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SHARP BOUNDS FOR SEIFFERT MEAN IN TERMS OF WEIGHTED POWER MEANS OF ARITHMETIC MEAN AND GEOMETRIC MEAN

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Abstract. For $a,b>0$ with $a \neq b$, let $P=(a-b)/(4\arctan\sqrt{a/b}-\pi)$, $A=(a+b)/2$, $G=\sqrt{ab}$ denote the Seiffert mean, arithmetic mean, geometric mean of a and b, respectively. In this paper, we present new sharp bounds for Seiffert P in terms of weighted power means of arithmetic mean A and geometric mean G:

$$\left(\tfrac{2}{3}A^{p_1}+\tfrac{1}{3}G^{p_1}\right)^{1/p_1} < P < \left(\tfrac{2}{3}A^{p_2}+\tfrac{1}{3}G^{p_2}\right)^{1/p_2},$$

where $p_1=4/5$ and $p_2=\log_{\pi/2}(3/2)$ are the best possible constants. Moreover, our sharp bounds for P are compared with other known ones, which yields a chain of inequalities involving Seiffert mean P.

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