Shape Priors for Level Set Representations

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Abstract. Level Set Representations, the pioneering framework introduced by Osher and Sethian [14] is the most common choice for the implementation of variational frameworks in Computer Vision since it is implicit, intrinsic, parameter and topology free. However, many Computer vision applications refer to entities with physical meanings that follow a shape form with a certain degree of variability. In this paper, we propose a novel energetic form to introduce shape constraints to level set representations. This formulation exploits all advantages of these representations resulting on a very elegant approach that can deal with a large number of parametric as well as continuous transformations. Furthermore, it can be combined with existing well known level set-based segmentation approaches leading to paradigms that can deal with noisy, occluded and missing or physically corrupted data. Encouraging experimental results are obtained using synthetic and real images.

1 Introduction

Level Set [14,13,19] and variational methods are increasingly considered by the vision community [17]. The application domain is wide and not restricted to image segmentation, restoration, impainting, tracking, shape from shading, 3D reconstruction [7], medical image segmentation [11], etc. These techniques have been exhaustively studied and also applied to other scientific domains like geometry, robotics, fluids, semiconductors designing, etc. [19].

Most of the mentioned applications share a common concern, tracking moving interfaces. Level Set representations are well suited computational methods to perform this task. They can be used to any dimension (curves, surfaces, hyper-surfaces, ...), are parameter free and can change naturally the topology. Moreover, they provide a natural way to estimate the geometric properties of the evolving interface.

Furthermore, they can deal with non-rigid objects and motions, since they refer to very local characteristics and can deform an interface pixel-wise. Opposite to that, they

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have a poor performance compared with parametric models when solid/rigid motions and objects are considered mainly because local propagations are quite sensitive to noise and fail to take fully advantage of *a priori* physical constraints like solid shape models. It is clear that evolving interfaces using level set representations is a powerful tool with certain strengths and some limitations. For example, this property (locality) is not helpful when the considered task refers to the extraction of solid objects, while it is a vital element when non-rigid motions and objects are considered.

Our visual space consists of objects from both categories. For example most of the active human organs cannot be considered solid, but at the same time their forms are well constrained within a family of shapes. This family cannot be fully characterized using parametric models. The use of level set-based methods is suitable for this kind of applications due to their ability of dealing with local deformations.

In this paper we consider a challenging application: constrain the level set representations to follow a shape global consistency while preserving the ability to capture local deformations. The most closely related work with our approach can be found in [4,6,10] and more recently in [21]. In [10] a two stage approach was proposed that integrates prior shape knowledge and visual information. During the first step, a segmentation solution is obtained according to a data-driven term, while during the second step a correction of the result is performed using a level set shape prior model that is obtained through a Principal Component Analysis. The same modeling technique is used in [21]. These two steps alternate until convergence is reached. In [4,21] a different technique was considered that refers to an optimization criterion with objective to recover a transformation that better maps the evolving interface to a shape prior term. This criterion is shape driven and in [4] aims at minimizing the Euclidean distance between the model and the prior while in [21] a region-driven criterion is considered that aims at minimizing a metric defined on the level set space. In [4], the shape prior was obtained by averaging the registered training examples and refers to a collection of points. Theoretical comparison of our approach with the ones proposed up to now in can be found in Section 5.

A novel mathematical functional is proposed in this paper that can account for global/local shape properties of the object to be recovered. This functional can be combined with any level set objective function under the assumption that a shape model with a certain degree of variability is available. Our approach consists of two stages. During the first stage a shape model is built directly on the level set space using a collection of samples. This model is constructed using a variational framework that creates a non-stationary pixel-wise model that accounts for shape variabilities. Then, this model is used as basis to introduce the shape prior in an energetic form. This prior aims at minimizing the non-stationary distance between the evolving interface and the shape model in terms of their level set representations. In order to demonstrate the performance of the proposed module, it is integrated with a data-driven variational method to perform image segmentation for physically corrupted and incomplete data.

The remainder of this paper is organized as follows: in section 2, we briefly present the level set representations. Section 3 is dedicated to the construction of the shape prior model using a a certain number of examples. In Section 4, we introduce our shape-prior energetic functional that is integrated with a data-driven variational framework. Finally, discussion and conclusions are part of Section 5.

2 Level Set Representations

Let us consider a parameterized closed evolving interface in a Euclidean plane $[C : [0,1] \rightarrow \mathcal{R}^2,\ p \rightarrow C(p)]$ and let $C(p,t)$ the family of interfaces generated by the propagation of the initial one $C_0(p)$ in the direction of the inward normal $\mathcal{N}$. Under the assumption that the propagation is guided by a scalar function $[F]$ of the geometric properties of the curve (i.e. curvature $\mathcal{K}$), we can have the following motion equation:

$$\begin{cases} C(p,0) &= C_0(p) \\ C_t(p) &= F(\mathcal{K}(p))\, \mathcal{N}(p) \end{cases}, \tag{1}$$

The implementation of this evolution can be done using a Lagrangian approach. In that case we produce the associated equations of motion for the position vector $(x,y) = C(p)$ and we update them according to a difference approximation scheme. As a consequence, the evolving interface cannot change its topology (with the exception of [12]).

To overcome this limitations, *Osher and Sethian* [14,13,19] have proposed to represent the evolving interface $C(p)$ with a zero-level set ($\phi = 0$) function of a surface z

$$\left[z = (x, y, \phi(x,y,t)) \in \mathcal{R}^3\right] \tag{2}$$

Deriving $\phi(x,y,t) = 0$ with respect to time and space (given [**eq.** (1)]) we obtain the following motion for the embedding surface $\phi()$:

$$\begin{cases} \phi\,(C_0(p), 0) &= 0 \\ \phi_t(p) &= F(\mathcal{K}(p))\, |\nabla\phi(p)| \end{cases}, \tag{3}$$

where $[|\nabla\phi|]$ is the norm of gradient and $[\mathcal{N} = -\frac{\nabla\phi}{|\nabla\phi|}]$.

Thus, we have established a connection between the family $C(p,t)$ and the family of one parameter surfaces $\phi(x,y,t)$ where the zero iso-surface of the function ϕ yields always to the evolving interface.

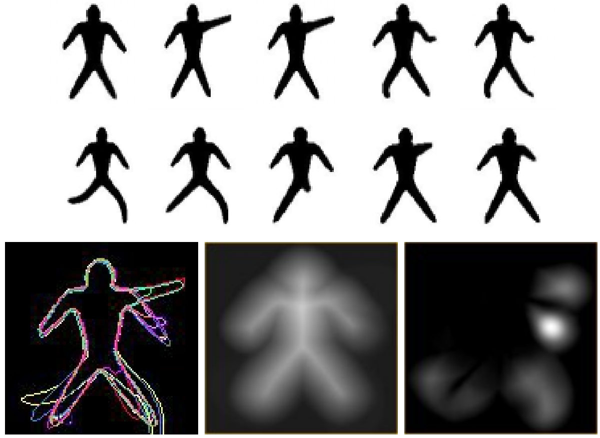
The embedding surface $\phi(p)$ remains a function as long as F is smooth and the evolving interface $C(p)$ can change topology. Additionally, numerical simulations on $\phi(p)$ may be developed trivially and intrinsic geometric properties of the evolving interface can be estimated directly from the level set function. Finally, the method can be easily extended to deal with problems in higher dimensions. A very common selection for the embedding function refers to the use of (Euclidean) distance transforms.

3 Shape Prior Model Construction

A vital component for most of the approaches that aimed at creating shape representations is the alignment of the training samples. Matching geometric shapes is an open as well as complex issue in computer vision that has been exhaustively studied. A complete review of the literature in shape matching can be found in [23].

(1)

(2)



(a) (b) (c)

Fig. 1. *(1) Trainning Samples, (2) Shape Prior Model, (a) Aligned Shapes, (b) Shape Prior Representation, (c) Model Variability.*

We consider a variational approach that is based on a shape-to-area principle for the alignment [16]. This framework exploits maximally the information of the level set representations. The central idea behind this approach is to perform the registration using the level set representations of the training examples. Therefore, we seek for shape-to-area transformations that best match the level set representations of the shapes of the training set. Thus, given a source shape $\mathcal{S}$ and a target shape $\mathcal{D}$ as well as their level set representations using distance transforms, registration is obtained by seeking a global transformation [A] with a scale component [s] that minimizes the following dissimilarity measure[1]:

$$E(\Phi_D, \Phi_S, A) = \iint_{\Omega} \left(s\Phi_S(x,y) - \Phi_D(A(x,y))\right)^2 dxdy$$

A detailed description and the algorithm and the extension to deal with local deformations can be found in [16]. The output of this procedure is a set of N level set representations (one for each training sample) [$\hat{\Phi}_i$] registered to an arbitrary selected (from the family of samples) reference shape [$\hat{\Phi}_0$]. The efficiency of this framework is shown in [**fig.** (1)].

The next step is the construction of the shape model, using the aligned contours. Point-based snake models [8], deformable models/templates [2], active shapes [5], level set representations [3], etc. are common selections. Although these representations are powerful enough to capture a certain number of local deformations, they require a large number of parameters to deal with important shape deformations. Moreover, they cannot deal with changes of topology (with the exception of level set methods [3] and the approach presented in [12]). Finally, their extension to describe structures of higher dimension than curves and surfaces is not trivial. In the Level Set literature, two models

[1] One can easily prove that level set representation are invariant to translation and rotation but not to scale variations. However, as it will be explained later in the paper, one can predict the effect of scale variations and the appearance of the scale parameter in the energy function.

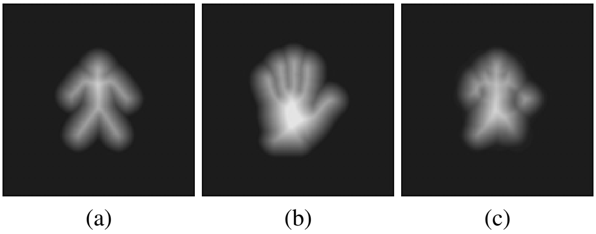


Fig. 2. *Construction of a Shape Prior Model using two very different aligned trainning samples. (a) Training Sample 1, (b) Training Sample 2, (c) Shape Prior Model.*

have been presented up to now. In [10,21] the use of a global statistical representation on the level set space was proposed while a different approach was considered in [4] where shapes are represented using a collection of points.

We consider a more challenging approach where the objective is to generate a shape model that accounts for local variations as well. In order to do that, we consider a stochastic framework with two unknown variables:

- The shape image, $\Phi_M(x,y)$,
- The local degrees (variability) of shape deformations $\sigma_M(x,y)$.

where each grid location can be described in the shape model using a Gaussian density function

$$p^M_{x,y}(\phi) = \frac{1}{\sqrt{2\pi}\sigma_M(x,y)} e^{-\frac{(\phi-\Phi_M(x,y))^2}{2\sigma^2_M(x,y)}}$$

A similar framework for a different purpose was proposed in [24].

The mean of this probability density function corresponds to the level set function, while the variance refers to the variation of the aligned samples in this location. On top of these assumptions, we impose the constraint that mean values of the shape model refer to a signed distance function (level set representation).

Thus given N aligned training samples (level set representations) where $\hat{\Phi}_i$ is the aligned transformation of Φ_i, we can construct a variational framework for the estimation of the BEST shape by seeking for the maximum likelihood of the local densities with respect to (Φ_M, σ_M):

$$E(\Phi_M, \sigma_M) = -\sum_{i=1}^{n} \iint_{x,y} \mathbf{log}\left[p^M_{x,y}(\hat{\Phi}_i(x,y))\right] dxdy$$
$$\text{SUBJECT TO THE CONSTRAINT}: |\nabla\Phi_M(x,y)|^2 = 1, \quad \forall(x,y) \in \Omega$$

Additionally, we can enforce spatial coherence on the variability estimates by adding a smoothness term. Since the constant term ($\sqrt{2\pi}$) does not affect the minimization procedure, the following functional is used:

$$
\begin{aligned}
E(\Phi_M, \sigma_M) =& (1-\alpha) \iint_\Omega \left(\left(\frac{d}{dx}\sigma_M(x,y) \right)^2 + \left(\frac{d}{dy}\sigma_M(x,y) \right)^2 \right) dxdy \\
&+ \alpha \iint_\Omega \sum_{i=1}^{n} \left(\mathbf{log}\left[\sigma_M(x,y)\right] + \frac{(\hat{\Phi}_i(x,y) - \Phi_M(x,y))^2}{2\sigma_M^2(x,y)} \right) dxdy \\
&\text{SUBJECT TO THE CONSTRAINT : } |\nabla \Phi_M(x,y)|^2 = 1, \quad \forall (x,y) \in \Omega
\end{aligned}
$$

where $[a]$ is a blending parameter between the two energy terms. The interpretation of the objective function is rather simple. The first component is data driven and aims at recovering a level set representation that best accounts for the training samples. The second term is a smoothness constraint on the representation variability. Neighborhood pixels for all registered examples of the training set have to exhibit similar variability properties.

The constrained optimization of this functional can be done using Lagrange multipliers and a gradient descent method. However, given the form of constraints (involvement of first and second order derivatives), we cannot obtain a closed form solution and prove that the conditions which guarantee the validity of Lagrange theorem are satisfied. Moreover, the number of unknown variables of the system is too high $O(N^2)$ and the system is quite unstable especially when there is large variability among training samples. A possible way to overcome this limitation that is currently investigated refers to the use of an augmented Lagrangian function, but even in that case the proof of validity and the initial conditions are open issues.

An alternative selection refers to a two step optimization process, that separates the two conditions. During the first step, we obtain the "optimal" solution according to the data driven terms, while during the second step we find the "optimal" projection of this solution to the manifold of acceptable solutions (distance functions).

Thus, the unknown variables are obtained by minimizing the previously defined data-driven objective function that preserves some regularity conditions:

$$
\begin{cases}
\frac{d}{dt}\Phi_M = \alpha \sum_{i=1}^{n} \frac{(\hat{\Phi}_i - \Phi_M)}{2\sigma_M^2} \\
\frac{d}{dt}\sigma_M = \alpha \sum_{i=1}^{n} \left[-\frac{1}{\sigma_M} + \frac{(\Phi - \Phi_M)^2}{\sigma_M^3} \right] + (1-\alpha) \left[\frac{\partial^2}{\partial x \partial x}\sigma_M + \frac{\partial^2}{\partial y \partial y}\sigma_M \right]
\end{cases}
$$

while the projection/correction to the manifold space of accepted solutions (Level Set Representations)[2] is done using a heavily considered Partial Differential Equation [20]:

$$
\left\{ \frac{d}{dt}\Phi_M = \left(1 - \operatorname{sgn}\left(\Phi_M^0\right)\right)\left(1 - |\nabla \Phi_M|\right) \right.
$$

where Φ_M^0 is the initial representation (data driven).

[2] The use of the data driven term will modify the evolving representation without respecting the constraint of being a distance function.

These two steps alternate until the system reaches a steady-state solution. Upon convergence of the system, we will obtain a level set representation model, that optimally expresses the properties of he training set using degrees of variability that are constrained to be locally smooth. As far as the initial conditions of the system are concerned, we use the level set representation of the reference sample while the variability estimates are set equal to one for the whole image plane. In order to avoid stability problems, one can replace the variability factors with

$$\sigma_M = 1 + \hat{\sigma}_M$$

and then seek for the estimates of $\hat{\sigma}_M$ that are constrained to be strictly positive. The performance of our method is demonstrated in [**fig.** (2)] where two very different training samples are used to generate a model that integrates information from both shapes. An example using training samples from the same family is shown in [**fig.** (1)].

4 Level Set Shape Priors

Let us now consider an image where an object with a shape form similar to the one of the training samples is present. Then, the objective is to recover the image area that corresponds to this object. At the very beginning, we will introduce our approach without using any data-driven term.

4.1 Shape-Driven Propagation

Let $\Phi : \Omega \times \mathcal{R}^+ \to \mathcal{R}^+$ be a Lipchitz function that refers to level set representation that is evolving over time $[t]$ given by,

$$\Phi(x,y;t) = \begin{cases} 0 & , (x,y) \in \partial\mathcal{R}(t) \\ +\,\mathcal{D}((x,y),\partial\mathcal{R}(t)) > 0 & , (x,y) \in \mathcal{R}(t) \\ -\,\mathcal{D}((x,y),\partial\mathcal{R}(t)) < 0 & , (x,y) \in [\Omega - \mathcal{R}(t)] \end{cases}$$

where $\partial\mathcal{R}(t)$ refers to the interface (boundaries) of $\mathcal{R}(t)$, $\mathcal{D}((x,y),\partial\mathcal{R}(t))$ **the minimum Euclidean distance between the pixel** (x,y) **and the interface** $\mathcal{R}(t)$ and t at time. Let us also define the approximations of Dirac and Heaviside [26] distributions as:

$$\delta_a(\phi) = \begin{cases} 0 & , |\phi| > \alpha \\ \frac{1}{2\alpha}\left(1 + \cos\left(\frac{\pi\phi}{a}\right)\right) & , |\phi| < \alpha \end{cases}$$

$$H_\alpha(\phi) = \begin{cases} 1 & , \phi > \alpha \\ 0 & , \phi < -\alpha \\ \frac{1}{2}\left(1 + \frac{\phi}{\alpha} + \frac{1}{\pi}\sin\left(\frac{\pi\phi}{a}\right)\right) & , |\phi| < \alpha \end{cases}$$

Then it can be shown easily that

$$\begin{array}{l} \{(x,y) \in \Omega : \lim_{\alpha \to 0^+} \left[H_\alpha\left(\Phi((x,y);t)\right)\right] = 1\} = \mathcal{R} \\ \{(x,y) \in \Omega : \lim_{\alpha \to 0^+} \left[\delta_\alpha(\Phi((x,y);t))\right] = 1\} = \partial\mathcal{R} \end{array}$$

Now, given an interface and (consequently) its level set representation, we would like to evolve it to recover a structure that respects some known shape properties $\Phi_M(x,y)$.

We assume that all instances of the evolving representation belong to the family shapes that is generated by applying all possible global transformations (according to a predefined model) to the prior shape model. This assumption is valid mainly for rigid objects.

Thus, given the current level set representation Φ, we can assume that there is an ideal transformation $A = (A_x, A_y)$ between the shape prior and the evolving representation. If we consider that noise does not effect our measures and the absence scale variations, then the optimal transformation will satisfy the following conditions,

$$\begin{cases} & (x,y) \rightarrow A(x,y) \\ \Phi(x,y) \approx \Phi_M(A(x,y)), & \forall (x,y) : H_a(\Phi(x,y)) \geq 0 \end{cases}$$

In that case, by considering a very simple optimization criterion like the sum of squared differences, the optimal transformation A should minimize the following functional:

$$E(\Phi, A) = \iint_{\Omega} H_\alpha(\Phi(x,y)) \left(\Phi(x,y) - \Phi_M(A(x,y))\right)^2 dxdy$$

In order to account for scale variations, we can assume a scale component [s] for the transformation A. It is straightforward to show that the level set representations are invariant to translation and rotation but not to scale variations. Given the properties of distance transforms from an interface, one can predict how scale changes will affect the information space: the level set representation values will be also scaled up/down according to the scale variable, resulting in the following scale/rotation/translation invariant criterion:

$$E(\Phi, A) = \iint_{\Omega} H_\alpha(\Phi(x,y)) \left(s\Phi(x,y) - \Phi_M(A(x,y))\right)^2 dxdy$$

Thus, we are seeking for a transformation that provides pixel-wise level set values correspondences between the evolving interface and the shape prior level set representation.

In order to minimize the above functional with respect to the evolving level set representation and the global linear transformation, we will assume (without loss of generality) that it is composed of $N+1$ motion parameters $A = [s, a_1, a_2, ..., a_N]$. Then, using the calculus of variations we obtain the following system of coupled equations:

$$\begin{cases} \frac{d}{dt}\Phi = -2\ s\ H_\alpha(\Phi)\left(s\Phi - \Phi_M(A)\right) + \delta_\alpha(\Phi)\left(s\Phi - \Phi_M(A)\right)^2 \\ \frac{d}{dt}s = -2\iint_{\Omega}\left[H_\alpha(\Phi)\left(s\Phi - \Phi_M(A)\right)\left(\Phi - \nabla\Phi_M(A)\cdot\frac{\partial}{\partial s}\left(A_x, A_y\right)\right)\right] \\ \forall j \in [1, N], \\ \frac{d}{dt}a_j = 2\iint_{\Omega}\left[H_\alpha(\Phi)\left(s\Phi - \Phi_M(A)\right)\left(\nabla\Phi_M(A)\cdot\frac{\partial}{\partial a_j}\left(A_x, A_y\right)\right)\right] \end{cases}$$

Let us now try to interpret the obtained motion equation for the time evolving level set representation term by term. We will consider $\underline{\Phi \rightarrow 0}$ to facilitate the interpretation:

– The first term $[-2sH_\alpha(\Phi)\left(s\Phi - \Phi_M(A)\right) = \Phi_M(A)]$ is positive when $\Phi_M(A)$ is positive. The physical meaning of this condition is that the projection of the considered pixel is interior to the shape prior interface. Therefore, the evolving interface has to expand locally resulting on a better fit with the model.

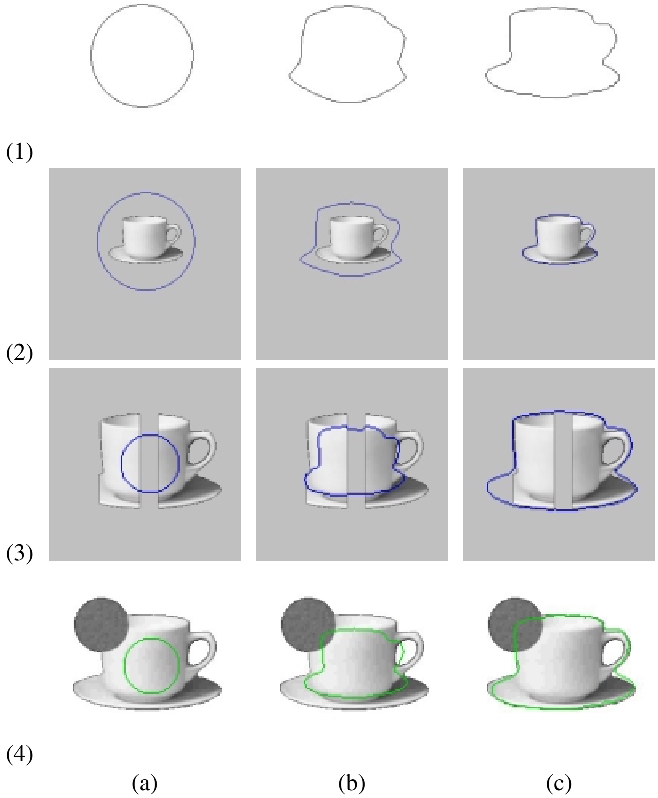


Fig. 3. *(1) Shape Prior Term, (2) Shape-Constrained Geodesic Active Contours, (3-4) Shape-Constrained Geodesic Active Regions, (a) Initial Contour, (b) Mid-Contour, (c) Final Contour. Different scales are used.*

– The second force aims at decreasing the length and the area defined by the evolving interface and consequently the value of the cost function. Therefore, one can ignore this component.

This simple/static model can have encouraging performance [**fig.** (3)]. However, it does not take into account the local shape variations and is constrained by a rigid transformation between the evolving representation and the shape prior model.

During the model construction, we have consider that the shape model can have some local degrees of variability. In that case the ideal transformation will map each value of current representation at the most probable value on the model:

$$\begin{cases} & (x, y) \to A(x, y) \\ \max_{x,y} \left\{ p^M_{A(x,y)} \left(s\Phi(x, y) \right) \right\} & \forall (x, y) : H_a(\Phi(x, y)) \geq 0 \end{cases}$$

The most probable transformation is the one ontained through the maximum likelihood for all pixels. Under the assumption that densities are independent across pixels, the minimization of the -log function of the maximum likelihood can be considered as global optimization criterion. This criterion refers to two set of unknown variables. The linear transformation A, and the level set function Φ:

$$E(\Phi, A) = -\iint_{\Omega} H_{\alpha}(\Phi(x,y))\log\left[p^{M}_{A(x,y)}(s\Phi(x,y))\right] dxdy$$
$$= \iint_{\Omega} H_{\alpha}(\Phi(x,y))\left[\log\left(\sigma_M(A(x,y))\right) + \frac{\left(s\Phi(x,y) - \Phi_M(A(x,y))\right)^2}{2\sigma^2_M(A(x,y))})\right] dxdy$$

The interpretation of this functional is rather simple; we seek a transformation and a level set representation that maximizes the posterior probability pixel-wise given the shape prior model. This model refers to a non-stationary measurement where pixels are considered according to the confidence of their projections in the shape prior model (variance term).

The minimization of this functional can be done using a gradient descent method:

$$\begin{cases} \frac{d}{dt}\Phi = -\, s\, H_{\alpha}(\Phi)\left[\frac{(s\Phi - \Phi_M(A))}{\sigma^2_M(A)})\right] - \delta_{\alpha}(\Phi)\left[\log\left(\sigma_M(A)\right) + \frac{(s\Phi - \Phi_M(A))^2}{2\sigma^2_M(A)})\right] \\ \frac{d}{dt}s = -2\iint_{\Omega} H_{\alpha}(\Phi)\left[\frac{1}{2\sigma_M(A)}\nabla\sigma_M(A)\cdot\frac{\partial}{\partial s}\left(A_x, A_y\right)\right. \\ \qquad + \frac{(s\Phi - \Phi_M(A))^2\left[\nabla\sigma_M(A)\cdot\frac{\partial}{\partial s}\left(A_x, A_y\right)\right]}{\sigma^3_M(A)} \\ \qquad \left. - \frac{(s\Phi - \Phi_M(A))\left(\Phi - \nabla\Phi_M(A)\cdot\frac{\partial}{\partial s}\left(A_x, A_y\right)\right)}{\sigma^2_M(A)}\right] \end{cases}$$

$$\begin{cases} \forall j \in [1, N], \\ \frac{d}{dt}a_j = -2\iint_{\Omega} H_{\alpha}(\Phi)\left[\frac{1}{2\sigma_M(A)}\nabla\sigma_M(A)\cdot\frac{\partial}{\partial a_j}\left(A_x, A_y\right) - \right. \\ \left. \frac{(s\Phi - \Phi_M(A))\left[\nabla\Phi_M(A)\cdot\frac{\partial}{\partial s}\left(A_x, A_y\right)\right]}{\sigma^2_M(A)} - \frac{(s\Phi - \Phi_M(A))^2\left[\nabla\sigma_M(A)\cdot\frac{\partial}{\partial a_j}\left(A_x, A_y\right)\right]}{\sigma^3_M(A)}\right] \end{cases}$$

We recall that the second term on the evolution of the level set representation is be ignored. Also due to the fact that the scale $[s]$ parameter appears in the cost function, a different motion equation is obtained in comparison with the other parameters of the registration model.

The obtained motion equations have the same interpretation with the ones presented earlier without the local variability factor. In the absence of data driven term, they will have the same behavior with the ones that do not account for local variability. On the other hand, the integration of the shape prior with data-driven terms can provide a soft-to-hard constraint. In order to demonstrate the efficiency of the proposed functional, we will integrate it with an existing well known data-driven variational framework for image segmentation.

4.2 Self-Constrained Geodesic Active Region

The geodesic active region model was originally proposed in [15], and aimed at combining boundary (in the form of Geodesic Active Contours) with some regional/global properties of the object to be recovered. This model makes the assumption that some *a priori* knowledge regarding the global region/statistical properties are available (intensities, optical flow, texture information, etc.)

The original model was defined on the image plane, and the obtained motion equation were implemented using level set mthods. Here, we will introduce a self-constrained version of this model, directly on the level set representation space. Thus, if we consider that some region-based image-based descriptor functions p_i are available that capture the intensity properties of each region, then the following objective functional

$$
\begin{aligned}
E(\Phi_i, A_i) = a \sum_{i=1}^{N} \iint_{\Omega} \delta_\alpha(\Phi_i(x,y)) g(|\nabla I(x,y)|) |\nabla \Phi_i(x,y)| \ + \\
b \sum_{i=1}^{N} \iint_{\Omega} [H_\alpha(\Phi_i(x,y)) g(p_i(I(x,y))) + (1 - H_\alpha(\Phi_i(x,y))) g(p_0(I(c,y)))] \ + \\
c \sum_{i=1}^{N} \iint_{\Omega} H_\alpha(\Phi_i(x,y)) \left[\log\left(\sigma_{M_i}(A_i(x,y))\right) + \frac{(s\,\Phi_i(x,y) - \Phi_{M,i}(A_i(x,y)))^2}{2\sigma^2_{M,i}(A_i} (x,y)) \right]
\end{aligned}
$$

where p_0 is the descriptor function that captures the background properties.

The minimization of this function with respect to the time evolving level set representations Φ_i can be done using the calculus of variations and the following equations are obtained:

$$
\begin{cases}
\forall i \in [1, N], \\
\frac{d}{dt}(\Phi_i) = a\,\delta_\alpha(\Phi_i)\,[g(I)\mathcal{K} - \nabla g(I) \nabla \Phi_i] + b\,\delta_\alpha(\Phi_i)\,[g(p_i(I)) - g(p_0(I))] - \\
\qquad c\,H_\alpha(\Phi_i)\,s\,\frac{(s\,\Phi_i - \Phi_{M,i}(A_i))}{\sigma^2_{M,i}(A)} - c\,\delta_\alpha(\Phi_i) \left[\log\left(\sigma_{M,i}(A_i) + \frac{(s\,\Phi_i - \Phi_{M,i}(A_i))^2}{2\sigma^2_{M,i}(A_i)} \right.\right]
\end{cases}
$$

that consist of three forces acting locally on the evolving interface all in the direction of the normal:

- An image-driven boundary force that shrinks the evolving interface (constrained by the curvature effect) towards the object boundaries,
- An image-driven region/statistical force that shrinks or expands the evolving interface towards the direction that optimizes the separation between the background pixels and the object pixels according to some predefined global statistical properties,
- A shape-driven force that shrinks or expands the evolving interface towards the direction that produces a segmentation result which satisfies some predefined shape constraints.

In the absence of regional information, we can consider the geodesic active contour model [3,9] that only makes use of boundary information.

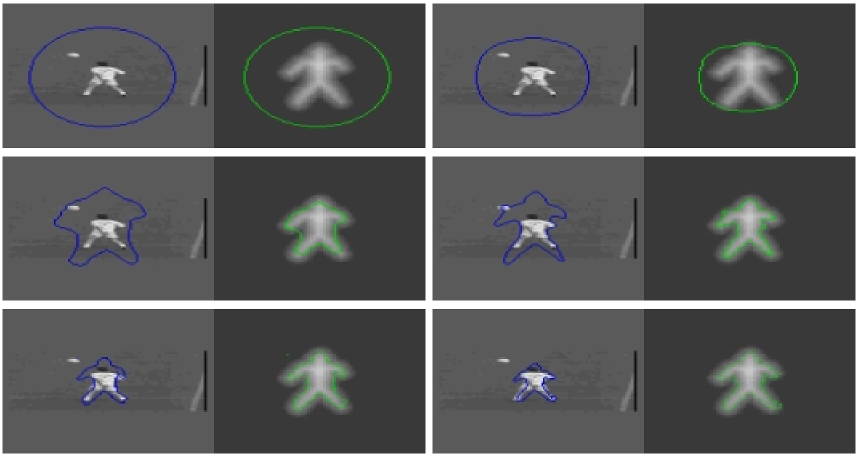


Fig. 4. *Self-Constrained Geodesic Active Regions (raster-scan format). The shape prior model was created using synthetic samples [***fig.** *3]). The projection of the evolving interface to the shape prior model is also shown.*

4.3 Implementation Issues

The last issue to be dealt with, is the numerical implementation of the proposed framework. The level set implementation is performed using the Narrow Band Method [1]. The essence of this method is to perform the level set propagation only within a limited zone (α parameter of the Dirac and Heaviside distributions) that is located around the latest position of the propagating contours (in the inward and outward direction). Thus, the working area is reduced significantly resulting on a significant decrease of the computational complexity per iteration. However, this method requires a frequent re-initialization of the level set functions that is performed using the Fast Marching algorithm [19]. A similar algorithm within the area of automatic control was proposed in [22].

5 Discussion and Summary

In this paper, we have proposed a novel approach for introducing shape priors into level set representations targeting 2D closed structures. Encouraging experimental results were obtained for real and synthetic images. Two key contributions are presented in this paper.

- The first, refers to new way of defining global-to-local shape prior models in the level set representations space according to probabilistic principles. This is obtained through a constrained variational framework that exploits maximally the information of the level set representations and can account for local degrees of variability.
- The second, refers to a novel energetic term that can account for shape priors in level set representations. This term is defined directly on the level set space and can deal with global transformations. Moreover, it can account for local variations due to the shape prior model.

Furthemore, this paper deals with registration and segmentation simultaneously. The objective is to recover a segmentation map that is in accordance with the shape prior model as well as a rigid registration between this map and the model. Due to the use of distance transforms [16] in the registration process, the method is robust to local deformations. This is due to the nature of this transformation that scales down local deformations when considere in a certain distance from the contour.

These two components have been integrated to an existing well known level set segmentation framework, the Geodesic Active Region model. The resulting functional refers to a joint optimization approach that can deal with important shape deformations, as well as with noisy physically corrupted and occluded data.

The proposed framework, to our understanding is favorably compared with the existing level set shape prior methods [4,10,21]. One can claim that our alignment method compared with the ones proposed up to now within this problem can have a better performance due to the information space that is used. Moreover, the proposed shape prior model can naturally account for local degrees of variability which is not the case for [4] and performs better than the model employed in [21]. Also, we claim that the construction of this model does not require a significant number of samples as the one proposed in [10,21]. Then, the shape prior term can account for a large variety of global transformations (opposite to [10]) as well as scale variations (opposite to [10,21]) and can deal with important local shape variations (opposite to [4]). Finally, we have proposed a robust method to estimate this transformation where a shape-to-area approach is considered that maximally exploits the information of the level set representations (opposite to [4] where pixel-wise shape correspondences are considered). Moreover, the extension of the proposed framework to deal with objects of any arbitrary dimension is trivial (opposite to [4]).

Regarding the computational cost of our approach, it is comparable (lower bound) with the one of the geodesic active region model. The most expensive part of the algorithm is the implementation of the level set propagation, and this part is common in both methods.

As far the future directions of this approach are considered, several issues remain open. A key characteristic of the Level Set Representations is the ability of changing the topology of the evolving interface. Although modeling and extracting separately multi-seeds objects can be naturally handled, we cannot detect multiple objects with different shape prior models by considering a single level set representation. At the same time, the behavior of the method is questionable when multiple initial seeds are used to recover a single object due to the assumption that there is a global transformation between the evolving representation and the shape prior model. This is a challenging perspective that will be explored in the near future. Also, the mathematical justification of the model is a step forward to be done. The use of this framework to deal simultaneously with segmentation and registration of medical volumes is a challenging application. In order to do that, investigating the use of faster numerical approximation techniques [25] to implement the proposed framework is a step to be done. Finally, the validation of the method is an open issue. Towards, this end the task of the segmentation of the left ventricle is considered where the shape prior holds a principle role during the segmentation process.

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