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SHARING THE GAINS FROM REGIONAL COOPERATION: A GAME THEORETIC APPLICATION TO PLANNING INVESTMENT IN ELECTRIC POWER

by

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I. INTRODUCTION

The distribution of the gains from cooperation between two or more economic agents is a problem that appears in several contexts in economics: bilateral monopoly, oligopoly theory, and the theory of regional cooperation.[1] Our problem concerns regional cooperation in planning investment in electric power, with reference to the four states of the Southern Electricity Region of India (Andhra Pradesh, Kerala, Mysore, and Tamil Nadu). We're interested in achieving a mutually acceptable basis for agreement, such that it's in each state's own interest to cooperate. At present, although regional planning has been accepted in principle by the Central Government of India and in greater or lesser degree by the various states, each State Electricity Board is planning on the assumption of self-sufficiency and there is little co-operation between the various Boards. Continuation of these policies will result in sub-optimal development of the electric power resources in the Region: a relative neglect of the excellent hydro resources in Mysore and Kerala, in **favor** of less economic non-hydro power resources in Andhra Pradesh and Tamil Nadu.[2]

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1. Segal [7] considers the problem of a "fair" distribution of the gains from co-operation among the members of the East African Customs Union. His analytic framework is similar to ours.

2. See Gately [2] for a fuller discussion of these points.

The costs of providing power to the Region, under various assumptions about the degree of cooperation among the states, are estimated in Section II, using a mixed-integer programming model of the investment planning problem. The degrees of cooperation examined include complete cooperation among all states, self-sufficiency for each state, and cooperation between some but not all states. The differences in the cost of providing power to the Region under the various assumptions measure the gains from cooperation..

In Section III various concepts from n-person game theory, such as the set of imputations and the core, are employed to clarify the notion of a mutually acceptable distribution of the gains from cooperation and to delimit those distributions satisfying certain minimal criteria. Since this analysis still leaves us with a large number of mutually acceptable distributions, we proceed in Section IV to consider several candidates proposed as the "best" division of the gains from cooperation. In Section V we offer some concluding comments.

II. MEASURING THE COSTS OF PROVIDING POWER TO THE REGION UNDER ALTERNATE ASSUMPTIONS ABOUT THE DEGREE OF COOPERATION.

We used a mixed-integer programming model of the investment planning problem[3] to estimate the cost of expanding and operating the electric power system in the Region, for each of the assumptions about the degree of co-operation. We divided the Region into three areas: Tamil Nadu (TN), Andhra Pradesh (AP), and the hybrid "state" Kerala-Mysore (KM); this was done to simplify the calculations in the investment planning problems.[4] With three areas, we had seven investment planning problems to solve: three under the assumption of self-sufficiency (one for each of the areas), three under the assumption of co-operation between only two areas (one for each of the three two-area combinations), and one for assumption of co-operation among all three areas.

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3. See Gately [2] for a complete description of the model and the results. Our model was an extension of the linear programming model used by Electricité de France; see Massé and Gibrat [4]. For a survey of similar models see Anderson [1].

4. Kerala and Mysore are in similar geographic position vis-a-vis Andhra Pradesh and Tamil Nadu; they are also quite similar with respect to electric power: both have all-hydro systems, as well as a sufficient number of attractive, untapped hydro sites so that non-hydro power sources need not be considered. Yet even with this simplification, the Regional investment planning model had 300 constraints, 800 variables (of which 92 were zero-one variables), and 3500 matrix entries; the solution required twenty-one minutes of execution time on an IBM 360/35 computer. Admittedly, this oversimplifies the problem of sharing the gains from co-operation, but our analytical techniques are easily extended to the case of four states, rather than three.

In each of the seven problems, a fifteen-year planning horizon was divided into five 3-year periods, for which a variety of investment and operating decisions were to be made, so as to minimize the cost of meeting projected demand levels without exceeding available capacity at any time. Investment choices in the Region included a variety of proposed hydro sites in each of the states,[5] lignite thermal power and nuclear power in Tamil Nadu, and conventional thermal power in Andhra Pradesh.[6] Operating costs were approximated by a simplified load dispatching routine, in which output level decisions[7] were made for the different plant types under various demand and hydrological supply conditions. (For those cases in which co-operation was assumed, interstate transmission capacity investment decisions and interstate transmission level decisions were also made.[8])

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5. Zero-one variables represented the build-or-don't build nature of the hydro site construction decision in each time period.

6. Non-hydro plant construction decisions were represented by continuous plant-size variables and zero-one fixed-charge variables reflecting scale economies in construction.

7. Continuous variables represented these decisions.

8. All of these decisions were represented by continuous variables.

The optimal values for the objective function for each of our seven investment planning problems are listed in Table 1. The total costs to the Region, and the breakdown of costs according to the area in which they were incurred,[9] are listed in Table 2 for the five alternate assumptions about the degree of co-operation (these are called the five coalition structures, or partitions of the three areas into coalitions[10]). The investment implications of the two extreme assumptions, self-sufficiency for each area and complete co-operation, are contrasted in Table 3. Note that the former results in much greater investment in Tamil Nadu and Andhra Pradesh, especially in non-hydro capacity.

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9. Interstate transmission costs are divided evenly between the two areas involved.

10. Here, as below, we follow the game-theoretic terminology of Luce and Raiffa [3].

TABLE 1

OPTIMAL VALUES OF THE OBJECTIVE FUNCTION FOR THE SEVEN INVESTMENT PLANNING PROBLEMS

(in Million Rupees of Present Value)

Investment Planning Problem	Optimal Value of the Objective Function
1. Tamil Nadu self-sufficient	5330
2. Andhra Pradesh self-sufficient	1870
3. Kerala-Mysore self-sufficient	860
4. Tamil Nadu cooperating with Kerala-Mysore	5020
5. Andhra Pradesh cooperating with Kerala-Mysore	1960
6. Tamil Nadu cooperating with Andhra Pradesh	6990
7. All three areas in co-operation	6530

TABLE 2

COSTS TO THE REGION AND COSTS INCURRED
IN EACH AREA FOR THE FIVE COALITION STRUCTURES
(in Million Rupees of Present Value)

Coalition Structure	Costs Incurred in			Total Cost in Region
	Tamil Nadu	Andhra Pradesh	Kerala-Mysore	
I. Self-sufficiency for each area: (TN), (AP), (KM)	5330	1870	860	8060
II. Cooperation between Tamil Nadu and Kerala-Mysore, self-sufficiency for Andhra Pradesh: (TN, KM), (AP)	2600	1870	2420	6890
III. Cooperation between Andhra Pradesh and Kerala-Mysore, self-sufficiency for Tamil Nadu: (AP, KM), (TN)	5330	480	1480	7290
IV. Cooperation between Tamil Nadu and Andhra Pradesh, self-sufficiency for Kerala-Mysore: (TN, AP), (KM)	5520	1470	860	7850
V. Complete cooperation (TN, AP, KM)	3010	1010	2510	6530

TABLE 3

INVESTMENT IN GENERATING CAPACITY IN EACH
AREA UNDER SELF-SUFFICIENCY AND UNDER REGIONAL PLANNING

(Amounts in Megawatts, MW)

3-year time period[a] t	Tamil Nadu		Andhra Pradesh		Kerala-Mysore	
	Self Sufficiency	Regional Planning	Self Sufficiency	Regional Planning	Self Sufficiency	Regional Planning
t=1	215 MW lignite 440 MW hydro	60 MW hydro	136 MW thermal	222 MW thermal 40 MW hydro		315 MW hydro
t=2	35 MW hydro	35 MW hydro	200 MW thermal	200 MW hydro	600 MW hydro	1812 MW hydro
t=3	894 MW nuclear	220 MW hydro	40 MW hydro		875 MW hydro	873 MW hydro
t=4	740 MW lignite 498 MW nuclear	454 MW lignite 1150 MW nuclear	393 MW nuclear			

a Non-hydro investment decisions are made for each of the first four 3-year time periods but not for the fifth, because the gestation lag would not allow the benefits of those investments to be explicitly considered by the model. For similar reasons, hydro investment decisions are made for each of the first three 3-year time periods only.

III. SOME MUTUALLY ACCEPTABLE WAYS OF SHARING THE GAINS FROM COOPERATION: IMPUTATIONS AND THE CORE

For each coalition, the gains to be shared among its members are measured by the difference between the total costs when its members cooperate and when they are self-sufficient. These differences, for each coalition of one, two, or three areas, constitute the characteristic function for the game (Table 4). (We assume here, as below, that money has the properties of a "transferable utility".[11])

While the gains from cooperation are always positive, there is always one area in which greater costs are incurred under cooperation than under self-sufficiency (cf. Table 2). These differences between the costs incurred in each area under self-sufficiency and under cooperation are the payoffs to to each area under the various coalition structure; the cost of self-sufficiency for each area is the point of zero payoff for that area and any cost below [above] that represents a positive [negative] payoff to that area.

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11. See Luce and Raiffa [3, sections 7.7 and 8.1]. Note that this does not imply any interpersonal comparison of utility.

TABLE 4

THE CHARACTERISTIC FUNCTION VALUE

FOR EACH COALITION

(in Million Rupees of Present Value)

Coalition	Characteristic Function Value
1. Tamil Nadu: (TN)	v(TN) = 0 (=5330 - 5330)
2. Andhra Pradesh: (AP)	v(AP) = 0 (=1870 - 1870)
3. Kerala-Mysore: (KM)	v(KM) = 0 (=860 - 860)
4. Tamil Nadu and Kerala-Mysore: (TN, KM)	v(TN,KM) = 1170 (=5330 + 860 - 5020)
5. Andhra Pradesh and Kerala-Mysore: (AP,KM)	v(AP,KM) = 770 (=1870 + 860 - 1960)
6. Tamil Nadu and Andhra Pradesh: (TN,AP)	v(TN,AP) = 210 (=5330 + 1870 - 6990)
7. Tamil Nadu, Andhra Pradesh, and Kerala-Mysore: (TN,AP,KM)	v(TN,AP,KM) = 1530 (=5330 + 1870 + 860 - 6530)

TABLE 5

PAYOFFS TO EACH AREA
UNDER THE VARIOUS COALITION STRUCTURES
(in Million Rupees of Present Value)

Coalition Structure	Payoff to Each Area			Sum for the Region
	Tamil Nadu	Andhra Pradesh	Kerala-Mysore	
I. (TN), (AP), (KM)	0	0	0	0
II. (TN, KM), (AP)	2730	0	-1560	1170 = v(TN,KM)
III. (AP, KM), (TN)	0	1390	-620	770 = v(AP,KM)
IV. (TN, AP), (KM)	-190	400	0	210 = v(TN,AP)
V. (TN, AP, KM)	2320	860	-1650	1530 = v(TN,AP,KM)

Clearly, any area with a negative payoff under a certain coalition structure would be irrational to cooperate unless it received a sufficiently large side payment from its coalition partner(s);[12] loosely speaking, the power-receiving area must at least cover the extra costs incurred in the power-transmitting area. The sum of an area's payoff plus [minus] the side payment(s) received from [made to] its coalition partners is called its final payment, denoted respectively P(TN), P(AP), and P(KM). The definition of individual rationality for a given area is that it should receive a final payment at least as great as it receives under self-sufficiency:

$$P(TN) \geq 0 = v(TN)$$

(1)
$$P(AP) \geq 0 = v(AP)$$

$$P(KM) \geq 0 = v(KM)$$

This notion of rationality could be extended to all three areas together: the sum of their final payments should be no less than what they could achieve under complete cooperation

(2')
$$P(TN) + P(AP) + P(KM) \geq 1530 = v(TN,AP,KM) \quad .$$

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12. Side payments are made in lump-sum rupees of present value, and have the properties of a "transferable utility", where the amount is conserved in the exchange.

However, since 1530 is the maximum gain possible, this condition amounts to

$$(2) \qquad P(TN) + P(AP) + P(KM) = 1530 = v(TN,AP,KM) .$$

Any vector of final payments {P(TN), P(AP), P(KM)} which satisfies condition (2) is feasible for the three-area coalition and Pareto optimal; if it also satisfies the conditions of (1) then it is called an <u>imputation</u>.

The notion of group rationality contained in condition (2') can also be extended to the three two-area coalitions. The sum of final payments to any two areas should be no less than what they could achieve without the cooperation of the third:

$$(3) \qquad \begin{aligned} P(TN) + P(KM) &\geq 1170 = v(TN,KM) \\ P(AP) + P(KM) &\geq 770 = v(AP,KM) \\ P(TN) + P(AP) &\geq 210 = v(TN,AP) \end{aligned}$$

If one of these conditions of (3) does not hold, then one area is getting more than its "fair share" of the benefits of cooperation; the other two could then refuse to cooperate with the third and both of them could benefit. All those imputations which satisfy the conditions of (3) constitute the <u>core</u> of the game.

An alternate definition of the core can be derived by subtracting[13] each of the conditions of (3) from condition (2); the core then is the set of imputations satisfying

$$
\begin{aligned}
&P(TN) \leq 760 = v(TN,AP,KM) - v(AP,KM) \\
(4) \qquad &P(AP) \leq 360 = v(TN,AP,KM) - v(TN,KM) \\
&P(KM) \leq 1320 = v(TN,AP,KM) - v(TN,AP)
\end{aligned}
$$

Geometrically, in the three-dimensional space of final payments, the set of imputations is the (triangular) intersection of the plane defined by condition (2) and the non-negative orthant, defined by the conditions of (1); in Figure 1, triangle LMN. The core is the intersection of the set of imputations and the three halfspaces defined by the conditions of (4); in Figure 1, the five-sided region bounded by points ABCDE.[14] The set of imputations is also depicted in the triangular graph of Figure 2, as well as several specific imputations of interest whose coordinates, corresponding side payments and

13. Note that when an inequality is multiplied by minus one the direction of the inequality reverses.

14. In Figure 1, the halfspace $P(TN) \leq 760$ is the area to the "left" of the plane defined by points $\beta\alpha\gamma\delta$; the halfspace $P(AP) \leq 360$ is the area below the plane defined by points $\mu\beta\lambda\delta$; the halfspace $P(KM) \leq 1320$ is the area on "this side" of the plane defined by $\lambda\epsilon\gamma\delta$

total final costs[15] are listed in Table 6.

Note that certain imputations, even within the core, result in negative total final costs to Kerala-Mysore: in Figure 2, any imputation to the right of a line through points G and H (P(KM) > 860). Tamil Nadu and Andhra Pradesh would consider such imputations "unfair" and "unacceptable" and they could threaten to not cooperate with Kerala-Mysore. One might argue that it would be irrational for them to actually carry out such a threat because at least one of them would then be worse off than under such an imputation. However, this assumes that the game is to be played once and only once; negotiations for regional cooperation typically can be re-opened at a later date so that they would be rational in carrying out, if necessary, a threat of (temporary) non-cooperation in the hope of achieving greater gains at a later negotiating session. So we could, perhaps, eliminate such imputations in the core from further consideration.

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15. The total final costs to a certain area under a certain imputation equals the costs incurred in that area under regional cooperation (3010 for TN, 1010 for AP, 2510 for KM) minus [plus] any side payments made to [received from] other areas.

Figure 1

THE SET OF IMPUTATIONS IN FINAL PAYMENTS SPACE

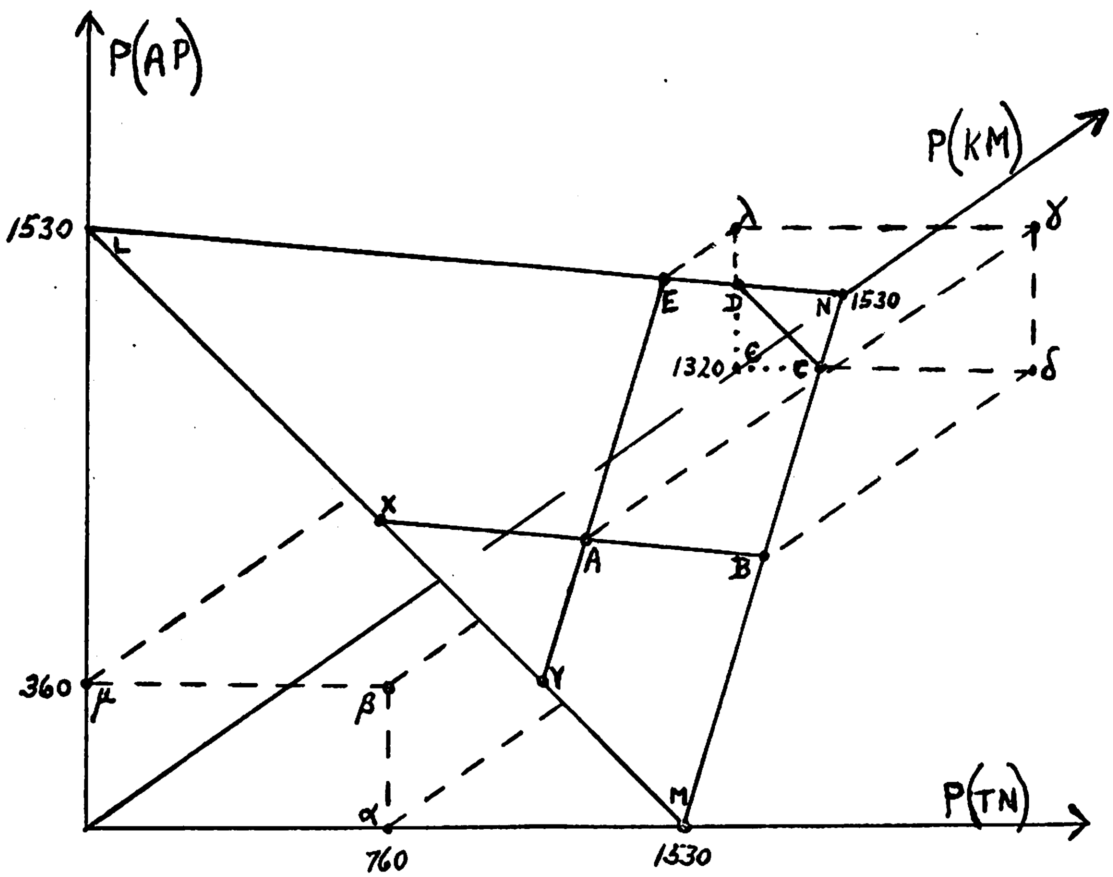

Figure 2

THE SET OF IMPUTATIONS

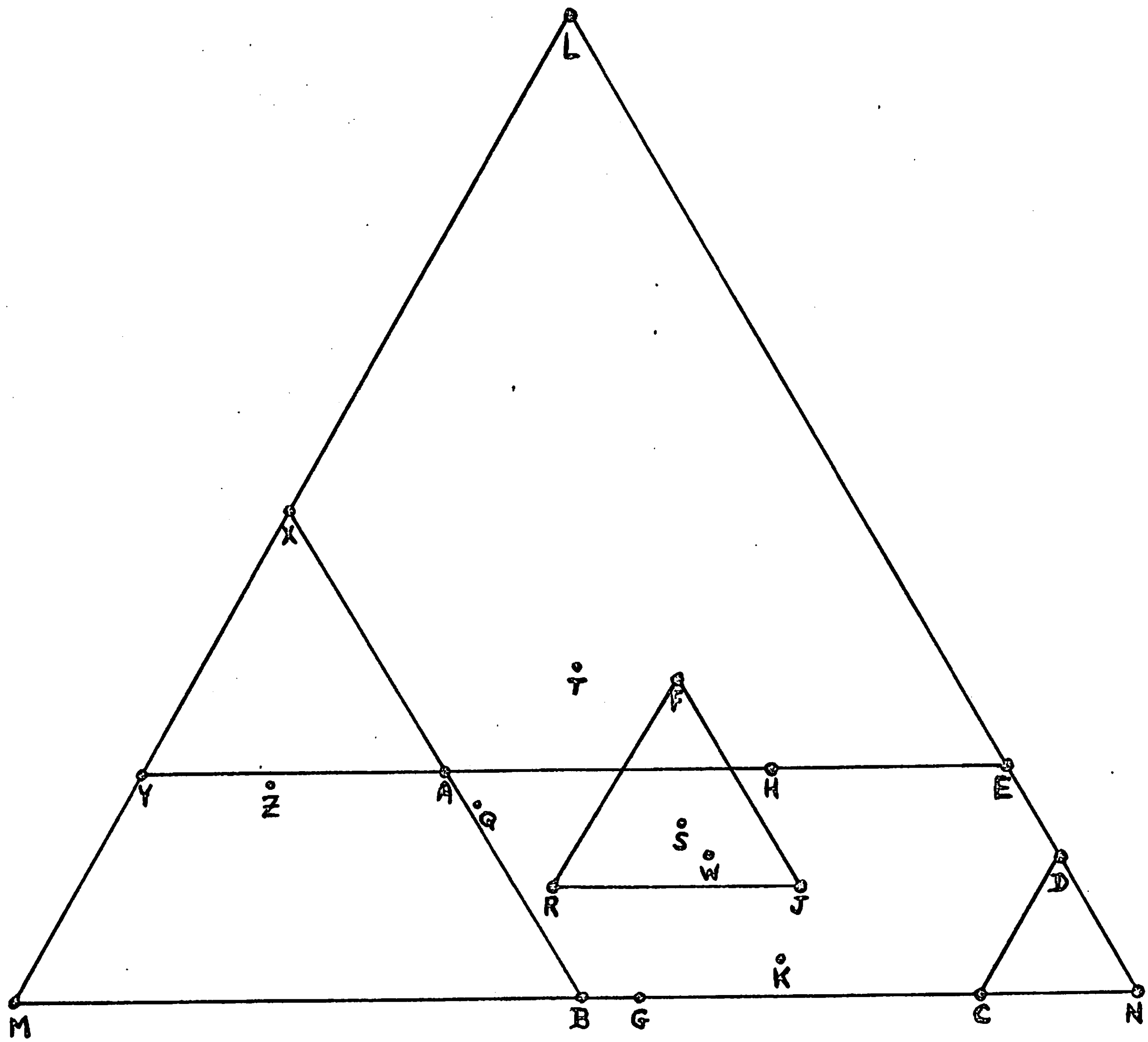

TABLE 6

VARIOUS IMPUTATIONS AND THEIR IMPLICATIONS

(in Million Rupees of Present Value)

Imputation	Final Payment to			Side Payments Made [Received] by			Total Final Costs		
	TN	AP	KM	TN	AP	KM	TN	AP	KM
A	760	360	410	1560	500	[2060]	4570	1510	450
B	760	0	770	1560	860	[2420]	4570	1870	90
C	210	0	1320	2110	860	[2970]	5120	1870	−460
D	0	210	1320	2320	650	[2970]	5330	1660	−460
E	0	360	1170	2320	500	[2820]	5330	1510	−310
F	380	490	660	1940	370	[2310]	4950	1380	200
G	670	0	860	1650	860	[2510]	4660	1870	0
H	310	360	860	2010	500	[2510]	5020	1510	0
J	380	180	970	1940	680	[2620]	4450	1690	−110
K	456⅔	56⅔	1016⅔	1863⅓	803⅓	[2666⅔]	4873⅓	1813⅓	−156⅔
L	0	1530	0	2320	[670]	[1650]	5330	340	860
M	1530	0	0	790	860	[1650]	3800	1870	860
N	0	0	1530	2320	860	[3180]	5330	1870	−670
Q	750	306	474	1570	554	[2124]	4580	1564	386
R	690	180	660	1630	680	[2310]	4640	1690	200
S	483⅓	283⅓	763⅓	1836⅔	576⅔	[2413⅓]	4846⅔	1586⅔	96⅔
T	510	510	510	1810	350	[2160]	4820	1360	350
W	477	225	828	1843	635	[2478]	4853	1645	32
X	760	770	0	1560	90	[1650]	4570	1100	860
Y	1170	360	0	1150	500	[1650]	4160	1510	860
Z	1010	356	164	1310	504	[1816]	4320	1514	696

Note also that, while the core places a "reasonable" upper limit on the final payment to each area, it does not guarantee a final payment greater than zero to either Tamil Nadu or Andhra Pradesh.[16] To eliminate imputations in which an area receives a final payment equal to or "close to" zero, we introduce an additional concept. For any imputation {P(TN), P(AP), P(KM)}, an area's <u>propensity to disrupt</u> the three-area agreement is the ratio of the combined loss of the other two areas to its own loss if it refused to agree and became self-sufficient:[17,18]

$$d_{TN} = \frac{P(AP) + P(KM) - v(AP,KM)}{P(TN)} = \frac{P(AP) + P(KM) - 770}{P(TN)}$$

$$d_{AP} = \frac{P(TN) + P(KM) - v(TN,KM)}{P(AP)} = \frac{P(TN) + P(KM) - 1170}{P(AP)}$$

$$d_{KM} = \frac{P(TN) + P(AP) - v(TN,AP)}{P(KM)} = \frac{P(TN) + P(AP) - 210}{P(KM)}$$

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16. Tamil Nadu [Andhra Pradesh] receives a zero final payment from core imputations on line DE [line BC] in Figures 1 and 2. The minimal final payment guaranteed by the core to Kerala-Mysore is 410, at imputation A.

17. Now that we are comparing one area's final payment with that of another, we are making interpersonal comparisons of utility.

18. To our knowledge of the game theory literature, this concept of a player's propensity to disrupt has not previously appeared.

The propensity would be negative when an area gets more than it would be allowed in the core; it approaches infinity as that area's final payment approaches zero. Those imputations where each area's propensity to disrupt is less than, say, one would be the area within triangle FJR in Figure 2.[19]

Although one would clearly have difficulty in justifying a precise choice of an "acceptable" upper limit for any area's propensity to disrupt (below which it would be satisfied with the agreement), the concept does have a certain intuitive plausibility and it can be used to eliminate those imputations in the core in which one area's propensity to disrupt is "too" high.[20]

Other concepts have been proposed in the literature to eliminate certain imputations and other sets of final payments from further consideration, namely the von Neumann-Morgenstern Solution, the bargaining set, and the kernel. In our example, however, the

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19. For any imputation outside this triangle, at least one area (and perhaps two areas) will lose an amount less than the combined loss of the other two areas if it refuses to cooperate.

20. Again, it can be argued that it would be rational for an area with a very high disruptive propensity to actually carry out a threat of (temporary) non-cooperation, in the hope of achieving gains at a later negotiating session.

von Neumann-Morgenstern Solution contains the core and other imputations as well,[21] and the set of imputations in the bargaining set is identical to the core.[22] The kernel contains only one imputation[23] (K), which is a member of the core and will be considered in the following section.

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21. Actually there are two such Solutions: one consisting of the union of the core and those imputations such that P(TN) = 760 (on line BAX in Figures 1 and 2), and the other consisting of the union of the core and those imputations with P(AP) = 360 (on line EAY in Figures 1 and 2). See Luce and Raiffa [3] for a discussion of the von Neumann-Morgenstern Solution and see Gately [2] for its application to our example.

22. There are other sets of final payments in the bargaining set, corresponding to coalition structures I, II, III, and IV, but they are not imputations. See Rapoport [6, Chapter 6], for the definition of the bargaining set we are using ($\mathcal{M}_1^{(i)}$ in the literature). See Owen [5, pp.185-191], for definitions of the six "bargaining sets": $\mathcal{M}$, $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}^{(i)}$, $\mathcal{M}_1^{(i)}$, and $\mathcal{M}_2^{(i)}$; for our example, the core is identical with the imputations contained in the first five and the entire set of imputations is contained in the sixth.

23. There are four other sets of final payments in the kernel, one for each of coalition structures I, II, III, and IV, but no other imputations. See Rapoport [6, Chapter 7] for a discussion of the kernel.

IV. SOME CANDIDATES FOR THE "BEST" WAY OF SHARING THE GAINS FROM COOPERATION

There are several imputations which might be suggested as candidates for the "best" division of the gains from cooperation. We shall examine six of these, some of them proposed quite arbitrarily: the imputation (T) which divides the total gains into three equal shares, the imputation (Q) which divides the total gains according to each area's share in the total demand for energy,[24] the imputation (Z) in which each area has the same ratio (about 81%) of total final costs under regional cooperation to its costs under self-sufficiency, the imputation (S) which is the "Shapley value" for the game, the imputation (K) in the kernel, and the imputation (W) which equalizes each area's "propensity to disrupt".

The equal-shares imputation (T) is a naively "equitable" candidate for the "best" distribution. It takes no account of the relative size or "bargaining strength" of the three parties involved. Nor is it a member of the core: Andhra Pradesh gets an amount so large that Tamil Nadu and Kerala-Mysore could both be better off by refusing to cooperate with it and dividing up the resulting benefits.

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24. The shares of the three areas in the total number of megawatt-hours demanded are roughly 49% for Tamil Nadu, 20% for Andhra Pradesh, and 31% for Kerala-Mysore.

The demand-share-weighted imputation (Q) uses energy demand as an index of the relative size of the three areas to divide up the total gains in an "equitable" manner. This imputation results in positive total final costs for each area and it is also a member of the core. While Kerala-Mysore receives a final payment about as low as possible within the core (and the other two areas about as high as possible),[25] it could not do better under any two-area agreement that either Tamil Nadu or Andhra Pradesh would accept. Kerala-Mysore could, of course, threaten not to cooperate at all and might carry out such a threat (temporarily) in hopes of a greater final payment at a later negotiating session.

Imputation Z uses an area's costs under self-sufficiency as an index of its size, weighting each area's share of the gains from cooperation by that index. Not surprisingly, Tamil Nadu with its high-cost sources of power does well under this criterion and, conversely, Kerala-Mysore does poorly. The imputation is not a member of the core: Kerala-Mysore could induce Andhra Pradesh into a two-area coalition such that each could be better off.

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25. Expressed differently, Kerala-Mysore has a relatively high propensity to disrupt ($d_{KM} = 1.79$) and the other two have quite low propensities ($d_{TN} = .013$, $d_{AP} = .176$).

The "Shapley value", imputation S, represents a more complicated notion of an "equitable" distribution,[26] and it is sometimes used as a benchmark of "fairness".[27] In our example, it is a member of the core and has positive total final costs for each area; in addition, each area's propensity to disrupt is less than unity. Such an imputation might therefore be a strong contender for the "best" division of the gains from cooperation. The Shapley value has, however, received critical comment in the literature: to quote Rapoport:

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26. It assumes that three-area coalition is formed by some sequence such as area i joining the two-area coalition (j,k), which in turn was formed by area j joining the one-area coalition (k); there are six possible sequences, each assumed equally likely. It also assumes that the area joining the coalition receives the entire increment resulting from his joining. The Shapley value for player i is his expected final payment under this procedure:

$$P(i) = \frac{1}{6}\{(v(i,j,k) - v(j,k)) + (v(i,j,k) - v(j,k)) + (v(i,j) - v(i)) + (v(i,k) - v(i)) + (v(i)) + (v(i))\}$$

See Luce and Raiffa [3] or Rapoport [6].

27. See Segal [7].

"... the Shapley value solution, besides determining a unique disbursement of the payoffs solely by the characteristic function of the game, has built into it a certain equity principle. This solution might therefore be a strong contender for the status of a 'normative' solution, i.e., one which 'rational players' ought to accept. Its weakness is precisely in that it derives entirely from the characteristic function of the game and not from what is 'behind' the characteristic function, i.e., the strategic structure of the game itself rather than the bargaining positions of the players in the process of coalition formation."[28]

In contrast with the equity principle built into the Shapley value solution, the kernel represents a "hardheaded" approach in evaluating each area's bargaining strength; to quote Rapoport again:

"... one could say that the Shapley value solution is based on the players' 'hopes' before they try to realize these hopes in bargaining procedures. As we have seen, other analyses (e.g., the theory of the kernel) tend to shatter some of these hopes".[29]

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28. Rapoport [6, p.113].

29. Ibid., p.133.

The imputation in the kernel[30] (K), which is a member of the core, awards a rather large share of the total gains to Kerala-Mysore (resulting in negative total final costs for that area), and hardly any share at all to Andhra Pradesh. As a result Andhra Pradesh has a very high propensity to disrupt ($d_{AP} = 5.35$) and would presumably be quite willing to carry out a threat of non-cooperation in the hope of receiving a larger share of the total gains. Furthermore, the rationale for selection of the kernel imputation is subject to serious criticism in our case,[31] so that we would not consider this imputation a very strong candidate for the "best" distribution of gains.

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30. The imputation in the kernel $\{P^*(TN), P^*(AP), P^*(KM)\}$ has the property that each area is "in equilibrium" with every other area, i.e., where one area's maximum hope of gain by excluding the second area from a two-area coalition with the third equals the second area's maximum hope of gain by excluding the first from a two-area coalition with the third:

for TN and AP,
$$1170 - P^*(KM) - P^*(TN) = 770 - P^*(KM) - P^*(AP)$$

for TN and KM,
$$210 - P^*(AP) - P^*(TN) = 770 - P^*(AP) - P^*(KM)$$

for AP and KM,
$$210 - P^*(TN) - P^*(AP) = 1170 - P^*(TN) - P^*(KM).$$

(This would, of course, involve interpersonal comparisons of utility). See Rapoport [6, Chapter 7].

31. In our case, each area's maximum hope of gain (footnote 30) is negative. Equating these "hopes" is thus a strange way of selecting a single imputation.

The last imputation we'll consider, imputation W, equalizes each area's propensity to disrupt the three-area agreement ($d_{TN} = d_{AP} = d_{KM}$), at a value of .595. In addition to being a member of the core and resulting in positive total final costs for each area, this imputation also equalizes the ratio of each area's final payment to its maximum final payment within the core.[32,33] This ratio could be suggested as a reasonable index of an area's "satisfaction", so that this imputation would then leave the three areas equally "satisfied". While this rationale admittedly involves interpersonal utility comparisons, it would appear at least as plausible as the rationale underlying any alternative candidate.[34]

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32. The value at which these ratios are equalized,

$$.625 = \frac{P(TN)}{760} = \frac{P(AP)}{360} = \frac{P(KM)}{1320},$$

is always equal to 1/1+d*, where d* is the common value (.595) of the propensities to disrupt.

33. Expressed differently, this imputation uses an area's maximum final payment within the core as an index of its "bargaining strength" and weights each area's share accordingly; for example,

$$P(TN) = \left(\frac{760}{760 + 360 + 1320}\right)(1530).$$

34. To our knowledge of the literature, the rationale of choosing this imputation has not previously appeared.

V. SOME CONCLUDING THOUGHTS

Having directed our attention in Section III to those imputations in the core, we proceeded in Section IV to examine six imputations proposed as candidates for the "best" division of the gains from cooperation. Of these we rejected two for not being in the core (imputations T and Z), and found two other imputations questionable in view of a high disruptive propentity for one area (very serious for imputation K, less serious for imputation Q). The two remaining candidates, imputations S and W, were both found acceptable on the grounds we considered. While the former, the Shapley value S, is often used as a "normative" solution, we have suggested a possible rationale for preferring the latter.

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