SHEAR FLOW DRIVEN COMBUSTION INSTABILITY; EVIDENCE, SIMULATION AND MODELING

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ABSTRACT

Combustion instability arises mostly due to the coupling between heat release dynamics and system acoustics; a condition elegantly stated by the Rayleigh criterion. In these cases, the acoustic field acts as *the resonator*, or pace maker, while the heat release dynamics, induced due to the impact of pressure or flow velocity perturbations on the equivalence ratio, flame length, residence time, shear layer, etc., supplies energy to support pressure oscillations. In this paper, we discuss another mechanism in which the resonator, or pace maker, may not be the acoustic field; instead it is an *absolutely unstable shear layer mode* acting as the source of sustained oscillations, which morph as large-scale eddies at a frequency different from acoustic modes. These perturb the flame motion and provide the energy to the acoustic field, acting here as an amplifier, to support pressure oscillations. We present evidence to the existence of this mechanism from several experiments where a pressure spectral peak can not be explained using acoustic analysis of the system, but instead matches the predicted absolute instability mode of the separated shear layer, and from numerical simulations. We also examine the impact of the mean velocity and temperature distribution on the frequency and growth rate to determine the dynamics leading to an absolute instability, and show that absolutely unstable modes are likely to arise at low equivalence ratios. In some cases, they can also be present at near stoichiometric conditions, i.e. only low and near unity stoichiometry can support an absolutely unstable mode. We use numerical simulation to distinguish between different unstable modes in the separating shear layer; the layer mode and the wake mode, and derive a reduced order model for the shear driven instability using POD analysis. Estimates of pressure amplitudes of fluid dynamic modes, when applied as forcing functions to the acoustic field, using this analysis compare favorably with experimental data.

INTRODUCTION

Combustion instability is observed under certain conditions of equivalence ratio, ϕ, incoming flow velocity, flame stabilization mechanisms, among other factors. In most cases, multiple modes, defined by different peaks in the pressure, heat release, or Rayleigh index spectra, co-exist. Most often, the acoustic field acts as the resonator for several of these modes, i.e., it defines each mode frequency as one of the possible longitudinal, transverse or bulk acoustic frequencies, and the velocity, pressure or temperature perturbation act as the driver for heat release dynamics through their impact on flame surface area, equivalence ratio, chemical time scales, etc. In other instances, one of the spectral frequencies may not agree with available acoustic frequencies, and "fluid dynamic" modes are thought to be the source of resonance. In this case, fluid dynamics instabilities define the mode frequency, perturb the heat release and feed energy into the acoustic field, which, under the circumstances, acts as an amplifier. Justifying the presence of these modes, predicting their frequency as function of the flow and other properties related to the equivalence ratio, and developing reduced models that capture their characteristics is the subject of this paper.

We review a number of experiments and numerical simulations. The emerging pattern in devices which use separating shear layers to stabilize premixed combustion indicates that two types of instability modes exist: (1) acoustically coupled modes in which the frequency scales with a longitudinal length and the acoustic velocity; and (2) shear flow driven modes where the frequency does not correspond to one of the system frequencies, but scales with the flow velocity and a large transverse length. We show that in the latter case, a similar frequency can be measured or identified in the corresponding non reacting flow, and argue that a shear flow mode can act as a resonator that support growing pressure oscillations in the reacting flow case, under certain values of the equivalence ratio.

Theoretical analysis starts with a nonreacting flow simulation of a separating shear layer that reveals two vortex-shedding patterns, and a spectral analysis of these simulations. To confirm the existence of these unstable modes and shed light on their origin, linear stability analysis of the mean velocity profiles is performed. The concept of *absolute instability* is incorporated to differentiate between a resonant or *absolutely unstable*, AU, mode, which persists independent of external forcing, and *convectively unstable*, CU, mode that requires continuous forcing. We extend the analysis to determine the impact of the density ratio and its distribution on the dynamics of the flow, as a surrogate for combustion. It is argued that while CU modes could couple with the acoustic field giving rise to resonance, the frequency in this case is that of the acoustic field, and only when the mode is AU will the instability frequency correspond to the shear driven mode, i.e., AU modes provide a frequency selection mechanism that explain some experimental results. Proper orthogonal decomposition is utilized to extract a reduced order non-linear model for this mode.

EVIDENCE:

The role of fluid dynamic modes in combustion instability studies involving separating shear layers over a rearward step, cavity, double expansion, and related configurations are now examined. In [1], a premixed flame stabilized over a step exhibited a low frequency mode which scaled to Strouhal number St = fH/U = 0.1, where f is the frequency (in Hz), H is the step height and U is the flow velocity at the step. In the simulation of [2], small and large structures were predicted and the pressure spectrum showed 0.3 < St < 0.75. In the step experiments of [3,4,5], a mode with St = 0.1 persisted downstream, even when large frequency forcing was applied, the mode also coincided with a quarter wave acoustic mode. In [6], some peaks whose frequency scaled linearly with the flow velocity, almost independent of the acoustic conditions, were measured; these were typically low frequency modes. In [7], a double expansion experiment was used, and peaks with a characteristic time = acoustic time + vortex "lifetime", were identified. Low frequency modes scaled with the flow velocity to St = 0.08 - 0.12. In [8], a double expansion experiment exhibited another form of fluid dynamic instability, a jet preferred mode with St = 0.26; a numerical simulation of a similar geometry in [9] predicted to St = 0.19. In the simulation of a premixed flame behind a bluff body [10], the Rayleigh index frequency doubled as the flow velocity was doubled, with St= 0.24. In numerical simulations, [11,12, 13], nonreacting flows exhibited a mode at St = 0.08 - 0.1; and reacting flows showed several unstable modes, with St = 0.08 - 0.1 among them, even when the flow was forced. The same nonreacting mode was observed in other studies, e.g., [14]. Flow visualizations in several of these studies reveal that the flame dynamics is dominated by large-scale vortices generated within the recirculation zone.

In the numerical simulations of a flow over a cavity in Ref. [11], two unstable flow modes were identified: (1) the *layer mode*, which scales with the shear layer thickness at the step, and (2) *the wake mode*, which scales with the step height. While both persist in nonreacting flow and reacting flow simulations [12,13], the wake mode whose St = 0.08, which becomes more significant as the length/depth ratio of the cavity increases, impacts the flame structure more strongly. It was also shown that while the premixed flame convolves around the outer edges of the large eddies, the distance between the vorticity layer and the flame increases downstream and at higher ϕ due to the increase in the burning velocity. We will show that this may have interesting implications as regards the overall dynamics of the flow.

In a recent experiment of premixed flame over a step, [15], the reacting flow pressure spectrum, shown in figure 1, exhibited three peaks at 48, 96 and 128 Hz, with the first and third being wide and short while the middle being narrow and high indicating that a possible fundamental difference in their origin might exist. Acoustic analysis reveals that the first and third peaks correspond closely to the 1/4 and 3/4 longitudinal modes of the combustor, while the second peak does not agree with identifiable acoustic modes. The two-dimensional acoustic wave equation was solved in the domain of the experiment with the temperature variation due to combustion introduced in the definition of the local speed of sound, and similar results were obtained, as shown in figure 2. The 96-Hz mode was not identified in the 2D simulation, and we concluded that this mode was not acoustic in origin, i.e., it must belong to a different resonator whose origin lies somewhere else in the flow dynamics. Results of the non-reacting flow in the same experiment show a peak near 105 Hz. The persistence of this nonacoustic mode in the nonreacting flow suggests that it may be shear driven. A theory in [16] shows that a forced-oscillation-driven combustion instability can achieve same order of magnitude pressure amplitudes as observed in the experiment.

Another interesting experimental observation is shown in figure 3, where the impact of the equivalence ratio on the pressure amplitude is shown. Both the 1/4 wave mode and the 96 mode amplitudes rise as the mixture is leaned out, while the opposite is true for the 3/4 wave mode, showing that ϕ has an impact on both the acoustic and the fluid dynamic mode.

The 96 Hz mode scales with the step height and average incoming flow velocity to a Strouhal number of 0.092. This falls within the values determined before for the "wake" mode in separating nonreacting and reacting shear layers. The proximity of the frequency in the non-reacting and reacting flows, and its apparent shear (nonacoustic) origin, suggests using nonreacting flow simulation in this configuration to analyze the origin of these oscillations. In several experimental investigations, and in some numerical studies, it was established that the dominant oscillations near the separation point, were approximately two-dimensional. Thus, we limit the analysis to a two-dimensional simulation. The results of these simulations will then be applied to examine the flow structure, its stability characteristics and finally to produce a reduced-order model.

SIMULATION

The two-dimensional simulations of a separating flow downstream of a rearward-facing step was performed using a vortex code. This code has been described extensively in the literature and several simulations of similar flow have been used to demonstrate its validity [11,17,18]. The code was applied to the case of 1:2 expansion, similar to the experiment in [15]. Following an initial transient, the flow settled into a stationary state in which repeated shedding of vortical structures at the step, followed by one or two merging to form a large eddy downstream was observed. The shedding was observed within a step height from the point of separation, while the large eddy forms within three step heights. Figure 4 shows a series of streamline plots, separated by normalized time of 0.4. The separating shear layer is defined by three time-dependent straight lines, the outer two lines delineate the boundaries of the shear layer while the middle shows its centerline, that is, the line connecting the centers of the eddies. It defines approximately the reattachment length. Other odes, utilizing different numerical methods, applied to the same problem have shown similar results, especially as regards the large scale vortex shedding and its frequencies [19,20].

The eddy shedding and pairing frequencies are determined using frequency domain analysis of local time traces to obtain the local dominant Strouhal numbers, defined as the frequency of the mode possessing maximum local amplitude. This value drops from 0.3, to close to 0.08 within 2-3 step height downstream of the step, similar trend was observed experimentally [21]. The drop is consistent with the eddy pairing mechanism described above where two and sometimes three eddies pair before leaving the recirculation zone. The smallest Strouhal number appears first in the middle of the recirculation zone. Plots of the amplitude of the corresponding frequency show a continuous increase of the dominant mode amplitude until the end of the recirculation zone, beyond which it begins to decay. Towards the end of the recirculation zone, the mode with the highest signature has a normalized frequency of 0.08.

STABILITY ANALYSIS

The self-sustained shedding of large-scale vortices forming half way through the recirculation zone points to the existence of a self-sustained unstable fluid dynamic mode, which persists, under some conditions, in the reacting flow and may act as a resonator for heat release dynamics. The numerical simulations confirm the existence of such resonator, but do not provide the theoretical justification for its presence. A theoretical framework for the investigation of shear flow dynamics is found in modern linear stability analysis in which unstable modes have been characterized as: (1) CU modes which grow while moving downstream away from their point of inception leaving behind decaying oscillations and stable flow (unless the perturbation is continuously applied); and, (2) AU modes which grow in place while spreading spatially contaminating the entire domain and "regenerating" the instability. The latter evolves into a self-sustained resonator that, once perturbed, continues to oscillate indefinitely (due to the transfer of energy from the mean flow to the perturbation). Detail of how to conduct this analysis can be found in Ref. [22,23].

This approach was applied to determine whether an absolute mode exists, and its frequency, in the separating shear layer flow described above. Analysis is performed as a function of the downstream location since the mean flow U(y,x) depends on the streamwise location. Results are shown in figure 5 where the growth rate of the local most unstable mode is shown. Close to the step and further downstream within the reattachment length, the modes are CU; almost half way within the recirculation zone is where AU modes can be sustained, with maximum growth rate found about 1.5 step heights. This is where in the numerical results we detect the formation of the large eddies. The AU mode frequency does not change for almost two-step heights downstream of its point of inception. This frequency, when normalized, leads to St = 0.083. This is the same value obtained in the experiment, and other results cited above for the "wake" mode.

To understand what flow characteristics give rise to this mode, we extend the study to a family of velocity profile defined as:

$$U(y) = \frac{\beta - 1}{2} + \frac{\beta + 1}{2}\tanh\left(\frac{y}{\delta}\right) - \left(\beta \tanh\left(\frac{y+1}{0.1}\right) + \tanh\left(\frac{y-1}{0.1}\right)\right). \qquad (1)$$

The last term was added to approximate the impact of the no-slip boundary conditions at the walls, and the first two terms describe the shear layer profile, where β represents the amount of backflow in the recirculation zone, while δ is the shear layer thickness. The impact of theses parameters on the frequency and growth rate of the instability are shown in figure 6. Switching from a CU to an AU mode occurs at large back flows and small shear layer thickness! This is consistent with the results obtained previously: closer to the step the shear layer thickness is small but the back flow is also very weak, further downstream, the back flow is stronger and the shear layer is still thin enough to support an AU mode, close to the end of the recirculation zone, the shear layer is much thicker and the back flow is becoming weaker.

In premixed reacting flows, the recirculation zone is made up of hot products separated from cold reactants by the flame front. To capture some of the impact of combustion on the unstable mode, we repeat the stability analysis while assuming the existence of a density or temperature profile superimposed on the velocity profile. The temperature distribution within the shear layer is taken as:

$$\theta(y) = \frac{1+\gamma}{2} + \frac{1-\gamma}{2}\tanh\left(\frac{y-\Delta}{\delta}\right), \qquad (2)$$

where, γ is the temperature ratio across the flame, and Δ is the local mean offset between the shear and temperature layers. The offset is the distance between the point of maximum shear and the flame front. We note here that: (1) this offset increases downstream as the flame front slopes upwards towards while the shear layer slopes downwards [9,24,25], and (2) as ϕ increases, both Δ and γ increase, with the flame moving further away from the shear layer. Results in figure 7 show that increasing γ improves the flow stability by reducing the growth rate of the AU mode, with higher temperature ratios leading to switching from an AU mode to a CU mode, which does not support self-sustained oscillations. Thus, low ϕ mixtures, with small Δ and γ, are more likely to support AU modes, i.e., leaning out/decreasing ϕ may lead to the onset of an AU mode and hence combustion instability at its frequency, consistent with the experimental results in figure 3 where the non acoustics mode was excited as ϕ was lowered. The results concerning the impact of γ are consistent with the notion that the momentum ratio across the layer, and not the velocity ratio, is what determine its stability.

As mentioned above, larger offset downstream are observed at higher ϕ, and that, as shown in figure 7, may lead to the reestablishment of an absolutely-unstable mode again (the arrow in the figure indicates qualitatively the impact of increasing ϕ). This is also consistent with experimental observations that combustion instability at the “shear driven” frequency, O(0.1), arises at low ϕ and near stoichiometry. In the latter case, both sides of the shear layer are flooded with hot gases.

REDUCED ORDER MODEL

Reduced order models of combustion have been developed for analysis and design of passive and active control systems. Here, we present preliminary results for a reduced-order model for the nonreacting shear layer described above, as a component of a more comprehensive combustion instability model that includes the acoustic field and heat release dynamics. Model reduction is accomplished using proper orthogonal decomposition, in which numerical results are used to construct a space of optimal basis functions that describe the different modes of the flow, and applying these functions to construct time-dependent ode’s that determine the amplitudes of the corresponding modes under different conditions. The eigenvalues and functions are obtained from the covariance matrix of the data evaluated at different time steps, while the ode’s are obtained from a Galerkin expansion of the dependent variables in these basis functions, and projecting the original Navier-Stokes equations onto their space [26]. The objectives of applying this procedure here are to: (1) confirm that the dominant mode in these data is that whose frequency is 0.083, and (2) examine the accuracy of a reduced model of this complex flow. The relative magnitudes of the covariance matrix eigenvalues are used to determine the significance of the corresponding mode.

Using one hundred snapshots for the flow, 100 eigenvalues obtained, as shown in figure 8. The magnitude of these eigenvalues decay very quickly, showing that the first few modes may be sufficient to describe the flow accurately. The frequency and damping ratio of the first two modes are St = 0.083 and -0.097, respectively, and for the second two modes are 0.23 and -0.0382, respectively. The frequencies are consistent with the spectral analysis of the numerical results, and the negative damping indicates that these are two growing modes. Figure 9 shows a comparison between the actual data and the POD reconstruction using 4, 6 and 10 modes, at two different points in the flow, where a node and antinode of the first mode exist. Clearly the reconstruction accuracy depends on the dominant dynamics at the location and the number of modes, the more significant the higher frequencies are, the larger the number of modes that should be used in the reconstruction.

CONCLUSIONS

Examination of reacting separating shear flow experiments and simulations reveal that some modes are not acoustic, and their frequencies scale with the step height and flow velocity to a Strouhal number of O(0.1). The mode is self-sustained, i.e., it must be supported by a fluid dynamic resonator. Numerical simulation and linear stability theory reproduce this mode, and justify its persistence by showing that it is absolutely unstable, i.e., once perturbed it grows into a self sustaining limit cycle. Numerical simulations demonstrate that this is a wake mode that forms almost half way within the recirculation zone, resulting from the pairing of smaller shear layer vortices. This mode becomes absolutely unstable when the shear layer is thin and the backflow is sufficiently strong. Under certain conditions, the presence of a density gradient consistent with a premixed flame may reduce the growth rate of this mode or turns it into a convectively unstable mode instead. The results suggest that combustion instability models describing the coupling between acoustics and heat release dynamics should be extended to include shear flow dynamics. We show that POD analysis can be used to construct a reduced order model for these modes. Coupling these flow models with acoustics and heat release dynamics will be pursued in future articles.

The pressure amplitudes measured in [15] can be estimated using these results as follows. According to [5] and [13], the heat release perturbation at the shear layer frequency, 96 Hz, may be assumed to be at 15% of the mean value. This perturbation can be used as a forcing function for two acoustic modes, whose frequencies are 48 and 124 Hz, at a location halfway within the recirculation zone, to calculate the resulting acoustic pressures. When adding the response of the two acoustic modes linearly, we predict 0.075 psi for the pressure oscillations amplitude, at 96 Hz; the experiment measured 0.07 psi.

ACKNOWLEDGEMENT

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FIGURE CAPTIONS

Figure 1. The power spectra of pressure fluctuations for the nonreacting and reacting flow conditions flow at: U = 26.5 m/s, H = 0.025 m, the Reynolds number is 45000 and the equivalence ratio is 0.73, results are taken from Ref [15].

Figure 2. The acoustic modes of the combustor in Ref [15], obtained from 2D numerical solution of the acoustic equations using ABACUS, conditions correspond to those described in figure 1.

Figure 3. The impact of the equivalence ratio on the pressure amplitude of the three modes identified in the experiment in Ref [15], whose frequencies are defined in figure 1.

Figure 4. Numerical simulations of the nonreacting flow over a step, Reynolds number 5000, showing the streamlines below a certain value to identify the dynamics of the vortical structures. Note the shedding of eddies at the step followed by pairing. Three lines are use to define an outer edge for the separating shear layer and its centerlines.

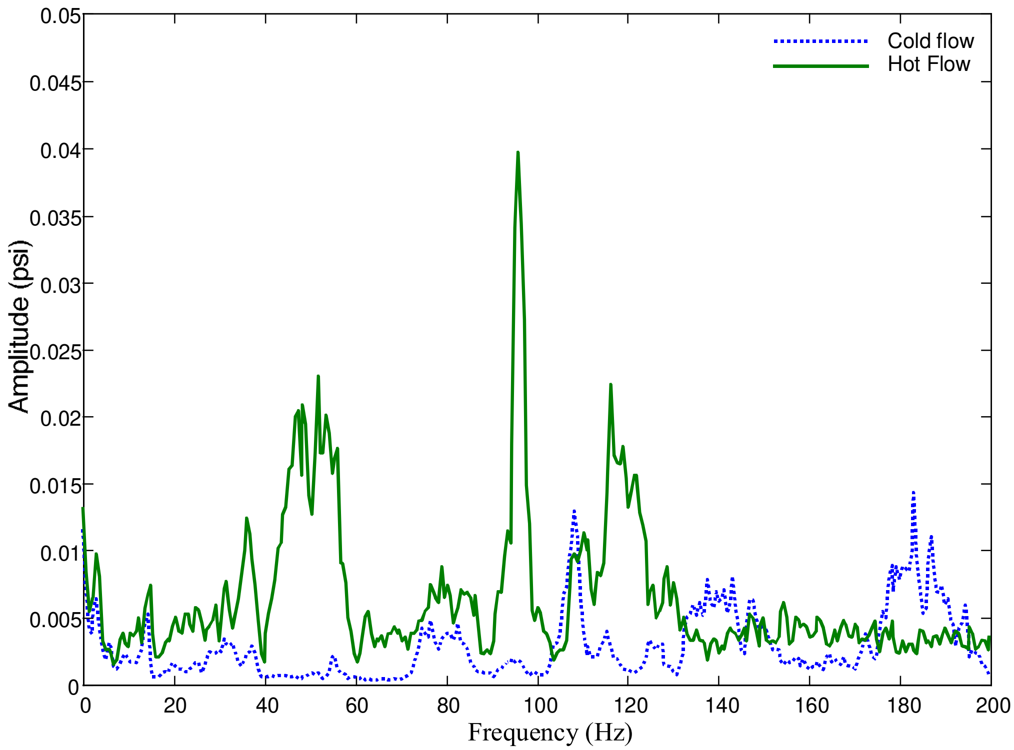
Figure 5. Local absolute mode frequencies for the mean velocity profiles obtained from the numerical simulation of a backward facing step flow shown in figure 4. In the parenthesis next to the data point, the normalized location of the section (x/H) is shown.

Figure 6. Absolute mode frequencies for the family of velocity profiles shown in equation (1), for different values of the shear layer thickness δ and the backflow β.

Figure 7. The impact of the temperature distribution on the properties of the absolute instability, shown in terms of the ratio of absolute growth rate to absolute frequency ($\omega_{0,i}/\omega_{0,r}$) for various values of temperature ratio and offset for the case with $\delta = 0.3$ and $\beta = 0.4$. The thick arrowed line shows a possible path for increasing the equivalence ratio.

Figure 8. The eigenvalues of the covariance matrix of the data set obtained from the numerical simulation of a backward facing step flow Re=5000, corresponding to the streamlines shown in figure 4.

Figure 9. Comparison between data and POD model for u' at (a) a node of the first mode located at (4.73,0.87) and (b) an anti node of the same mode, (5.67,0.33). The POD model uses 4 , 6 and 10 modes to reconstruct the data.



Cold flow
Hot Flow
Amplitude (psi)
Frequency (Hz)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0
20
40
60
80
100
120
140
160
180
200

figure 1

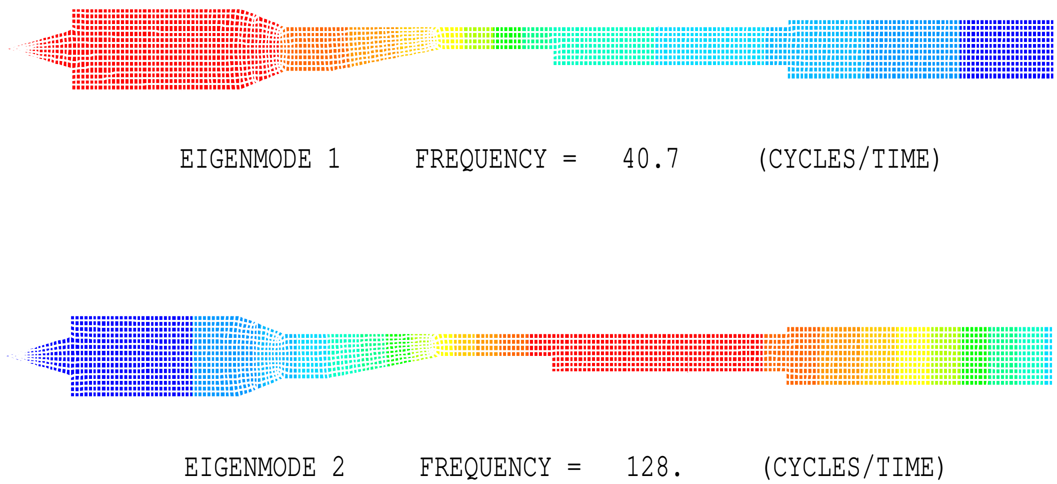
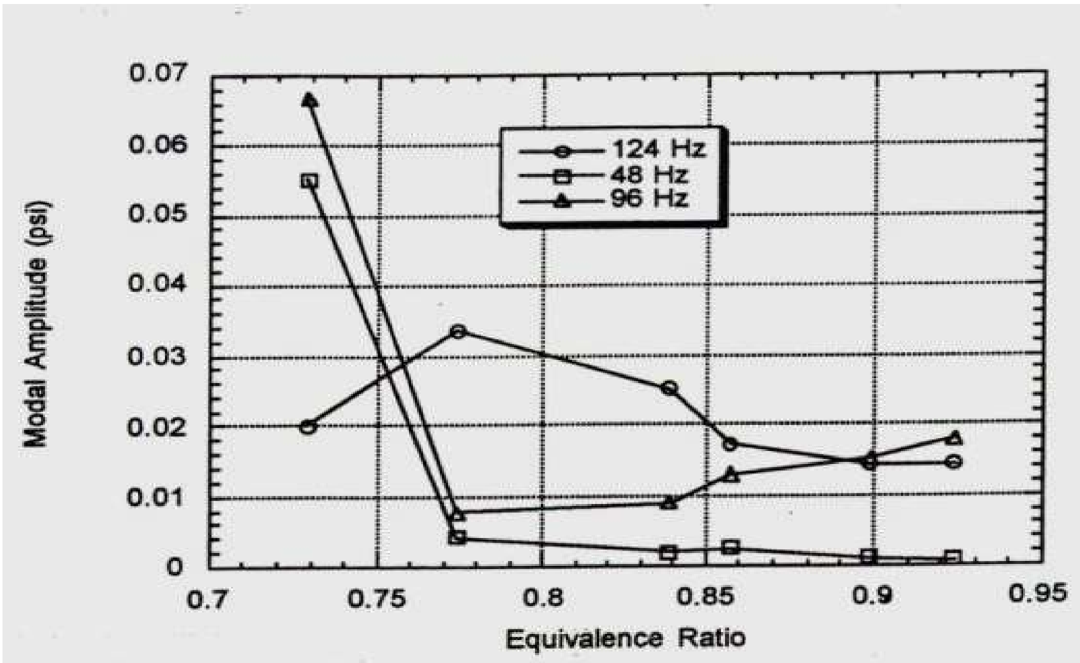
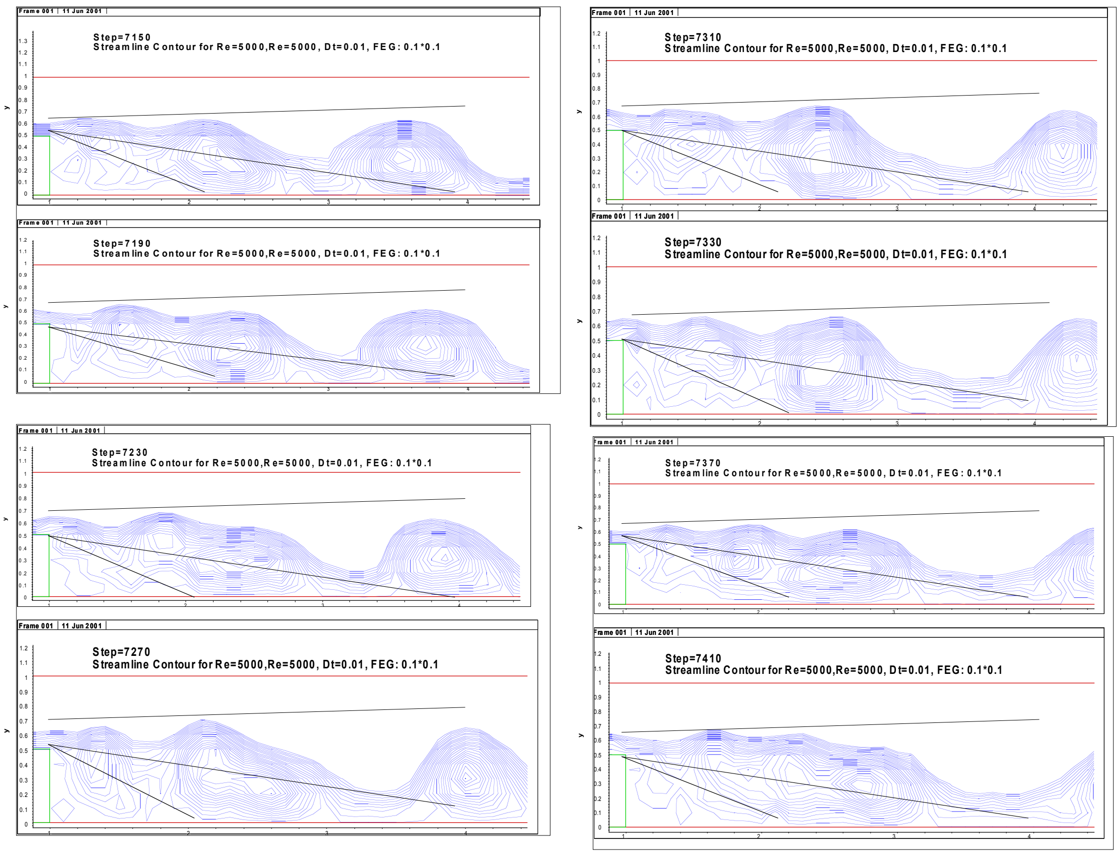

EIGENMODE 1 FREQUENCY = 40.7 (CYCLES/TIME)
EIGENMODE 2 FREQUENCY = 128. (CYCLES/TIME)

figure 2



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Modal Amplitude (psi)
124 Hz
48 Hz
96 Hz
0.7
0.75
0.8
0.85
0.9
0.95
Equivalence Ratio

figure 3


Frame 001 | 11 Jun 2001
Step=7150
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
Frame 001 | 11 Jun 2001
Step=7190
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
Frame 001 | 11 Jun 2001
Step=7230
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
Frame 001 | 11 Jun 2001
Step=7270
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
Frame 001 | 11 Jun 2001
Step=7310
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
Frame 001 | 11 Jun 2001
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Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
Frame 001 | 11 Jun 2001
Step=7370
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1
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Step=7410
Streamline Contour for Re=5000,Re=5000, Dt=0.01, FEG: 0.1*0.1

figure 4

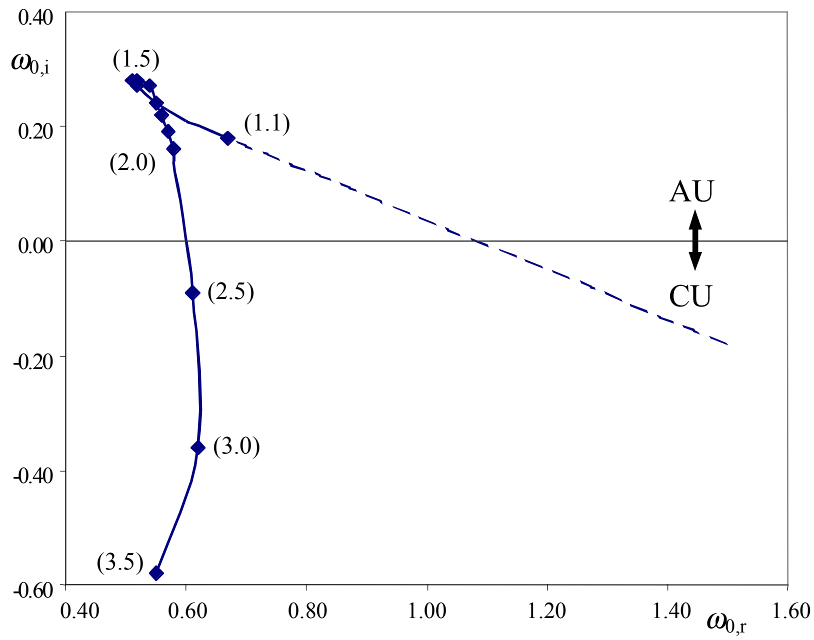
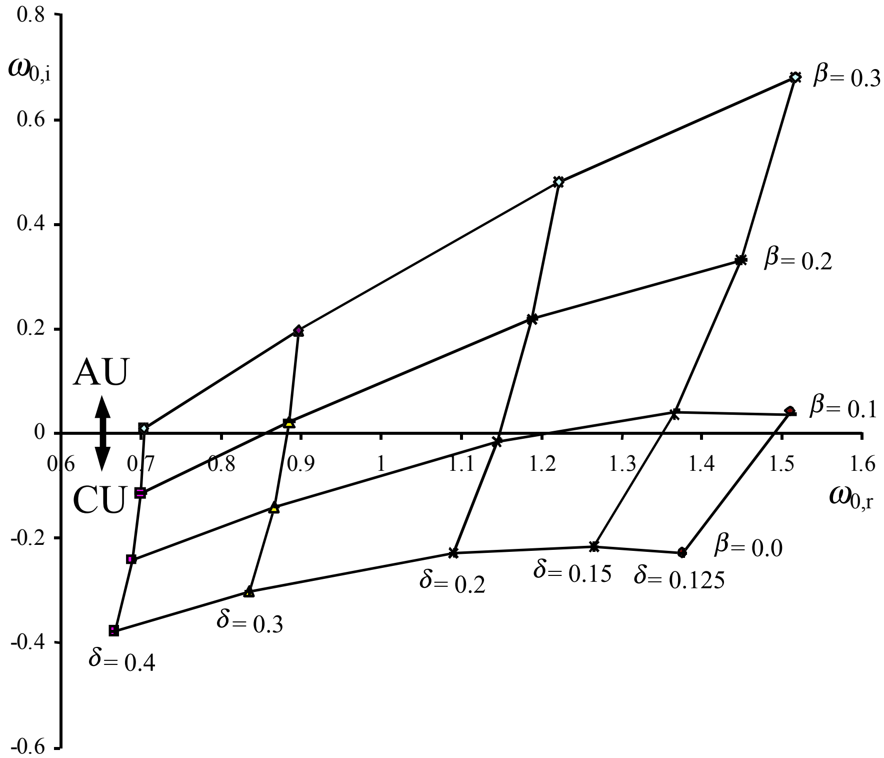
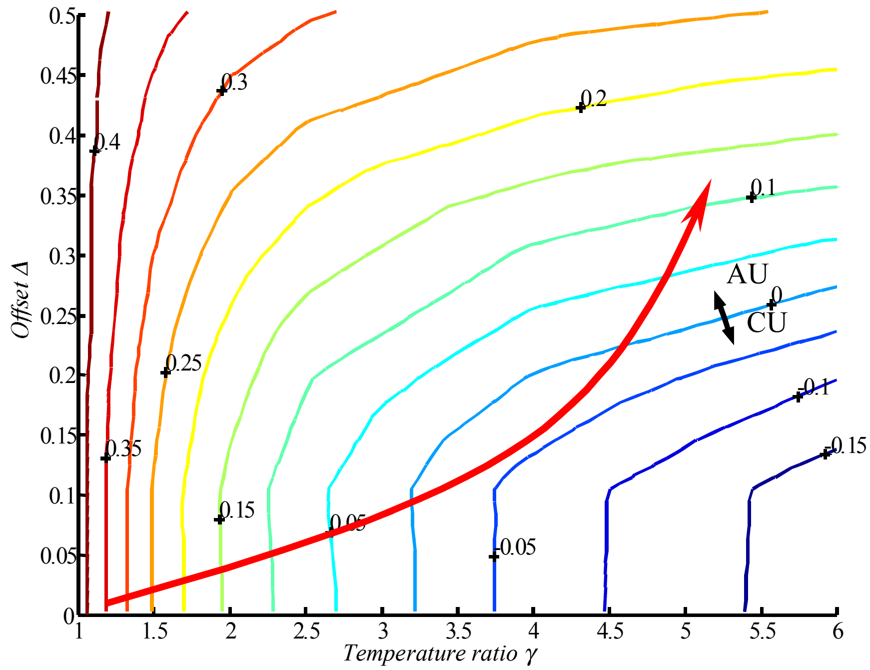

0.40
$\omega_{0,i}$
0.20
0.00
-0.20
-0.40
-0.60
0.40
0.60
0.80
1.00
1.20
1.40
1.60
$\omega_{0,r}$
(1.5)
(1.1)
(2.0)
(2.5)
(3.0)
(3.5)
AU
CU

figure 5



0.8
$\omega_{0,i}$
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
$\omega_{0,r}$
AU
CU
$\beta = 0.3$
$\beta = 0.2$
$\beta = 0.1$
$\beta = 0.0$
$\delta = 0.4$
$\delta = 0.3$
$\delta = 0.2$
$\delta = 0.15$
$\delta = 0.125$

figure 6



Offset Δ
Temperature ratio γ
0.4
0.3
0.35
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
AU
CU

figure 7

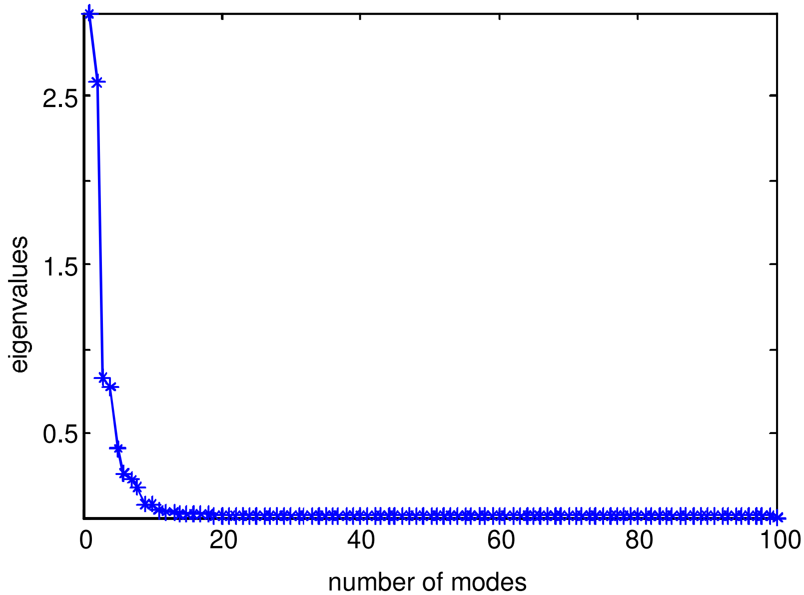


figure 8

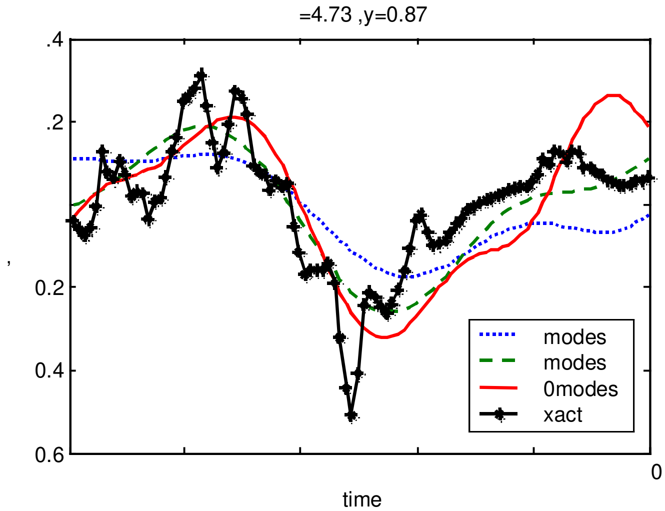


figure 9a

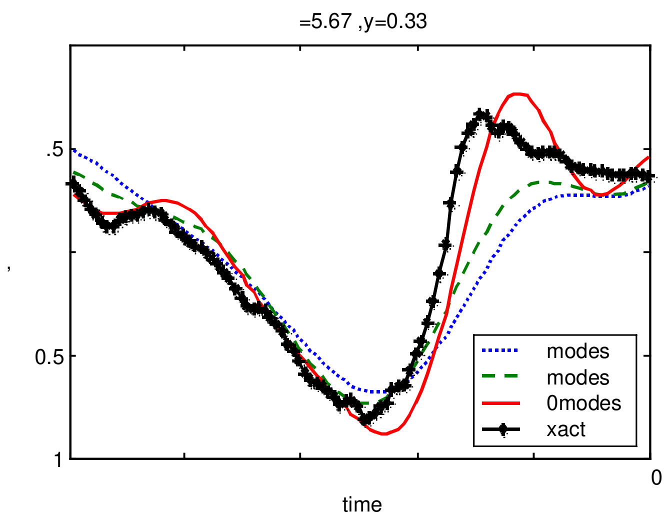


Figure 9b

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