Shape Tracking of Extended Objects and Group Targets with Star-Convex RHMs

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***Abstract*—This paper is about tracking an extended object or a group target, which gives rise to a varying number of measurements from different measurement sources. For this purpose, the shape of the target is tracked in addition to its kinematics. The target extent is modeled with a new approach called *Random Hypersurface Model* (*RHM*) that assumes varying measurement sources to lie on scaled versions of the shape boundaries. In this paper, a star-convex *RHM* is introduced for tracking star-convex shape approximations of targets. Bayesian inference for star-convex *RHMs* is performed by means of a Gaussian-assumed state estimator allowing for an efficient recursive closed-form measurement update. Simulations demonstrate the performance of this approach for typical extended object and group tracking scenarios.**

Keywords: Target tracking, shape tracking, extended objects, group targets.

I. Introduction

A typical modeling assumption in target tracking is that the target is a mathematical point without an extent. However, in real-world tracking systems, there are essentially two major scenarios in which this standard assumption is not suitable.

First, the resolution of modern sensor devices (such as radar devices) is often higher than the spatial extent of a target object (see Fig. 1). As a consequence, several unknown points, i.e., measurement sources, on the target object may be resolved during a single scan. These measurement sources may vary from scan to scan and their locations depend on the shape of the target but also on more complex target-dependent properties (such as the nature of the surface) or even the target-to-sensor geometry.

Second, a collectively moving group of point targets can be considered as a single entity as there is high interdependency between the individual group members that is specified by the group behavior. Also, in this case the measurement sources vary from scan to scan and their locations highly depend on the properties of the group (such as the inter-group geometry).

In this sense, an *extended object* can be defined as a set of measurement sources that share a common property, e.g., dynamic behavior or a state variable. If the set consists of a finite set of measurement sources, we call it a *group target* [1]. However, in case of a continuous set of measurement sources, it is called an *extended object*. According to this definition, extended object and group tracking consists of tracking the set of measurement sources forming the target.

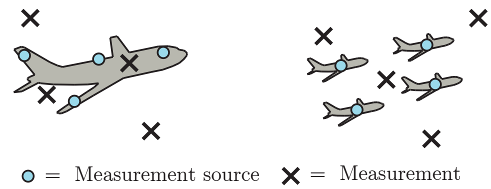


Figure 1: Extended object (left) and group target (right).

The traditional approach to extended object and group target tracking models an extended target explicitly as a discrete set of measurement sources with a common bulk component [2]–[5]. The tracking problem then consists of estimating the locations of the measurement sources together with the bulk component. Such an explicit target model is suitable if proper models for the measurement sources are available and the sensor resolution is high enough to resolve the measurement sources.

In this work, an alternative approach to the problem of extended object and group tracking is pursued. The basic idea is to track the shape of the target rather than the location of the measurement sources [6]–[8]. For instance, in the first scenario, the measurement sources on the airplane may depend on several unpredictable factors, e.g., the target surface or the sensor-to-target geometry, so that explicit models for the measurement sources are not available. In the second scenario, resolution conflicts and a high number of closely-spaced targets may render it hard or even impossible to track individual point targets. Hence, in both cases it is suitable to track the shape of the target.

This non-standard target tracking problem raises new challenges: First, in general, less information about the location and number of the measurement sources on the modeled shape is available. This results form the fact that the true shape and its properties are typically unknown. Occlusions and a specific sensor-to-target geometry can further influence the measurement sources. Even when only a single measurement per time step from a target is received, it must be possible to estimate the shape. Furthermore, in case of a group target, only a discrete set of measurement sources is possible at all. Hence, a big challenge is that the assumptions made on the measurement sources are justified and the target extent model has to be robust to modeling errors. Second, a recursive Bayesian state estimator for extended object tracking requires the description of uncertain geometric shapes and leads to a hierarchical nonlinear model that first specifies the generation of measurement sources and then the generation of measurements. As a consequence, sophisticated nonlinear state estimation techniques are required for dealing with this non-standard high-dimensional estimation problem.

Existing extended object tracking methods employ basic shapes such as ellipses [7], [8], sticks [6], [9], rectangles or (known) Gaussian mixtures [6] for modeling the target. Basic shapes such as ellipses are suitable in case of high measurement noise and a few available measurements. However, in case of rather low measurement noise (compared to the target extent), it may be possible to extract more detailed shape information from the measurements. Detailed shape information is highly relevant for the entire tracking system, because it can be used for

- target classification, e.g., type of airplane or formation,
- improving the motion model,
- track management, e.g., splitting and merging of groups,
- data association, and
- sensor management and planning algorithms.

A. Contributions

The main contribution of this paper is a Bayesian method for tracking star-convex shape approximations of extended objects and group targets based on noisy position measurements. For this purpose, a so-called *Random Hypersurface Model* is employed for modeling the target extent, and a Gaussian-assumed Bayesian state estimator is derived for an efficient recursive measurement update.

To the best of our knowledge, this is the first extended object tracking method allowing for *explicitly* estimating such detailed geometric shape information.

B. Related Work

A recent approach to modeling extended targets uses so-called spatial distribution models [6], [10], where each measurement source is assumed to be a random draw from a known object-dependent probability distribution. In [6], [9], [10], stick targets and (known) Gaussian mixtures have been used as a spatial distribution. In general, it would be possible, but computationally challenging, to employ a spatial distribution for estimating the parameters of complex geometric shapes. Spatial distributions also have been employed within PHD filters for tracking multiple extended objects in a cluttered environment [11], [12]. An approach for extended object tracking based on spatial cluster processes is introduced in [13].

In [7], [14], the uncertainty about an elliptic extent is expressed by means of a random symmetric positive definite matrix. The original approach [7], [14] does not incorporate measurement noise. However, an extension, which is also able to deal with measurement noise was developed in [15]. Furthermore, an extension to multiple extended objects based on the PMHT [16] has been developed in [17]. A detailed comparison of the random matrix approach and *RHMs* for ellipses is given in [18].

Related problems to extended object tracking can also be found in computer vision. For instance, curve fitting [19]–[21] deals with fitting a curve to noisy data points. However, there it is assumed that the data points, i.e., the measurements, only stem from the boundary of the shape. In a similar way, in [22], polynomials are fitted to noisy measurements from a radar device in order to track road lanes. Active contour models developed in [23] are used for tracking the contour of an object in an image. There, feature detection algorithms are employed in order to detect boundary points. Based on these features, an aggregated observation for updating the prior is constructed.

The tracking of contaminant clouds is considered in [24] and extended object tracking based on down-range extent measurements is treated in [25].

C. Overview

The remainder of this paper is structured as follows: In Section II, a general model for extended target tracking within a Bayesian framework is introduced. Subsequently, in Section III, a particular target extent model for star-convex shapes based on a *Random Hypersurface Model* is presented. Based on this model, a Bayesian state estimator for star-convex target shapes is derived in Section IV. The resulting estimator is evaluated in Section V by typical extended target and group target tracking examples. Finally, this paper is concluded in Section VI.

II. Modeling Extended Targets

As we aim at developing a recursive Bayesian state estimator for extended objects, a probabilistic model of the target is required. For this purpose, the state to be estimated, a measurement model specifying the measurement generation process, and a dynamic model for the temporal evolution of the state have to be specified.

In this work, the target shape is represented with a parameter vector $\underline{p}_k$, where k is the time index. The target shape itself is then denoted by the set $\mathcal{S}(\underline{p}_k)$.

Example 1 (Circle)**.** A circular shape in two-dimensional space can be represented by $\underline{p}_k = \left[\underline{m}_k^T, r_k\right]^T$, where $\underline{m}_k$ is the center and r_k the radius. The target shape is given by $\mathcal{S}(\underline{p}_k) = \{\underline{z} \mid \underline{z} \in \mathbb{R}^2 \text{ and } ||\underline{z} - \underline{m}_k||_2 \leq r_k\}$.

The entire state vector of the extended target at time step k to be tracked is a random vector $\underline{\boldsymbol{x}}_k = \left[\underline{\boldsymbol{p}}_k^T, \dots\right]^T$ that consists of the shape vector and possible further state variables, e.g., the velocity.

A. Measurement Model

At each time step k, a set of n_k position measurements $\{\hat{\underline{y}}_{k,l}\}_{l=1}^{n_k}$ becomes available. Here, we assume that the measurements are generated independently. This allows us to estimate the shape of a target even if only a single measurement

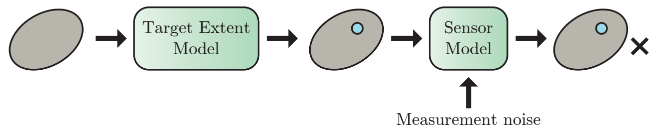


Figure 2: Measurement model for extended objects.

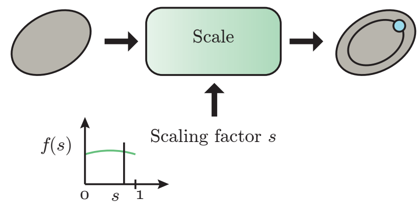


Figure 3: Random Hypersurface Model for an ellipse.

per time step is available. Several measurements per time step can be treated sequentially. Hence, for given $\mathcal{S}(\underline{p}_k)$, the measurement model specifies how a single measurement $\hat{\underline{y}}_{k,l}$ is obtained. The measurement model consists of two parts, the *target extent model* and the *sensor model* (see Fig. 2).

Target Extent Model: For a given shape $\mathcal{S}(\underline{p}_k)$, the target extent model specifies the location of the *measurement source* $\underline{z}_{k,l}$ (see Fig. 2). Note that we do not want to estimate the locations of the measurement sources explicitly. Such an implicit model also avoids the explicit treatment of resolution conflicts (see also [6], [14]). In this work, we employ a so-called *Random Hypersurface Model* for the target extent, which is introduced in the following Section III.

Sensor Model: Given a measurement source $\underline{z}_{k,l}$, the sensor model gives the measurement $\hat{\underline{y}}_{k,l}$, where it is assumed that measurements are generated independently. Here in this work, we focus on Cartesian measurements corrupted with Gaussian noise, i.e., the measurement $\hat{\underline{y}}_{k,l}$ is a noisy observation of the measurement source $\underline{z}_{k,l}$ according to

$$\hat{\underline{y}}_{k,l} = \underline{z}_{k,l} + \underline{\boldsymbol{v}}_{k,l} \ , \tag{1}$$

where the noise term $\underline{\boldsymbol{v}}_{k,l}$ is zero-mean white Gaussian noise with covariance matrix $\Sigma^v_{k,l}$. Note that most relevant sensors can be formalized within this model. For instance, angle-distance measurements can be transformed to Cartesian measurements [26].

Extensions of the Model: The above introduced model can easily be extended to incorporate more information, e.g., the number of measurements received per time step may depend on the object extension.

B. Dynamic Model

The dynamic model is a probabilistic model for the temporal evolution of the target state $\underline{\boldsymbol{x}}_k$. In contrast to a point target, also the temporal evolution of the shape has to be modeled, as the shape vector is part of the state vector. Here in this work, no restrictions on the dynamic model are imposed. A general dynamic model may be characterized by means of a system equation

$$\underline{\boldsymbol{x}}_{k+1} = a_k(\underline{\boldsymbol{x}}_k, \underline{u}_k, \underline{\boldsymbol{w}}_k) \ , \tag{2}$$

where $a_k(\cdot)$ is the system function, $\underline{u}_k$ is the system input, and $\underline{\boldsymbol{w}}_k$ the system noise.

C. Bayesian State Estimator

Based on the measurement and system model, a recursive Bayesian state estimator for the state $\underline{\boldsymbol{x}}_k$ can be developed. As several measurements per time step may be received, the standard notation for a Bayesian state estimator has to be slightly extended. In the following, we denote the probability density for the state vector $\underline{\boldsymbol{x}}_k$ after the incorporation of all measurements up to time step $k-1$ and the measurements $\hat{\underline{y}}_{k,1}, \ldots, \hat{\underline{y}}_{k,l}$ with $f_{k,l}(\underline{x}_k)$.

Time Update: The state vector evolves according to a Markov model characterized by the conditional density function $f(\underline{x}_k|\underline{x}_{k-1})$ derived from the system equation (2). The prediction $f_{k,0}(\underline{x}_k)$ for time step k thus results from the Chapman-Kolomogorov equation

$$f_{k,0}(\underline{x}_k) = \int f(\underline{x}_k|\underline{x}_{k-1}) \cdot f_{k-1,n_{k-1}}(\underline{x}_{k-1}) d\underline{x}_{k-1} \ .$$

Measurement Update: The prediction $f_{k,0}(\underline{x}_k)$ is updated with the set of measurements $\{\hat{\underline{y}}_{k,l}\}_{l=1}^{n_k}$ according to Bayes' rule. Because the measurement generation process is assumed to be independent for consecutive measurements, they can be incorporated recursively according to

$$f_{k,l}(\underline{x}_k) = \alpha_{k,l} \cdot f^L(\hat{\underline{y}}_{k,l}|\underline{x}_k) \cdot f_{k,l-1}(\underline{x}_k) \ ,$$

where $f^L(\hat{\underline{y}}_{k,l}|\underline{x}_k)$ is a single measurement likelihood function and $\alpha_{k,l}$ is a normalization factor.

Gaussian Assumption: In this work, all probability densities are approximated with Gaussian densities, i.e., $f_{k,l}(\underline{x}_k) \approx \mathcal{N}(\underline{x}_k - \underline{\mu}^x_{k,l}, \Sigma^x_{k,l})$, leading to a so-called Gaussian-assumed estimator.

III. Random Hypersurface Model (RHM)

A *Random Hypersurface Model* [8], [27] is a target extent model that specifies the location of a single measurement source for a given target shape. In this work, we use a particular *RHM* for star-convex shapes.

Definition 1 (Star-Convex)**.** A set $\mathcal{S} \subset \mathbb{R}^N$ is *star-convex* with center $\underline{m}$ iff each line segment from $\underline{m}$ to any point in $\mathcal{S}$ is contained in $\mathcal{S}$.

Given a star-convex shape, a measurement source is assumed to be an element of a randomly scaled version of the shape boundary (see Fig. 3). Hence, the process of generating a *single* measurement source is modeled as follows: Given a star-convex shape, first a scaled version of the shape boundary is generated, while leaving the center unchanged. Second, the measurement source is selected from the scaled boundary according to an arbitrary, unknown rule.

The scaling factor is specified by an independent random draw from a one-dimensional probability density function (see the green-colored function in Fig. 3). This probability density has to be specified in advance. It is part of the target extent model and it can be assumed to be independent of the target shape. A formal definition of a *Random Hypersurface Model* is given in the following:

Definition 2 (Random Hypersurface Model)**.** Given is a star-convex shape $\mathcal{S}(\underline{p}_k)$ with parameter vector $\underline{p}_k$ and center $\underline{m}_k$. If $\tilde{s}_{k,l}$ is a random draw from the one-dimensional random variable $\boldsymbol{s}_{k,l}$, the measurement source $\underline{z}_{k,l}$ is an element of the scaled boundary

$$\underline{m}_k + \tilde{s}_{k,l} \cdot \left(\bar{\mathcal{S}}(\underline{p}_k) - \underline{m}_k \right) \ ,$$

where $\bar{\mathcal{S}}(\underline{p}_k)$ denotes the bound of $\mathcal{S}(\underline{p}_k)$.[1]

Remark 1. The restriction to star-convex shapes ensures that $\underline{z}_{k,l} \in \mathcal{S}(\underline{p}_k)$, i.e., the shape is contractible. Scaling the object boundaries can also be interpreted as a homotopy from a point (the object center) to the object boundary.

Remark 2. For fixed scaling factor, e.g., $\tilde{s}_{k,l} = 1$, each measurement source lies on the boundary of the shape, i.e., $\underline{z}_{k,l} \in \bar{\mathcal{S}}(\underline{p}_k)$. Actually, for a static extended object, the model is then equivalent to the well-known *functional model* for fitting curves to noisy data [21]. In this sense, a *Random Hypersurface Model* encompasses curve fitting. Furthermore, it is interesting to note that an *RHM* becomes a *spatial distribution model* [6], [10] if the measurement source is assumed to be drawn randomly from the scaled boundary. Hence, one can say that an *RHM* encompasses many spatial distributions.

An *RHM* as described above is just a forward model, which together with the sensor model gives a rule for mapping the hidden state (the shape parameters) to the observed measurement. In order to use an *RHM* for extended object tracking, the following steps have to be performed:

- A particular geometric shape $\mathcal{S}(\underline{p}_k)$ with suitable parametrization $\underline{p}_k$ has to be chosen. The set $\mathcal{S}(\underline{p}_k)$ can be represented for instance as the solution of an implicit equation [8] or in parametric form (see Section IV).
- A particular one-dimensional probability distribution for the random scaling factor $\boldsymbol{s}_{k,l}$ must be chosen.
- In order to perform backward inference, i.e., infer the hidden state from the observed measurements, a Bayesian state estimator has to be designed.

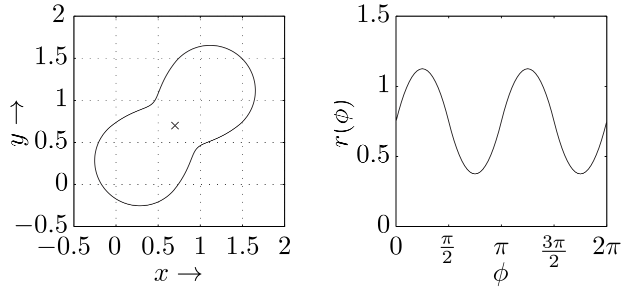
Random Hypersurface Models have already been used for tracking elliptic target shapes in [8]. In the following Section IV, a particular extended object tracking algorithm for star-convex shapes based on an *RHM* is presented.

A. Advantages

The main advantages of *Random Hypersurface Models* compared to other state-of-the art extended object tracking methods are the following:

- The target shape can be modeled as a basic shape like an ellipse [8], but also as general star-convex shape (see Section IV).

1 Set operations are defined element-wise.



(a) Star-convex shape. (b) Polar representation.

Figure 4: Example for representing the shape of star-convex extended objects.

- Bayesian inference for *RHMs* is computationally tractable even for complex star-convex shapes. By means of a Gaussian-assumed Bayesian state estimator based on analytic moment calculation, a closed-form recursive measurement update can be performed. As the state of the extended object is represented with a Gaussian distributed random vector, *RHMs* lend themselves to be embedded into other Bayesian tracking algorithms, e.g., data association algorithms.
- The uncertainty of the estimated shape is directly available due to the Gaussian state representation.
- *RHMs* are robust target models (see the simulations in Section V and [8]). Precise shape approximations are obtained even if the modeled shape does not coincide exactly with the true shape. The only required target-dependent information is the probability distribution of the one-dimensional scaling factor.

IV. RHM for Star-Convex Shapes

In the following, a *Random Hypersurface Model* for tracking a full star-convex extended object is presented. For this purpose, a star-convex shape is represented with a radial function that specifies the distance from the center to the boundary point for a given angle. By means of parameterizing the radial function with Fourier coefficients [28], an efficient Gaussian-assumed Bayesian state estimator that recursively estimates the radial function and the center of the extended object can be derived. For the sake of simplicity, we restrict our discussion to two-dimensional space, generalization to higher dimensions is straightforward.

A. Parametric Representation of Star-Convex Shapes

A star-shaped extended object $\mathcal{S}(\underline{p}_k)$ is represented in *parametric form* with the help of a so-called shape signature, which is a one-dimensional function representing the shape bounds [28]. In particular, a radial function $r(\phi)$ [28], [29], which gives the distance from the center to a contour point depending on the angle ϕ, is employed to get a polar representation of the contour (see Fig. 4). This representation is suitable for *RHMs* as it is restricted to star-convex shapes and the incorporation of the scaling factor is easy (scaling the shape corresponds to scaling the radial function).

If the radial function $r(\phi)$ is characterized by a parameter vector $\underline{b}_k$ and the center of the object is denoted with $\underline{m}_k$, the complete shape parameter vector becomes $\underline{p}_k = \left[\underline{b}_k^T, \underline{m}_k^T\right]^T$ and the extended object is given by

$$\mathcal{S}(\underline{p}_k) = \{s \cdot r(\underline{b}_k, \phi) \cdot \underline{e}(\phi) + \underline{m}_k \mid \phi \in [0, 2\pi] \text{ and } s \in [0,1]\} \ , \quad (3)$$

where $\underline{e}(\phi) := \begin{bmatrix}\cos(\phi)\\ \sin(\phi)\end{bmatrix}$ is the unit vector with angle ϕ. In this work, the radial function [28] is expanded as Fourier series in ϕ. However, in general other representations such as splines [23] are also suitable. Assuming $r(\underline{b}_k, \phi)$ to be periodic in ϕ with period $[0, 2\pi]$, the Fourier series expansion of degree N_F becomes

$$r(\underline{b}_k, \phi) = \frac{a_k^{(0)}}{2} + \sum_{j=1\ldots N_F} a_k^{(j)} \cos(j\phi) + b_k^{(j)} \sin(j\phi) \ , \quad (4)$$

where the parameter vector $\underline{b}_k$ is given by

$$\underline{b}_k = \left[a_k^{(0)}, a_k^{(1)}, b_k^{(1)}, \ldots a_k^{(N_F)}, b_k^{(N_F)}\right]^T \ .$$

Note that (4) becomes linear in $\underline{b}_k$ for fixed ϕ, i.e.,

$$r(\underline{b}_k, \phi) = \mathbf{R}(\phi) \cdot \underline{b}_k \ ,$$

where

$$\mathbf{R}(\phi) \ = \ [1, \cos(\phi), \sin(\phi), \ldots, \cos(N_F\phi), \sin(N_F\phi)] \ .$$

Fourier coefficients with lower indices encode information about the coarse features of the shape, while Fourier coefficients with higher indices give information about finer details.

Remark 3. As the uncertainty about the Fourier coefficients is represented with a Gaussian distribution, a full uncertainty description of the shape is available, e.g., it is easy to derive sigma bounds for a shape estimate.

B. Bayesian State Estimator

Having defined a particular representation for a star-convex extended object, the next step is to derive a recursive Bayesian state estimator. With (1) and (3), the measurement equation $h(\cdot,\cdot,\cdot)$ becomes

$$\begin{aligned}\hat{\underline{y}}_{k,l} &= \underline{z}_{k,l} + \underline{\boldsymbol{v}}_{k,l} \\ &= \boldsymbol{s}_{k,l} \cdot r(\underline{\boldsymbol{b}}_k, \phi_{k,l}) \cdot \underline{e}(\phi_{k,l}) + \underline{\boldsymbol{m}}_k + \underline{\boldsymbol{v}}_{k,l} \quad (5)\\ &:= h(\underline{\boldsymbol{x}}_k, \underline{\boldsymbol{v}}_{k,l}, \boldsymbol{s}_{k,l}) \ ,\end{aligned}$$

which maps the state $\underline{\boldsymbol{x}}_k$, the measurement noise $\underline{\boldsymbol{v}}_{k,l}$, and the scaling factor $\boldsymbol{s}_{k,l}$ to the measurement $\hat{\underline{y}}_{k,l}$. The term $\phi_{k,l}$ denotes the (unknown) angle between the vector from the center to the measurement source $\underline{z}_{k,l}$ and the x-axis. If $r(\underline{\boldsymbol{b}}_k, \phi_{k,l})$ becomes linear for given $\phi_{k,l}$ as in the case of Fourier descriptors, the measurement equation turns out to be

$$\hat{\underline{y}}_{k,l} = \boldsymbol{s}_{k,l} \cdot \mathbf{R}(\phi_{k,l}) \cdot \underline{\boldsymbol{b}}_k \cdot \underline{e}(\phi_{k,l}) + \underline{\boldsymbol{m}}_k + \underline{\boldsymbol{v}}_{k,l} \ . \quad (6)$$

Unfortunately, the angle $\phi_{k,l}$ is unknown in (6). However, we can substitute the unknown value of $\phi_{k,l}$ by a proper point estimate. In general, a proper point estimate is given by the most likely angle $\phi_{k,l}$. In case of isotropic measurement noise, a proper point estimate $\phi_{k,l}$ is given by the angle between the vector from the current shape center estimate $\underline{\mu}_{k,l-1}^m$ to the measurement $\hat{\underline{y}}_{k,l}$ and the x-axis

$$\hat{\phi}_{k,l} := \angle\left(\hat{\underline{y}}_{k,l} - \underline{\mu}_{k,l-1}^m, \underline{e}_x\right) \ .$$

For a given measurement $\hat{\underline{y}}_{k,l}$, (6) can be seen as a constraint on the values of $\underline{\boldsymbol{x}}_k$, $\underline{\boldsymbol{v}}_{k,l}$ and $\boldsymbol{s}_{k,l}$, so that we can perform algebraic manipulations on (6). Algebraic manipulations on (6) can be used for reducing the influence of $\hat{\phi}_{k,l}$ on $\underline{\boldsymbol{x}}_k$ as follows

$$\begin{aligned}\hat{\underline{y}}_{k,l} - \underline{\boldsymbol{m}}_k &= \boldsymbol{s}_{k,l} \cdot \underline{e}(\hat{\phi}_{k,l}) \cdot \mathbf{R}(\hat{\phi}_{k,l}) \cdot \underline{\boldsymbol{b}}_k + \underline{\boldsymbol{v}}_{k,l} \ , \\ ||\hat{\underline{y}}_{k,l} - \underline{\boldsymbol{m}}_k||^2 &= \boldsymbol{s}_{k,l}^2 \cdot ||\mathbf{R}(\hat{\phi}_{k,l}) \cdot \underline{\boldsymbol{b}}_k||^2 + \quad (7)\\ &\quad 2\boldsymbol{s}_{k,l}\mathbf{R}(\hat{\phi}_{k,l})\underline{\boldsymbol{b}}_k\underline{e}(\hat{\phi}_{k,l})^T\underline{\boldsymbol{v}}_{k,l} + ||\underline{\boldsymbol{v}}_{k,l}||^2 \ .\end{aligned}$$

Based on (7), the following new measurement equation $h^*(\cdot,\cdot,\cdot,\cdot)$ is obtained

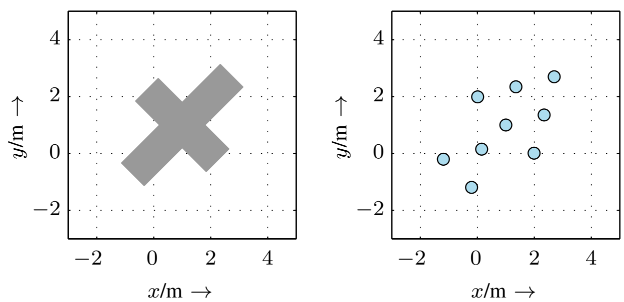
$$\begin{aligned}0 &= h^*(\underline{\boldsymbol{x}}_k, \underline{\boldsymbol{v}}_{k,l}, \boldsymbol{s}_{k,l}, \hat{\underline{y}}_{k,l}) \quad (8)\\ &= \boldsymbol{s}_{k,l}^2 \cdot ||\mathbf{R}(\hat{\phi}_{k,l}) \cdot \underline{\boldsymbol{b}}_k||^2 + \\ &\quad 2\boldsymbol{s}_{k,l}\mathbf{R}(\hat{\phi}_{k,l})\underline{\boldsymbol{b}}_k\underline{e}(\hat{\phi}_{k,l})^T\underline{\boldsymbol{v}}_{k,l} + ||\underline{\boldsymbol{v}}_{k,l}||^2 - \\ &\quad ||\hat{\underline{y}}_{k,l} - \underline{\boldsymbol{m}}_k||^2 \ ,\end{aligned}$$

which maps the state $\underline{\boldsymbol{x}}_k$, the measurement noise $\underline{\boldsymbol{v}}_{k,l}$, the scaling factor $\boldsymbol{s}_{k,l}$, and the measurement $\hat{\underline{y}}_{k,l}$ to a pseudo-measurement 0.

The obtained final measurement equation (8) only involves quadratic terms and thus, is easy to handle with standard filtering techniques: Given the density $f_{k,l-1}(\underline{x}_k) = \mathcal{N}(\underline{x}_k - \underline{\mu}_{k,l-1}^x, \Sigma_{k,l-1}^x)$, the posterior density $f_{k,l}(\underline{x}_k) \approx \mathcal{N}(\underline{x}_k - \underline{\mu}_{k,l}^x, \Sigma_{k,l}^x)$ having received the measurement $\hat{\underline{y}}_{k,l}$ can be calculated by means of a Gaussian state estimator such as the EKF [30] or the UKF [30]. A measurement update can even be performed by means of analytic moment calculation as described in [31] in order to obtain an optimal closed-form measurement update. Naturally, also non-Gaussian estimators could be used for performing Bayesian inference based on the measurement equation (8). Note that the likelihood function $f^L(\hat{\underline{y}}_{k,l}|\underline{x}_k)$ results from the measurement equation (8).

V. Evaluation

An evaluation of *RHMs* for star-convex shapes is performed in two scenarios: In the first scenario the shape of a fixed non-moving extended object and a group is estimated. The second scenario demonstrates the practicability of *RHMs* by means of an extended object moving according to a constant velocity model.



(a) Target 1: Extended Object. (b) Target 2: Group Target.

Figure 5: Targets in the example.

A. Static Extended Object

In the following, a single target with a fixed position and shape is considered, where 300 measurements are sequentially received from the target. Simulations are performed with low measurement noise level $\Sigma^v_{k,1} = \mathrm{diag}(0.2, 0.2)$, medium noise level $\Sigma^v_{k,1} = \mathrm{diag}(0.4, 0.4)$, and high noise level $\Sigma^v_{k,1} = \mathrm{diag}(0.6, 0.6)$. Furthermore, an extended object of an aircraft-like shape and a group target as depicted in Fig. 5 are used in the simulations. The measurement sources are uniformly drawn from the target surface in Fig. 5a and uniformly drawn from the group members in Fig. 5b.

The shape of the targets is estimated with an *RHM* for star-convex shapes implemented according to the procedure introduced in Section IV-B and a UKF [30]. Herein, the radial function is represented with 15 Fourier descriptors and the scaling factor is assumed to be Gaussian distributed with mean 0.7 and variance 0.02 for both targets. The parameters of the shape are a priori set to a Gaussian with mean $\left[0.5, 0.5, 2, 0, \ldots, 0\right]^T$ and covariance matrix $\mathrm{diag}(0.4, 0.4, 0.3, 0.3, \ldots, 0.3)$, i.e., an uncertain circle with radius 2 and center $\left[0.5, 0.5\right]^T$.

The parameters of the estimated shape are averaged over 20 Monte-Carlo runs. The resulting point estimates for the target shapes are depicted in Fig. 6. Note that the uncertainty of shapes has not been plotted. For getting an impression of the magnitude of the measurement noise, the measurements of a particular run are also given in Fig. 6. It is important to note that this is just done for visualization. The estimator incorporates the measurements recursively, because in practical applications the target state evolves over time. It can be seen that for both targets the shape is estimated precisely with an *RHM*. With increasing measurement noise, the details of the shape slightly vanish. This is an intuitive result as with low sensor resolution, details are harder to resolve and more measurements are required. Hence, these examples show that detailed object information can be extracted with *RHMs* for star-convex shapes and that they are a robust target model.

B. Tracking a Moving Extended Object

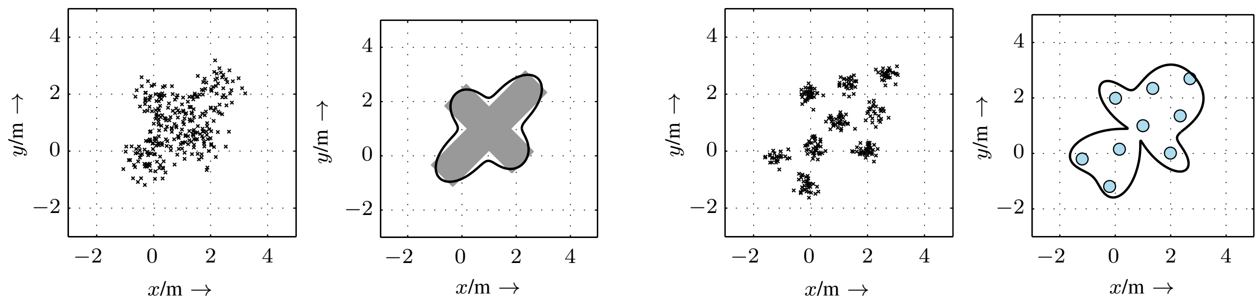
In the second scenario, an extended object is tracked by means of an *RHM* for star-convex objects and a constant velocity model for the target motion. The extended object moves along the trajectory depicted in Fig. 7. The number of measurements received from the target is Poisson distributed with mean 6 and the measurement noise is zero-mean Gaussian with covariance matrix $\Sigma^v_{k,l} = \mathrm{diag}(0.2, 0.2)$.

The state to be tracked is $\underline{\boldsymbol{x}}_k = \left[\underline{\boldsymbol{p}}_k^T, \boldsymbol{x}^v_k, \boldsymbol{y}^v_k\right]^T$, where $\left[\boldsymbol{x}^v_k, \boldsymbol{y}^v_k\right]^T$ is the velocity vector and $\underline{\boldsymbol{p}}_k$ are the shape parameters given by 15 Fourier descriptors and the center. As the extended object is assumed to evolve according to a constant velocity model, the system equation is $\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{w}}_k$, where $\mathbf{A}_k = \mathrm{diag}(\mathbf{I}_{15}, \mathbf{A}^{cv}_k)$ with $\mathbf{A}^{cv}_k = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \end{bmatrix}$ and $\mathbf{I}_{15}$ is the identity matrix of dimension 15. The system noise is zero-mean Gaussian noise with covariance matrix $\mathbf{C}^w_k = \mathrm{diag}(0.03 \cdot \mathbf{I}_{15}, \mathbf{C}^{cv}_k)$ with $\mathbf{C}^{cv}_k = q \begin{bmatrix} \frac{T^3}{3}\mathbf{I}_2 & \frac{T^2}{2}\mathbf{I}_2 \\ \frac{T^2}{2}\mathbf{I}_2 & T\mathbf{I}_2 \end{bmatrix}$ and $q = 0.3$. Hence, the center of the object evolves according to a constant velocity model and the shape parameters are just made more uncertain over time in order to capture shape changes. Again, the shape of the target is tracked with an *RHM* for star-convex extended objects and a UKF [30] for the measurement update according to (8). The scaling factor is Gaussian distributed with mean 0.7 and variance 0.02. The estimated shape (averaged over 20 time steps) is depicted in Fig. 7 for two snippets of the trajectory. It can be seen that the shape of the extended object is tracked well, even when the shape changes its orientation. In Fig. 7, the example measurements show that naïve approaches for estimating a shape would be bound to fail, e.g., directly computing an enclosing shape of the measurements is infeasible as the measurements are noise-corrupted, and only a few measurements may be available per time step. It is even possible that only one measurement per time step is available. Altogether, this scenario demonstrates that *RHMs* are suitable for tracking the star-convex shape of a moving extended object by means of constant velocity model.

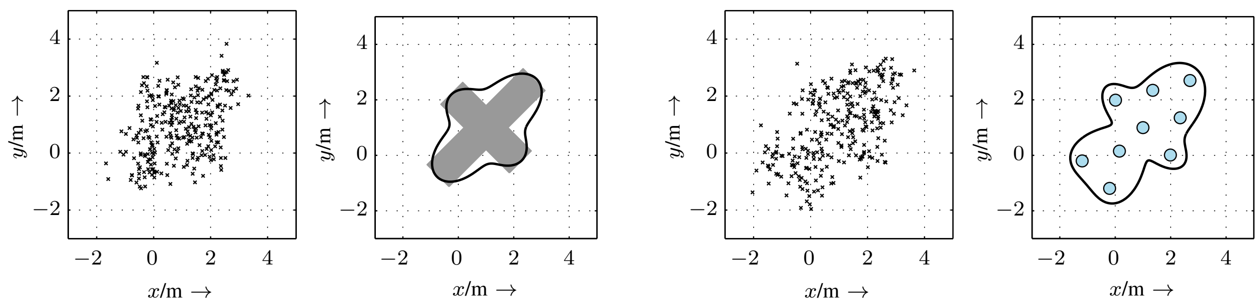
VI. Conclusions and Future Work

This work presented a Bayesian tracking algorithm for star-convex shapes of extended targets based on an *RHM*. The approach allows for tracking detailed shape information while being computationally tractable at the same time. Simulation results have shown the practicability of the approach.

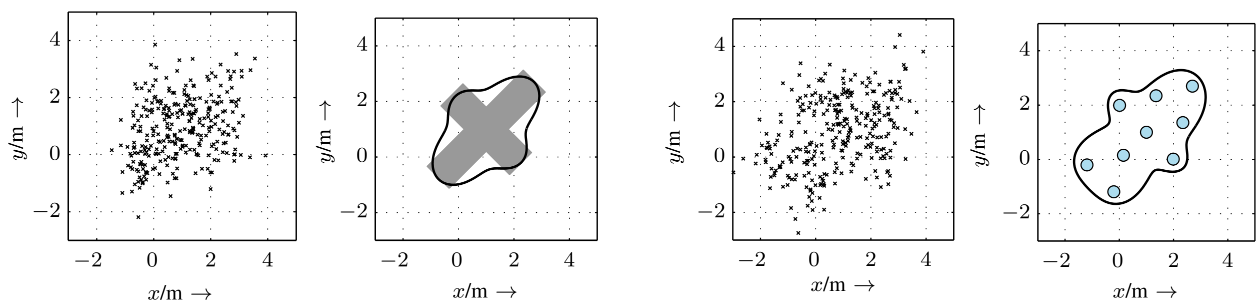
The capability of estimating detailed shape information paves the way for new possibilities concerning entire tracking systems: For instance, a mechanism for adapting the complexity of the used shape description is required. In case of high measurement noise and high kinematic noise, only a rather coarse description of the target can be inferred. Furthermore, only the size and orientation of a target can be tracked if the shape is known. Finally, detailed shape information are useful for, e.g., target classification and track management. *RHMs* can also be embedded into multi-target tracking algorithms in order to track multi-extended objects.



(a) Low noise level.



(b) Medium noise level.



(c) High noise level.

Figure 6: Example measurements for a particular run and point estimates for the shape (averaged over 20 runs).

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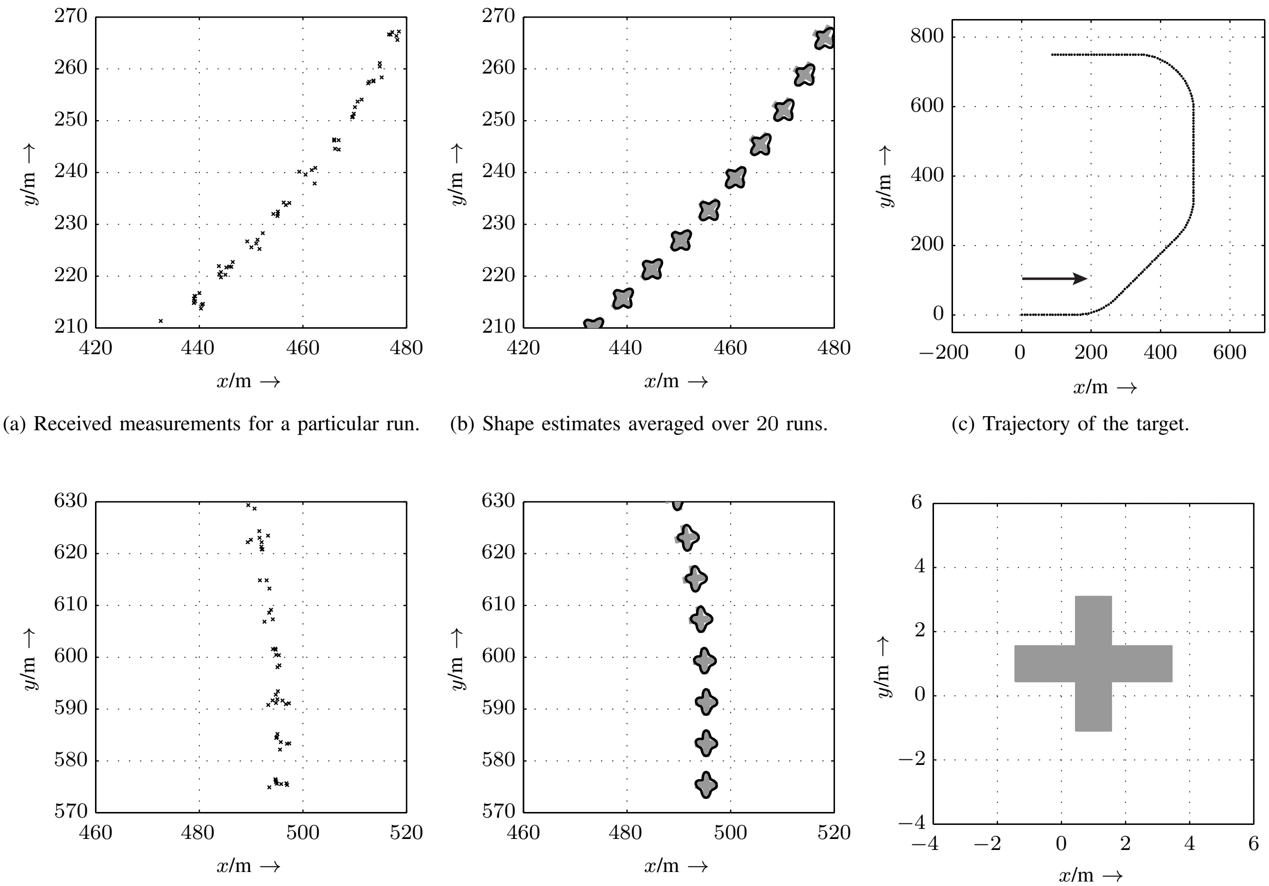

(a) Received measurements for a particular run. (b) Shape estimates averaged over 20 runs. (c) Trajectory of the target.

(d) Received measurements for a particular run. (e) Shape estimates averaged over 20 runs. (f) Target shape.

Figure 7: Tracking an extended object with an *RHM* for star-convex objects and a constant velocity model.

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