

SHARP WILKER AND HUYGENS TYPE INEQUALITIES FOR TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS

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Abstract. In this paper, we prove Wilker and Huygens type inequalities for inverse trigonometric functions. This solves two conjectures proposed by Chao-Ping Chen. Also, we present new sharp Wilker and Huygens type inequalities for trigonometric functions.

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