.; Mahmood, W.; Pal, M.; Mashwani, W.K. Extension of GRA method for multiattribute group decision making problem under linguistic Pythagorean fuzzy setting with incomplete weight information. *Int. J. Intell. Syst.* **2022**, *37*, 9726–9749. [CrossRef]

51. Ali, J.; Bashir, Z.; Rashid, T.; Mashwani, W.K. A q-rung orthopair hesitant fuzzy stochastic method based on regret theory with unknown weight information. *J. Ambient. Intell. Humaniz.* **2022**. [CrossRef]

52. Kukker, A.; Sharma, R. Genetic Algorithm-Optimized Fuzzy Lyapunov Reinforcement Learning for Nonlinear Systems. *Arab. J. Sci. Eng.* **2020**, *45*, 1629–1638. [CrossRef]

53. Kukker, A.; Sharma, R. Stochastic Genetic Algorithm-Assisted Fuzzy Q-Learning for Robotic Manipulators. *Arab. J. Sci. Eng.* **2021**, *46*, 9527–9539. [CrossRef]

54. Kumar, A.; Sharma, R. Linguistic Lyapunov reinforcement learning control for robotic manipulators. *Neurocomputing* **2018**, *272*, 84–95. [CrossRef]

55. Kumar, A.; Sharma, R. Fuzzy Lyapunov Reinforcement Learning for Non-Linear Systems. *ISA Trans.* **2017**, *67*, 151–159. [CrossRef]

56. Amador-Angulo, L.; Castillo, O.; Pulido, M. Comparison of fuzzy controllers for the water tank with type-1 and type-2 fuzzy logic. In Proceedings of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), Edmonton, AB, Canada, 24–28 June 2013; pp. 1062–1067.

57. Sola, H.B.; Fernandez, J.; Hagras, H.; Herrera, F.; Pagola, M.; Barrenechea, E. Interval Type-2 Fuzzy Sets are Generalization of Interval-Valued Fuzzy Sets: Toward a Wider View on Their Relationship. *IEEE Trans. Fuzzy Syst.* **2015**, *23*, 1876–1882. [CrossRef]

58. Coupland, S.; John, R. Geometric type-1 and type-2 fuzzy logic systems. *IEEE Trans. Fuzzy Syst.* **2007**, *15*, 3–15. [CrossRef]

59. Mendel, J.M.; Liu, F.; Zhai, D. α-plane representation for type2 fuzzy sets: Theory and applications. *IEEE Trans. Fuzzy Syst.* **2009**, *17*, 1189–1207. [CrossRef]

60. Pedrycz, W.; Song, M. Granular fuzzy models: A study in knowledge management in fuzzy modeling. *Int. J. Approx. Reason.* **2012**, *53*, 1061–1079. [CrossRef]

61. Pedrycz, W. From fuzzy sets to shadowed sets: Interpretation and computing. *Int. J. Intell. Syst.* **2009**, *24*, 48–61. [CrossRef]

62. Mitra, S.; Pedrycz, W.; Barman, B. Shadowed c-means: Integrating fuzzy and rough clustering. *Pattern Recognit.* **2010**, *43*, 1282–1291. [CrossRef]

63. Wijayasekara, D.; Linda, O.; Manic, M. Shadowed Type-2 Fuzzy Logic Systems. In Proceedings of the 2013 IEEE Symposium on Advances in Type-2 Fuzzy Logic Systems (T2FUZZ), Singapore, 16–19 April 2013; pp. 15–22.

64. Gonzalez, C.I.; Melin, P.; Castillo, O.; Juarez, D.; Castro, J.R. Toward general type-2 fuzzy logic systems based on shadowed sets. In *Advances in Fuzzy Logic and Technology*; Springer: Cham, Switzerland, 2017; pp. 131–142.

65. Castillo, O.; Peraza, C.; Ochoa, P.; Amador-Angulo, L.; Melin, P.; Park, Y.; Geem, Z.W. Shadowed Type-2 Fuzzy Systems for Dynamic Parameter Adaptation in Harmony Search and Differential Evolution for Optimal Design of Fuzzy Controllers. *Mathematics* **2021**, *9*, 2439. [CrossRef]

66. Castillo, O.; Melin, P.; Valdez, F.; Soria, J.; Ontiveros-Robles, E.; Peraza, C.; Ochoa, P. Shadowed Type-2 Fuzzy Systems for Dynamic Parameter Adaptation in Harmony Search and Differential Evolution Algorithms. *Algorithms* **2019**, *12*, 17. [CrossRef]

67. Ontiveros-Robles, E.; Melin, P. A hybrid design of shadowed type-2 fuzzy inference systems applied in diagnosis problems. *Eng. Appl. Artif. Intell.* **2019**, *86*, 43–55. [CrossRef]

68. Chatterjee, K.; De, A.; Felix, T.S. Chan. Real time traffic delay optimization using shadowed type-2 fuzzy rule base. *Appl. Soft Comput.* **2019**, *74*, 226–241. [CrossRef]

69. Ochoa, P.; Castillo, O.; Melin, P.; Soria, J.S. Differential Evolution with Shadowed and General Type-2 Fuzzy Systems for Dynamic Parameter Adaptation in Optimal Design of Fuzzy Controllers. *Axioms* **2021**, *10*, 194. [CrossRef]

70. Yang, X.-S.; Deb, S. Cuckoo search via Lèvy flights. In Proceedings of the World Congress on Nature & Biologically Inspired Computing (NABIC '09), Coimbatore, India, 9–11 December 2009; pp. 210–214.

71. Walton, S.; Hassan, O.; Morgan, K.; Brown, M.R. A review of the development and applications of the Cuckoo search algorithm. In *Swarm Intelligence and Bio-Inspired Computation Theory and Applications*; Yang, X.-S., Cui, Z., Xiao, R., Gandomi, A.H., Karamanoglu, M., Eds.; Elsevier: San Diego, CA, USA, 2013; pp. 257–271.

72. Yang, X.-S.; Deb, S. Engineering optimization by cuckoo search. *Int. J. Math. Model. Numer. Optim.* **2010**, *1*, 330–343.

73. Yang, X.S. Flower pollination algorithm for global optimization. In *Unconventional Computation and Natural Computation*; Springer: Berlin, Germany, 2012; Volume 7445, pp. 240–249.

74. Glover, B.J. *Understanding Flowers and Flowering: An Integrated Approach*; Oxford University Press: Oxford, UK, 2007.

75. Patel, H.R.; Shah, V.A. A metaheuristic approach for interval type-2 fuzzy fractional order fault-tolerant controller for a class of uncertain nonlinear system. *Automatika* **2022**, *63*, 656–675. [CrossRef]

76. Patel, H.R.; Shah, V.A. Type-2 fuzzy logic applications designed for active parameter adaptation in me-taheuristic algorithm for fuzzy fault-tolerant controller. *Int. J. Intell. Comput. Cybern.* **2022**, *ahead-of-print*. [CrossRef]

77. Willmer, P. *Pollination and Floral Ecology*; Princeton University Press: Princeton, NJ, USA, 2011.

78. Balasubramani, K.; Marcus, K. A Study on Flower Pollination Algorithm and Its Applications. *Int. J. Appl. Innov. Eng. Manag. (IJAIEM)* **2014**, *3*, 320–325.

79. Abdel-Raouf, O.; Abdel-Baset, M. A new hybrid flower pollination algorithm for solving constrained global optimization problems. *Int. J. Appl. Oper.-Res.* **2014**, *4*, 1–13.

80. Raval, S., Patel, H.R., Patel, S., Shah, V.A. Passive Fault-Tolerant Control Scheme for Nonlinear Level Control System with Parameter Uncertainty and Actuator Fault. In *Applications of Fuzzy Techniques. NAFIPS 2022. Lecture Notes in Networks and Systems*; Dick, S., Kreinovich, V., Lingras, P., Eds.; Springer: Cham, Switzerland, 2023; Volume 500, pp. 229–242. [CrossRef]

81. Raval, S., Patel, H.R., Shah, V.A. Neural Network-Based Control Framework for SISO Uncertain System: Passive Fault Tolerant Approach. In *Intelligent and Fuzzy Techniques: Smart and Innovative Solutions, INFUS 2020*; Advances in Intelligent Systems and Computing; Kahraman, C., Cevik Onar, S., Oztaysi, B., Sari, I., Cebi, S., Tolga, A., Eds.; Springer: Cham, Switzerland, 2021; Volume 1197, pp. 1039–1047. [CrossRef]

82. Patel, H.R., Shah, V.A. Fault-Tolerant Controller Comparative Study and Analysis for Benchmark Two-Tank Interacting Level Control System. *SN Comput. Sci.*, **2021**, *2*, 145–154. [CrossRef]

83. Patel, H.R.; Shah, V.A. Integrated Design of Model-Based Passive Fault-Tolerant Control for Nonlinear Systems Based on PID and Fuzzy Control. In *Soft Computing: Theories and Applications. Advances in Intelligent Systems and Computing*; Pant, M., Sharma, T., Verma, O., Singla, R., Sikander, A., Eds.; Springer: Singapore, 2020; Volume 1053, pp. 155–169. [CrossRef]

84. Patel, H.R.; Shah, V.A. A Novel Design of Centralized Fractional order PID Controller and its Optimal Time Domain Tuning: A hybrid two interacting conical frustum tank level process case study. In Proceedings of the Memorias del Congreso Nacional de Control Autom A̧tico (CNCA 2019), Puebla, Mexico, 23–25 October 2019; pp. 754–761.

85. Patel, H.R.; Shah, V.A. Fault Detection and Diagnosis Methods in Power Generation Plants- The Indian Power Generation Sector Perspective: An Introductory Review *J. Energy & Manag.* **2018**, *2*, 31–49.

86. Patel, H.R.; Shah, V.A. Passive Fault Tolerant Control System Using Feed-forward Neural Network for Two-Tank Interacting Conical Level Control System Against Partial Actuator Failures and Disturbances, *IFAC-PapersOnLine*, **2019** *52*, 141–146. [CrossRef]

87. Patel, H.R.; Shah, V.A. Passive Fault-Tolerant Tracking for Nonlinear System with intermittent Fault and Time Delay. *IFAC-PapersOnLine* **2019** *52*, 200–205. [CrossRef]

88. Patel, H.R.; Shah, V.A. Passive fault-tolerant control system design with tracking control against major system faults: Application to a canonical tank level system. In *Communication and Computing Systems*; CRC Press: Boca Raton, FL, USA, 2019; pp. 98–104.

89. Patel, H.R.; Shah, V.A. Decentralized stable and robust fault-tolerant PI plus fuzzy control of MIMO systems: A quadruple tank case study, *Int. J. Smart Sens. Intell. Syst.* **2019**, *122*, 1–20. [CrossRef]

90. Lakshmanaprabu, S.K.; Wahid, N.A.; Banu Sabura, U. Design of Centralized Fractional order PI Controller for Two Interacting Conical Frustum Tank Level Process. *J. Appl. Fluid Mech.* **2017**, *10*, 1–23.

91. Patel, H.R.; Shah, V.A. Fault Tolerant Control Systems: A Passive Approaches for Single Tank Level Control System *J. Instrum. Control. Eng.* **2018**, *6*, 1–8.

92. Patel, H.R.; Shah, V.A. A passive fault-tolerant control strategy for a non-linear system: An application to the two tank conical non-interacting level control system. *Maskay* **2019**, *9*, 1–8. [CrossRef]

93. Valdez, F.; Melin, P.; Castillo, O. An improved evolutionary method with fuzzy logic for combining particle swarm optimization and genetic algorithms. *Appl. Soft Comput.* **2011**, *11*, 2625–2632. [CrossRef]

94. Olivas, F.; Valdez, F.; Melin, P.; Sombra, A.; Castillo, O. Interval type-2 fuzzy logic for dynamic parameter adaptation in a modified gravitational search algorithm. *Inf. Sci.* **2019**, *476*, 159–175. [CrossRef]

95. Eftimov, T.; Korošec, P.; Seljak, B.K. A Novel Approach to statistical comparison of meta-heuristic stochastic optimization algorithms using deep statistics. *Inf. Sci.* **2017**, *417*, 186–215. [CrossRef]

96. Ontiveros-Robles, E.; Melin, P.; Castillo, O. Comparative analysis of noise robustness of type 2 fuzzy logic controllers. *Kybernetika* **2018**, *54*, 175–201. [CrossRef]