

## SHARP TRIANGLE INEQUALITY AND ITS REVERSE IN BANACH SPACES

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*Abstract.* We shall present a sharp triangle inequality and its reverse inequality with  $n$  elements in a Banach space  $X$ , or equivalently we shall estimate the difference  $\sum_{j=1}^n \|x_j\| - \|\sum_{j=1}^n x_j\|$  for given  $x_1, x_2, \dots, x_n$  in  $X$ , where equality attainedness will be characterized. Several applications will be given.

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*Key words and phrases:* triangle inequality, sharp triangle inequality, reverse triangle inequality, strictly convex Banach space, uniform non- $\ell_1^n$ -ness.

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