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Shear Viscosity from Gauss-Bonnet Gravity with a Dilaton Coupling

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Abstract

We calculate the shear viscosity of field theories with gravity duals of Gauss-Bonnet gravity with a non-trivial dilaton using AdS/CFT. We find that the dilaton filed has a non-trivial contribution to the ratio of shear viscosity over entropy density and after imposing causal constraint for the boundary field theory, the new lower bound $4/25\pi$, obtained from pure Gauss-Bonnet gravity, may have a small violation.

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1 Introduction

The development of AdS/CFT correspondence [1, 2, 3, 4] provides an efficient way to study the hydrodynamic properties of strongly coupled gauge field theories. A remarkable example is the calculation of the shear viscosity [5, 6, 7, 8] for conformal field theories with gravity dual descriptions. The ratio of the shear viscosity over entropy density is calculated to be $1/4\pi$ for a large variety of conformal field theories with gravity duals in the large N limit, with or without chemical potentials [9, 10, 11, 12, 13]. With large N corrections, the ratio of shear viscosity over entropy density was found to have a positive correction to $1/4\pi$ [14, 15, 16, 17, 18, 19, 20]. Along with the fact that all known substances including water and liquid helium as well as the quark-gluon plasma created at Relativistic Heavy Ion Collider (RHIC) have a larger shear viscosity over entropy density ratio than $1/4\pi$, it was conjectured that $1/4\pi$ is a universal lower bound for all materials, which is called the Kovtun-Starinets-Son (KSS) bound [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

However, in Refs. [32, 33, 34] the authors considered R^2 higher derivative gravity corrections and found that the modification of the ratio of shear viscosity over entropy density to the conjectured bound is negative, which means that the lower bound is violated in this condition. The higher derivative gravity corrections they considered can be seen as generated from stringy corrections given the vastness of the string landscape. A new lower bound, $4/25\pi$ which is smaller than $1/4\pi$, is proposed, based on the causality of dual field theory.

In Refs. [35, 36] it was conjectured that the shear viscosity is fully determined by the effective coupling of the transverse gravitons on the horizon in the dual gravity description. This was confirmed in Ref. [38] using the scalar membrane paradigm approach and also in Ref. [45] by directly calculating the on-shell action of the transverse gravitons. In these calculations, the effective action of the transverse gravitons in a given background in generalized gravity theories was assumed to be a minimally coupled massless scalar with an effective coupling which depends on the radial coordinate, while in the Einstein gravity, the effective coupling is just a constant. However, it cannot be directly seen just from the action of the dual gravity theory whether the effective action of the transverse gravitons really takes the form assumed there. Furthermore the choice of coordinate system of the background black hole geometry also affects the form of the action of the transverse gravitons because this formalism is not covariant under coordinate transformations. Thus to consider more general gravity theories, we use an effective action of the transverse gravitons with a slightly generalized form of coupling. In this new formalism, we define a new three-dimensional effective metric $\widetilde{g}_{\mu\nu}$ and the transverse gravitons are minimally coupled to this new effective metric. Following the same procedure in Ref. [45], we find that the shear viscosity of the dual field theory can also be calculated and it is no longer fully determined by the effective coupling of gravitons in the general case.

The concrete expression of the action of the transverse gravitons for a given gravity theory still needs to be calculated explicitly. For the Einstein and Gauss-Bonnet gravity with scalars and vectors coupled only to the ordinary derivatives of metrics, the expression of the effective action of the transverse gravitons has been found to have the same dependence on the background metric as the pure Einstein and Gauss-Bonnet gravity, respectively [45]. However, this class of the theories are not the low-energy effective theories of the strings, which always contain the dilaton with nontrivial couplings. In this paper, we calculate the effective action of the transverse gravitons for Gauss-Bonnet gravity coupled to a nontrivial dilaton field [46], and examine how the above results are modified. In fact we will find that the effective action of the transverse gravitons is not the same as pure Gauss-Bonnet gravity.

In this nontrivial example, after careful analysis we find that the causal constraint that should be imposed to make sure the boundary field theory does not violate causality is still simple. With the constraint, we find that the ratio of η/s may have a small violation to the new lower bound proposed in Ref. [32].

In the remainder of this paper, we first give a brief calculation of the shear viscosity given the form of the effective action of the transverse gravitons in Sec. 2. In Sec. 3 we calculate the effective metric for Gauss-Bonnet gravity coupled with a nontrivial dilaton field. In Sec. 4 we give the causal constraints and the analysis of the value of the ratio. Sec. 5 gives the conclusions and discussions.

2 Shear viscosity

In this section we calculate the shear viscosity of the dual field theory given the form of the effective action of the transverse gravitons. We use the Kubo formula

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega i}\Big(G^A_{xy,xy}(\omega,0) - G^R_{xy,xy}(\omega,0)\Big), \tag{1}$$

where η is the shear viscosity and the retarded Green's function is defined by

$$G^R_{\mu\nu,\lambda\rho}(k) = -i\int d^4x e^{-ik\cdot x}\theta(t)\langle[T_{\mu\nu}(x), T_{\lambda\rho}(0)]\rangle. \tag{2}$$

These are defined on the field theory side. The advanced Green's function can be related to the retarded Green's function of energy momentum tensor by $G^A_{\mu\nu,\lambda\rho}(k) = G^R_{\mu\nu,\lambda\rho}(k)^*$. Using the field operator correspondence, the Green function of energy momentum tensors of the field theory can be calculated through the effective action of gravitons of the dual gravity theory.

For simplicity, here we choose spatial coordinates so that the momentum of the perturbation points along the z-axis. The perturbations can be written as $h_{\mu\nu} = h_{\mu\nu}(t, z, u)$ with the radial coordinate u. In this basis, there are three groups of gravity perturbations, each of which is decoupled from the others: the scalar, vector and tensor perturbations [40]. Here we use the simplest one, the tensor perturbation h_{xy}. We use ϕ to denote this perturbation with one index raised $\phi = h^x_y$ and write ϕ in a basis as $\phi(t, u, z) = \phi(u)e^{-i\omega t+ipz}$.

We can then get the effective action of this transverse graviton by keeping terms to the second order of ϕ in the gravity action. For the Einstein gravity, the effective action of the transverse gravitons is always

$$S = \frac{1}{16\pi G}\Big(-\frac{1}{2}\Big)\int d^5x\sqrt{-g}(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi), \tag{3}$$

when matter fields are coupled to the metric minimally [45], where $g_{\mu\nu}$ is the metric of the background black hole solution. For general gravity theories coupled to matter fields minimally or non-minimally, the action of the transverse gravitons is no longer of the form (3). In Refs. [38, 45] a deformed form of the effective action

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}K_{eff}(u)(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi), \tag{4}$$

is studied, where $K_{eff}(u)$ is an effective coupling constant. There might be some extra terms appearing in (4) like $N(u)\partial_z\phi\partial_z\phi$ where $N(u)$ is a function regular at $u = 1$ (black hole horizon), and these terms will not affect the value of η. With the addition of the extra terms, the $K_{eff}(u)$ in (4) is not a scalar under the general coordinate transformation, so the expression in (4) is not a real covariant form. Generally, the factors in front of $g^{tt}\partial_t\phi\partial_t\phi$ and $g^{uu}\partial_u\phi\partial_u\phi$ are not always the same and in these cases the effective action of the transverse gravitons cannot be written in the form of (4). It is difficult to determine whether the effective action of the transverse gravitons takes such a form as in (4) for a generic gravity theory. Thus in the following of this section, we use a general form of the expression of the effective action of the transverse gravitons, which is valid generally and does not depend on the choice of coordinate system.

For gravity theories in which the transverse gravitons can be decoupled from other perturbations, a general form of the effective bulk action of the transverse gravitons can be written as

$$S = \frac{V_{x,y}}{16\pi G}\Big(-\frac{1}{2}\Big)\int d^3x\sqrt{-\widetilde{g}}(\widetilde{K}(u)\widetilde{g}^{MN}\widetilde{\nabla}_M\phi\widetilde{\nabla}_N\phi + m^2\phi^2) \tag{5}$$

up to some total derivatives, where $\widetilde{g}_{MN}$ is a three-dimensional effective metric, m is an effective mass which can be any function of the radial coordinate and $\widetilde{\nabla}_M$ is the covariant differential using the metric $\widetilde{g}_{MN}$. Here we use the effective metric $\widetilde{g}_{MN}$ to denote the factor in front of $\widetilde{\nabla}_M\phi\widetilde{\nabla}_N\phi$ in the action of graviton. $\phi = h^x_y$ is a scalar in the three dimensions of t, u and z, while it is not a scalar in the whole five-dimensional system. Because we have assumed that h^x_y only depends on the coordinates t, u and z, the effective action of h^x_y can be viewed as a deduced three-dimensional action where the other two directions can be integrated out. Thus this is not the dimensional reduction in the usual sense, and the Newton constant in (5) is still the five-dimensional Newton constant. We write the action in the three-dimensional form so that the action (5) takes a general covariant form and $\widetilde{K}(u)$ is a scalar under general coordinate transformations. In the following of this paper, we use $g_{\mu\nu}$, (μ, $\nu = t$, u, x, y, z) to denote the metric of the five-dimensional background and $\widetilde{g}_{MN}$, (M, $N = t$, u, z) to denote the three-dimensional effective metric. The effective action for the transverse gravitons in the Einstein gravity can be obtained from (5) by choosing $\widetilde{g}^{MN} = g^{MN}$ for M, $N = t$, u, z and $K(u) = \sqrt{-g}/\sqrt{-\widetilde{g}}$. Thus in this case $\widetilde{K}(u)$ comes from the determinant of the metric of the x and y directions. Note that here $\widetilde{K}(u)$ is not the same one as $K_{eff}(u)$ in (4). In fact this $\widetilde{K}(u)$ can be absorbed into $\widetilde{g}_{MN}$ and be eliminated to give a minimally coupled action

$$S = \frac{V_{x,y}}{16\pi G}\Big(-\frac{1}{2}\Big)\int d^3x\sqrt{-\overline{g}}(\overline{g}^{MN}\partial_M\phi\partial_N\phi + \bar{m}^2\phi^2), \tag{6}$$

with

$$\overline{g}^{MN} = \widetilde{K}^{-2}(u)\widetilde{g}^{MN}, \tag{7}$$

and

$$\bar{m}^2 = \widetilde{K}(u)^{-3}m^2. \tag{8}$$

In the following calculation of this and the next section, we will still keep $\widetilde{K}(u)$ in order to have $\widetilde{g}^{MN} = g^{MN}$ for M, $N = t$, u, z in the case of the Einstein gravity.

Before we continue to calculate the shear viscosity, we first write down the background metric. In this paper we mainly focus on the case of Ricci-flat black hole backgrounds. The case for black holes with hyperbolic horizon topology has been discussed recently [41, 42]. We assume that the background black hole solution is of the form

$$ds^2 = -g(u)(1-u)dt^2 + \frac{1}{h(u)(1-u)}du^2 + \frac{r_+^2}{ul^2}(d\vec{x}^2), \tag{9}$$

where the horizon of the black hole locates at $u = 1$ and the AdS boundary is at $u = 0$, $h(u)$, $g(u)$ are functions of u, regular at $u = 1$ and l is related to the cosmological constant Λ by $l = \sqrt{-6/\Lambda}$. r_+ is the black hole horizon radius in the radial coordinate r, which has a relation to the coordinate u used here through $u = \frac{r_+^2}{r^2}$. In the usual Einstein gravity with the cosmological constant Λ, l is just the AdS radius, but it could be different from the effective AdS radius in more general gravity theories, for example, Einstein-Gauss-Bonnet theory with a dilaton [46], which we will consider below. Of course, one may take l as the effective AdS radius in those gravity theories, and this will not change the result. In what follows, for simplicity, we will keep $l = \sqrt{-6/\Lambda}$ even in the case of Einstein-Gauss-Bonnet theory with a dilaton. In addition, let us mention that $h(u)$ and $g(u)$ should be regular at the horizon. This implies that the Ricci-flat black hole solution we consider here is a nonextremal one.

Here we choose a diagonal background metric and we will argue that generally the effective three dimensional metric $\widetilde{g}_{MN}$ which appears in the effective action of transverse gravitons should also be diagonal even when Gauss-Bonnet terms are present. Gauss-Bonnet gravity as well as general Lovelock gravity have the property that the Einstein equations of motion contain at most second derivatives of the metric. Thus in the effective action of transverse gravitons (6) for pure Gauss-Bonnet gravity there are also at most second derivatives of ϕ, where higher derivative terms can only exist in the total derivative terms which do not affect our result. To get the effective action of transverse gravitons, we should expand the gravity action to the second order of ϕ and with the reason stated above we only need to keep terms up to the second derivatives of ϕ. By detailed calculation of the expansion of Riemann tensors with respect to ϕ, we can see that the possible non-zero coefficients of non-diagonal terms can only come from the Gauss-Bonnet term in the case of Gauss-Bonnet gravity. We give a simple illustration here to show that these coefficients are all zero given the assumption that the background metric is diagonal. We denote the three components of the Gauss-Bonnet term by $R_{a_1a_2a_3a_4}R_{a_5a_6a_7a_8}g^{b_1b_2}g^{b_3b_4}g^{b_5b_6}g^{b_7b_8}$, where the indices a_i and $b_i = t,\ u,\ x,\ y,\ z$ should be contracted for $i = 1, \cdots, 8$ and the metric here is the background metric. There are three kinds of perturbations of these terms which can give non-diagonal terms in (6) and these three kinds of perturbations can be written out as $\delta_{(1)}R_{a_1a_2a_3a_4}\delta_{(1)}R_{a_5a_6a_7a_8}g^{b_1b_2}g^{b_3b_4}g^{b_5b_6}g^{b_7b_8}$, $\delta_{(2)}R_{a_1a_2a_3a_4}R_{a_5a_6a_7a_8}g^{b_1b_2}g^{b_3b_4}g^{b_5b_6}g^{b_7b_8}$, and $\delta_{(1)}R_{a_1a_2a_3a_4}R_{a_5a_6a_7a_8}\delta_{(1)}g^{b_1b_2}g^{b_3b_4}g^{b_5b_6}g^{b_7b_8}$, where δ_j means to keep terms of the j-th order of ϕ. By careful analysis of the non-zero components of $\delta_{(j)}R_{a_1a_2a_3a_4}$, we find that for the case of the first two kinds of perturbations, there are always indices which appear an odd number of times in a_i while because the background metric is diagonal, all indices should appear an even number of times in b_i and for the last kind of perturbation, some indices appear an odd number of times in a_i and b_1, b_2 while they have to appear an even number of times in b_i for i from 3 to 8. Thus the indices can't be properly contracted and all these three kinds of perturbations can't exist, so the effective metric of $\widetilde{g}_{MN}$ should be diagonal. When matter fields are coupled to the Gauss-Bonnet term, as long as the

Einstein equation of motion contains at most second derivatives of the metric component g_{xy} as a function of t, u, z and the perturbation of the transverse gravitons can get decoupled from the perturbation of matter fields, the arguments above are still valid. Here the Einstein equation of motion can contain higher derivative terms by counting both the matter fields and the metric but it can only contain at most second derivative terms of the metric component g_{xy}. In the following of this paper, we only consider this kind of gravity theory. This argument can also be used to show that $\widetilde{g}_{MN}$ would also be diagonal for Lovelock gravity, which we do not display explicitly here.

We then follow the procedure in Ref. [45] to calculate η. We write the action of the transverse gravitons in the momentum space

$$S = \frac{V_{x,y}}{16\pi G}\Big(-\frac{1}{2}\Big)\int \frac{dwdp}{(2\pi)^2}du\sqrt{-\widetilde{g}}\Big(\widetilde{K}(u)(\widetilde{g}^{uu}\phi'\phi' + w^2\widetilde{g}^{tt}\phi^2 + p^2\widetilde{g}^{zz}\phi^2) + m^2\phi^2\Big), \tag{10}$$

by expanding

$$\phi(t,u,z) = \int \frac{dwdp}{(2\pi)^2}\phi(u;k)e^{-iwt+ipz}, \quad k = (w,0,0,p), \quad \phi(u;-k) = \phi^*(u;k), \tag{11}$$

where the prime stands for the derivative with respect to u and the ϕ^2 terms should be recognized as $\phi^*\phi$. For the Einstein gravity, $\widetilde{g}_{MN} = g_{MN}$ for M, $N = t$, u, z, $\widetilde{K}(u) = r_+^2/ul^2$ and $m = 0$. Generally, $\widetilde{K}(u)$ is a regular function at $u = 1$ and $\widetilde{g}_{MN}$ is a diagonal metric similar to the background metric. The equation of motion of the transverse gravitons can be obtained from the action (10) as

$$\phi''(u,k) + A(u)\phi'(u,k) + B(u)\phi(u,k) = 0, \tag{12}$$

where

$$A(u) = \frac{(\sqrt{-\widetilde{g}}\widetilde{K}(u)\widetilde{g}^{uu})'}{\sqrt{-\widetilde{g}}\widetilde{K}(u)\widetilde{g}^{uu}}, \tag{13}$$

and

$$B(u) = -\widetilde{g}_{uu}\Big(\widetilde{g}^{tt}w^2 + \widetilde{g}^{zz}p^2 + \frac{m^2}{\widetilde{K}(u)}\Big). \tag{14}$$

In order to solve the equation, we should make a detailed analysis of the property of $\widetilde{g}_{uu}$, $\widetilde{g}_{tt}$, and $B(u)$. The gravity action we consider should at least contain an Einstein term $\sqrt{-g}R$ and there may be other terms like $\sqrt{-g}R_{GB}^2$ and so on. The contribution to the effective action of the transverse gravitons can be obtained separately from these action terms and they should be summed. Thus $\widetilde{g}_{MN}$ should at least contain a contribution of g_{MN} which comes from the Einstein term. Generally $\widetilde{g}_{uu}$ and $\widetilde{g}^{tt}$ should have poles at most of the same order as g_{uu} and g^{tt}, i.e. poles of the first order. We isolate the parts of the first order pole in $\widetilde{g}_{uu}$ and $\widetilde{g}^{tt}$ denoted as

$\widetilde{g}^1_{uu}/(1-u)$ and $\widetilde{g}^{tt}_1/(1-u)$ respectively, where $\widetilde{g}^1_{uu}$ and $\widetilde{g}^{tt}_1$ are finite at $u=1$. We then follow the standard procedure to solve Eq. (12). First we impose the incoming boundary condition at the horizon so that

$$\phi(u) = (1-u)^{-i\beta w} F(u), \tag{15}$$

where $F(u)$ is regular at the horizon. β can be calculated by considering (12) in the limit $u \to 1$, which leads to

$$\beta = \sqrt{-\widetilde{g}^1_{uu}\widetilde{g}^{tt}_1}\Big|_{u=1}. \tag{16}$$

Note here that $\widetilde{g}^{zz}$ does not have any poles just like g^{zz}, so it does not affect the value of β. Also we assume m^2 has poles at most of the first order at the horizon, so m^2 does not affect the value of β either. Because we only need to know the $w \to 0$ behavior of this graviton, we can expand the solution as

$$F(u) = 1 + i\beta w F_0(u) + O(w^2) + O(p^2). \tag{17}$$

By expanding Eq. (12) to the first order of w, we get the equation of $F_0(u)$:

$$F_0''(u) + A(u)F_0'(u) + \frac{1}{(1-u)^2} + \frac{A(u)}{1-u} = 0. \tag{18}$$

The solution of this function is already given in Ref. [45] and it can be uniquely determined by imposing the regular boundary conditions. We can determine the derivative of the solution as

$$F_0'(u) = \frac{S(1)}{1-u}\Big(\frac{1}{S(u)} - \frac{1}{S(1)}\Big), \tag{19}$$

where $S(u) = (\sqrt{-\widetilde{g}}\widetilde{K}(u)\widetilde{g}^{uu})/(1-u)$ and $S(1) = \lim_{u\to 1} S(u)$. Using the same arguments as in the appendix of Ref. [45], we find that the on-shell action is

$$S_{on-shell} = \frac{V_{x,y}}{16\pi G}\Big(-\frac{1}{2}\Big)\int \frac{dwdp}{(2\pi)^2} du \Big(\sqrt{-\widetilde{g}}\widetilde{K}(u)\widetilde{g}^{uu}\phi'\phi\Big)' \tag{20}$$

after the Gibbons-Hawking surface contribution has been counted. Integrating this action gives

$$S_{on-shell} = \frac{V_{x,y}}{16\pi G}\Big(-\frac{1}{2}\Big)\int \frac{dwdp}{(2\pi)^2}\Big(\sqrt{-\widetilde{g}}\widetilde{K}(u)\widetilde{g}^{uu}\phi'\phi\Big)\Big|^{u=0}_{u=1}. \tag{21}$$

Then the shear viscosity can be calculated using the Kubo formula (1) to be

$$\eta = \frac{1}{16\pi G}\lim_{w\to 0}\frac{-\sqrt{-\widetilde{g}}\widetilde{K}(u)\widetilde{g}^{uu}\phi'\phi|_{u=0}}{iw} = \frac{1}{16\pi G}(\beta S(1)). \tag{22}$$

After substituting β in (16) and $S(u)$ we find that

$$\eta = \frac{1}{16\pi G}\Big(\sqrt{\widetilde{g}_{zz}}\widetilde{K}(u)\Big)\Big|_{u=1}. \tag{23}$$

For the black hole backgrounds where the area formula of black hole entropy still holds, the entropy density is

$$s = \frac{1}{4G}\frac{r_+^3}{l^3}, \tag{24}$$

and the ratio of shear viscosity over entropy density is

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{l^3}{r_+^3}\Big(\sqrt{\widetilde{g}_{zz}}\widetilde{K}(u)\Big)\Big|_{u=1}. \tag{25}$$

Thus we obtained a general formula to calculate the ratio of shear viscosity over entropy density. It can be checked that for those actions which can be written in the form (4), the ratio of η/s reproduces the dependence on the effective coupling on the horizon obtained in Ref. [45].

3 Non-trivial dilaton

In the previous section, we have given a general formula for the shear viscosity of the dual field theory. Using this formula, we can read the ratio of shear viscosity over entropy density from the effective action of the transverse gravitons in the gravity description. However, the exact form of the effective action of the transverse gravitons still has to be calculated case by case. In Ref. [45] we know that for the Einstein and Gauss-Bonnet gravity coupled with matter fields minimally, the effective action of the transverse gravitons is not affected by the matter fields, but the arguments in Ref. [45] do not apply to gravity theories with non-minimally coupled matter fields. In this section we calculate the effective action of the transverse gravitons for Gauss-Bonnet gravity with a non-minimally coupled dilaton and use the formula (23) to obtain the shear viscosity of the dual field theory.

The action we consider in this section is [46]

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\Big[R - \frac{1}{2}\nabla_\mu\phi_d\nabla^\mu\phi_d + \frac{\lambda l^2}{2}e^{-\gamma\phi_d}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2) - 2\Lambda e^{\tau\phi_d}\Big], \tag{26}$$

where λ is the Gauss-Bonnet coupling, l is the AdS radius, γ and τ are constants and ϕ_d is the dilaton field. This action can be obtained by transforming the string frame action with a nontrivial dilaton to the Einstein frame. The ten-dimensional critical string theory predicts $\gamma = 1/2$ and in this paper we follow [46] to leave γ unfixed. The term $2\Lambda e^{\tau\phi_d}$ is the effective cosmological term produced by a nontrivial dilaton, and $\tau = 5/2$ if we assume that the term comes from the expectation value of the RR 10-form in type IIB superstring theory. However there are also other sources of this term and here we leave τ unfixed also. In fact the effective cosmological term can be replaced by a more general potential $V(\phi)$, which does not affect the result. The unique requirement is to have an asymptotically AdS black hole solution with such a potential. In addition, to make sure that the gravity regime is valid, we have to impose the condition that $\lambda \ll 1$, ϕ_d not too large and ϕ_d changes slowly along the radial coordinate u.

We also assume the Ricci-flat black hole solution (9) and calculate the effective action of the transverse gravitons on this background. It can be checked directly from the first-order Einstein and Klein-Gordon equation of motion that the transverse gravitons can get decoupled from other perturbations. Then we can get the effective action of transverse gravitons by keeping quadratic terms of ϕ in the original action (26) and find that we have at most second derivatives of ϕ up to this order. Thus the action of the transverse gravitons can be written in the form (5) and the effective three-dimensional metric for this specific case is

$$\widetilde{g}^{uu} = \Big[1 + \frac{\lambda l^2}{2} e^{-\gamma\phi_d}\Big(\frac{g^{uu} g'_{tt}}{u g_{tt}} + \frac{2\gamma\phi'_d g^{uu} g'_{tt}}{g_{tt}} - \frac{2\gamma g^{uu}\phi'_d}{u}\Big)\Big] g^{uu}, \tag{27}$$

$$\widetilde{g}^{tt} = \Big[1 + \frac{\lambda l^2}{2} e^{-\gamma\phi_d}\Big(\frac{g^{uu\prime}}{u} - \frac{3g^{uu}}{u^2} - \frac{2\gamma g^{uu}\phi'_d}{u} - 4\gamma^2 g^{uu}{\phi'_d}^2 + 4\gamma g^{uu}\phi''_d + 2\gamma\phi'_d g^{uu\prime}\Big)\Big] g^{tt}, \tag{28}$$

and

$$\begin{aligned}\widetilde{g}^{zz} = \Big[1 + \frac{\lambda l^2}{2} e^{-\gamma\phi_d}\Big(- \frac{g^{uu\prime} {g_{tt}}'}{g_{tt}} + \frac{g^{uu} {g'_{tt}}^2}{g_{tt}^2} + \frac{2\gamma g^{uu} {g_{tt}}' \phi'_d}{g_{tt}} - \frac{2 g^{uu} {g_{tt}}''}{g_{tt}} + 4\gamma g^{uu} {\phi_d}''\\ +2\gamma g^{uu\prime} {\phi_d}' - 4\gamma^2 g^{uu} {\phi'_d}^2\Big)\Big] g^{zz},\end{aligned} \tag{29}$$

where the metric $g^{\mu\nu}$ denotes the background metric. After a lengthy calculation, it is found that the m^2 term vanishes if we use the Einstein equations for the background metric.

In fact, the effective action of the transverse gravitons for this gravity theory (26) can be written as

$$S = \frac{1}{16\pi G}\Big(-\frac{1}{2}\Big)\int d^5x\sqrt{-g}\widetilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{30}$$

where $g^{\mu\nu}$ here is a five dimensional metric with $\widetilde{g}^{\mu\nu} = \widetilde{g}^{\mu\nu}$ for μ, $\nu = t$, u, z and $\widetilde{g}^{\mu\nu} = g^{\mu\nu}$ for μ, $\nu = x$, y, as a non-covariant form, so $\widetilde{K}(u) = \sqrt{-g}/\sqrt{-\widetilde{g}}$, where $g^{\mu\nu}$ denotes the background five-dimensional metric (9). Thus the shear viscosity can be obtained using the formula (23) as

$$\eta = \frac{1}{16\pi G}\frac{r_+^3}{l^3}\Big(1 - \frac{\lambda l^2}{2} e^{-\gamma\phi_d(1)} h(1)\Big(1 + 2\gamma{\phi_d}'(1)\Big)\Big). \tag{31}$$

For this Ricci-flat black hole, the entropy still obeys the Bekenstein-Hawking entropy area law [44, 46], so the entropy density can be easily written as

$$s = \frac{1}{4G}\frac{r_+^3}{l^3}. \tag{32}$$

Thus the ratio of shear viscosity over entropy density is

$$\frac{\eta}{s} = \frac{1}{4\pi}\Big(1 - \frac{\lambda l^2}{2} e^{-\gamma\phi_d(1)} h(1)\Big(1 + 2\gamma{\phi_d}'(1)\Big)\Big). \tag{33}$$

Here the dilaton ϕ_d should be regular at the horizon. The formula (33) for η/s is valid even when other scalar or vector fields are present as long as those fields are minimally coupled to the ordinary derivatives of the background metric. Here to analyze the concrete value of η/s we concentrate on the action (26) without other matter fields coupled. Thus from the Einstein equation of motion of the action (26) [46]

$$h(1) = \frac{8}{l^2} e^{\tau\phi_d(1)}, \tag{34}$$

we can express the ratio of η/s as

$$\frac{\eta}{s} = \frac{1}{4\pi}\Big(1 - 4\lambda e^{(\tau-\gamma)\phi_d(1)}\Big(1 + 2\gamma{\phi_d}'(1)\Big)\Big). \tag{35}$$

It is easy to check that when $\phi_d(1) \to 0$, (35) reduces to $(1-4\lambda)/4\pi$ for the pure Gauss-Bonnet case. However, it is worth noting that the pure Gauss-Bonnet black hole solution without the dilaton is not a solution of equations of motion for the action (26) by simply requiring the dilaton being a constant, because the constant dilaton field ϕ_d does not satisfy its Klein-Gordon equation with the pure Gauss-Bonnet black hole metric (in fact, in order to satisfy the Klein-Gordon equation with a constant dilaton, one has to impose the additional condition $\gamma = 0$). In addition, let us mention here that the ratio (35) looks dependent on the dilaton and its derivative on the horizon. In fact, the dependence of the derivative of the dilaton field can be eliminated by its equation of motion. The Klein-Gordon equation gives [46]

$${\phi_d}'(1) = -12\gamma\lambda e^{(\tau-\gamma)\phi_d(1)} + \frac{3}{2}\tau, \tag{36}$$

on the horizon. After substituting (36) into (35), we get

$$\frac{\eta}{s} = \frac{1}{4\pi}\Big(1 - 4\lambda e^{(\tau-\gamma)\phi_d(1)}\Big(1 - 24\gamma^2\lambda e^{(\tau-\gamma)\phi_d(1)} + 3\gamma\tau\Big)\Big). \tag{37}$$

This is our main result. Note that (37) is valid for all Ricci-flat solutions of the action (26). When $\phi_d = 0$ and $\gamma = 0$, it reduces to the one for pure Gauss-Bonnet gravity without dilaton field.

4 Causal constraints

It was discovered that the KSS bound can be violated in theories with higher order gravity corrections [32], but the causality on the boundary field can give a constraint to the parameter and thus we can have a new lower bound on η/s [33, 32]. Here we also discuss the causal problem on the boundary field theory to see what kind of constraints we can get on the η/s ratio.

Following the method used in Ref. [33], we need to write down the graviton equation of motion in the form of

$$\bar{g}^{MN}\bar{\nabla}_M\bar{\nabla}_N\phi = 0, \tag{38}$$

where $\bar{g}^{\mu\nu}$ is the effective metric defined in (7) (different from the $\widetilde{g}_{MN}$ we used above) describing the motion of graviton, and $\bar{\nabla}$ is the derivative operator using the metric $\bar{g}^{MN}$. The equation of motion (38) can be directly derived from the action (6) with $\bar{m} = 0$. Then we apply the standard geometrical optics approximation in the large momentum limit. To be more explicit, we write the wave function in the form $\phi = \phi_{en}(t,u,z)e^{i\theta(t,u,z)}$, where ϕ_{en} denotes a slowly changing envelope function and θ is a rapidly varying phase function. Inserting this into (38), we obtain at leading order

$$\frac{dx^M}{ds}\frac{dx^N}{ds}\bar{g}_{MN} = 0, \tag{39}$$

with the identification $\frac{dx^M}{ds} \equiv \bar{g}^{MN}k_N \equiv \bar{g}^{MN}\bar{\nabla}_N\theta$.

This equation describes a classical particle moving in the spacetime with a metric $\bar{g}_{MN}$, which is no longer the same as the background metric. In the new spacetime $\bar{g}_{MN}$, there are still translation symmetries in the t and z directions, so $\omega = i\bar{\nabla}_t\theta$ and $q = -i\bar{\nabla}_z\theta$ are conserved integrals of motion along the geodesic. Then Eq. (39) can be expanded as

$$\bar{g}^{tt}\omega^2 + \bar{g}^{zz}q^2 + \bar{g}_{uu}(\frac{du}{ds})^2 = 0,$$

which can be rewritten as

$$(\frac{du}{ds})^2 = (-\bar{g}^{tt}\bar{g}^{uu}q^2)[\frac{\omega^2}{q^2} - \frac{\bar{g}^{zz}}{-\bar{g}^{tt}}]. \tag{40}$$

In the following discussion we assume $q^2 > 0$ and the term $(-\bar{g}^{tt}\bar{g}^{uu}q^2)$ is always larger than zero, so we can rescale s as $\tilde{s} = s\sqrt{-\bar{g}^{tt}\bar{g}^{uu}q^2}$ to absorb this term and get

$$(\frac{du}{d\tilde{s}})^2 = \frac{\omega^2}{q^2} - \frac{\bar{g}^{zz}}{-\bar{g}^{tt}}. \tag{41}$$

This equation describes a one-dimensional system with a particle of energy $\frac{\omega^2}{q^2}$ moving in a potential given by $\frac{\bar{g}^{zz}}{-\bar{g}^{tt}}$. This will correspond to a bouncing geodesic starting and ending at the boundary [33]. Note that unlike the case in Ref. [33], although $\tilde{s}$ is not an affine parameter, we can still get the bouncing geodesics.

Our next task is the same as in the case without the dilaton [33]. Along a bouncing geodesic, we have

$$\triangle t(\alpha) = 2\int_0^{u_{turn}(\alpha)} \frac{\dot{t}}{\dot{u}} du = 2\int_0^{u_{turn}(\alpha)} \sqrt{\frac{-\bar{g}^{tt}}{\bar{g}^{uu}}} \frac{\alpha}{\sqrt{\alpha^2 - c_g^2}} du, \tag{42}$$

$$\triangle z(\alpha) = 2\int_0^{u_{turn}(\alpha)} \frac{\dot{z}}{\dot{u}} du = 2\int_0^{u_{turn}(\alpha)} \sqrt{\frac{-\bar{g}^{tt}}{\bar{g}^{uu}}} \frac{c_g^2}{\sqrt{\alpha^2 - c_g^2}} du, \tag{43}$$

where

$$\alpha = \frac{\omega}{q}, \quad c_g^2 = \frac{\bar{g}^{zz}}{-\bar{g}^{tt}},$$

and dot denotes the derivative with respect to s.

The graviton moving along the bouncing geodesic will hover near u_{turn} if $\alpha \to c_{g,max}$. So if c_g has a maximal value which is larger than 1 in the bulk region $0 < u < 1$, the ratio $\frac{\triangle z}{\triangle t}$ can be larger than 1. This means causality violation of the boundary field theory, as $\frac{\triangle z}{\triangle t}$ describes the effective velocity of the graviton moving from one point on the boundary, along the bouncing geodesic, to another point on the boundary. Because c_g is zero on the horizon and is set to 1 on the boundary, we can have a peak of c_g which is larger than 1 if

$$\left.\frac{\partial c_g^2}{\partial u}\right|_{u\to 0} = \lim_{u\to 0} \frac{\partial[\bar{g}^{zz}/(-\bar{g}^{tt})]}{\partial u} = \lim_{u\to 0} \frac{\partial[\widetilde{g}^{zz}/(-\widetilde{g}^{tt})]}{\partial u} > 0. \tag{44}$$

This condition is sufficient but not necessary for causality violation.

Let us now calculate c_g^2 at $u \to 0$ to give the constraint. According to Ref. [46], the spacetime is asymptotically AdS, and we have

$$\left. g_{tt}\right|_{u\to 0} = -\frac{a_1}{u} + a_2 u + \cdots, \tag{45}$$

$$\left. g^{uu}\right|_{u\to 0} = b_1 u^2 + b_2 u^4 + \cdots, \tag{46}$$

$$\left. \phi_d\right|_{u\to 0} = \phi_0 + \phi_1 u^{2+\epsilon} + \cdots, \tag{47}$$

where ϵ is a positive constant depending on some parameters in this theory, and ϕ_0 and ϕ_1 are two constants, assuming that a nonrational term does not contribute. By using the Einstein equations and Klein-Gordon equation, we can fix those expansion parameters as

$$\begin{aligned} a_1 &= \frac{r_+^2}{l_{\text{eff}}^2} N^2, \quad a_2 = \frac{2M}{r_+^2} N^2, \\ b_1 &= \frac{4}{l_{\text{eff}}^2}, \quad b_2 = -\frac{8M}{r_+^4}, \end{aligned} \tag{48}$$

where $l_{\text{eff}}^{-2} = \frac{1-\sqrt{1-4\lambda e^{(\tau-\gamma)\phi_0}}}{2\lambda l^2 e^{-\gamma\phi_0}}$, where M is the mass parameter of the black hole solution and r_+ is the horizon radius. Here $N^2 = l_{\text{eff}}^2/l^2$ is introduced in order to make the bulk metric conformal to a Minkowski spacetime on the boundary. We can use these asymptotic expansions as well as the formulas (28) and (29) to expand c_g^2 to order of u^2 near the boundary

$$\begin{aligned}
c_g^2\Big|_{u\to 0} &= \frac{g^{zz}}{-g^{tt}}\cdot\frac{1-\frac{\lambda l^2}{2}e^{-\gamma\phi_0}(b_1-b_2u^2)+4b_1\frac{a_2}{a_1}\frac{\lambda l^2}{2}e^{-\gamma\phi_0}u^2+O(u^{2+\epsilon})}{1-\frac{\lambda l^2}{2}e^{-\gamma\phi_0}(b_1-b_2u^2)+O(u^{2+\epsilon})} \\
&= \Big(1-\frac{a_2}{a_1}u^2+O(u^3)\Big)\Big(1+\frac{4b_1\frac{a_2}{a_1}\frac{\lambda l^2}{2}e^{-\gamma\phi_0}}{1-\frac{\lambda l^2}{2}e^{-\gamma\phi_0}b_1}u^2+O(u^3)\Big) \\
&= 1+\frac{a_2}{a_1}\left(\frac{2b_1\lambda l^2 e^{-\gamma\phi_0}}{1-\frac{1}{2}b_1\lambda l^2 e^{-\gamma\phi_0}}-1\right)u^2+O(u^3). \qquad (49)
\end{aligned}$$

The terms with ϕ_d' or ϕ_d'' are $O(u^{2+\epsilon})$ terms. Then in order to get $\frac{\partial c_g^2}{\partial u}>0$ near the boundary, we must have

$$\frac{a_2}{a_1}\Big[\frac{2b_1\lambda l^2 e^{-\gamma\phi_0}}{1-\frac{1}{2}b_1\lambda l^2 e^{-\gamma\phi_0}}-1\Big]>0. \qquad (50)$$

Substituting a_1, a_2 and b_1, we get the condition for causal violation

$$\lambda e^{(\tau-\gamma)\phi_0}>0.09. \qquad (51)$$

Thus in order not to have causal violation, we have to impose the condition that $\lambda e^{(\tau-\gamma)\phi_0}<0.09$. This is almost the same as the constraint for Gauss-Bonnet gravity without nontrivial dilaton (in which $\lambda<0.09$), except for a shift to λ brought by the dilaton field on the boundary. This is because the Gauss-Bonnet black hole solution with nontrivial dilaton has the same asymptotic behavior as the case without the dilaton field.

With this causal constraint, we can come back to analyze the result (37). We rewrite (37) as

$$\frac{\eta}{s}=\frac{1}{4\pi}\Big(1-4\lambda e^{(\tau-\gamma)\phi_0}e^{(\tau-\gamma)[\phi_d(1)-\phi_0]}\Big(1-24\gamma^2\lambda e^{(\tau-\gamma)\phi_d(1)}+3\gamma\tau\Big)\Big), \qquad (52)$$

where the constraint is $\lambda e^{(\tau-\gamma)\phi_0}<0.09$. Remember that to make sure the gravity regime is valid, we should assume that $e^{\tau\phi_d}$ and $e^{-\gamma\phi_d}$ are of the order $O(1)$ in the bulk and $\lambda\ll 1$. Also we have to assume that $e^{\phi_0}\ll 1$ in order to make sure the string coupling constant $\ll 1$, and luckily this can be checked to be the case after imposing the Klein-Gordon equation of the dilaton field at the boundary. The term $-24\gamma^2\lambda e^{(\tau-\gamma)\phi_d(1)}$ can be neglected compared to $3\gamma\tau$. Thus it is easy to see that the new lower bound imposed in Ref. [32] can be absolutely violated for solutions with $(\tau-\gamma)[\phi_d(1)-\phi_0]>0$ and $3\gamma\tau>0$. Because we must have $\tau-\gamma<0$ in order for the black hole solutions to exist [46], the new lower bound imposed in Ref. [32] can be violated for solutions with $\phi_d(1)-\phi_0<0$ and $3\gamma\tau>0$. This kind of solutions indeed exists, as shown in Ref. [46]. Both $e^{(\tau-\gamma)[\phi_d(1)-\phi_0]}$ and $3\gamma\tau$ are of the order $O(1)$, so the violation is small, of the order $O(\lambda)$.

5 Conclusions and Discussions

In this paper we have calculated the shear viscosity of field theories with gravity duals of Gauss-Bonnet gravity with a nontrivial dilaton using AdS/CFT and found that the ratio of the shear viscosity over entropy density explicitly depends on the dilaton field on the black hole horizon. Also we have discussed the causal violation condition of the dual field theory and found that it is the same as the case without the dilaton field in the sense of rescaling the Gauss-Bonnet coupling and effective cosmological constant by the dilaton field at the boundary. After imposing causal constraint for the boundary field theory, we have found that the new lower bound $4/25\pi$ may have a small violation due to the nontrivial dilaton.

In a recent paper [47], it was argued that the KSS bound would be violated for super conformal field theories with non-equal central charges. They also showed that the scalars and vectors coupled to the Gauss-Bonnet gravity only affect the value of the shear viscosity to the order $O(\lambda^2)$. However, in our case we find that the nontrivial coupled dilaton field affects the value of the shear viscosity at the order $O(\lambda)$. They are not inconsistent with each other. In Ref. [47], the authors considered perturbative solutions due to scalar fields and higher derivative curvature terms, based on a five-dimensional AdS black hole solution. In that case, the scalars should acquire an expectation value of order $O(\lambda)$ in the AdS vacuum and black hole backgrounds, so the scalars only affect the value of shear viscosity at the order of $O(\lambda^2)$. In our case, we considered exact solutions of equations of motion. The scalar field is of the order $O(1)$, and the shear viscosity is determined by the value of the scalar on the horizon. The violation to the new lower bound can be of order $O(\lambda)$. Note that the dual field theory will be a conformal field theory only at the UV boundary. Therefore it is expected that the bulk viscosity for the dual field theory of this black hole solution is also nonzero. It would be interesting to calculate the bulk viscosity in this background using the sound mode to see if the bulk viscosity has any nontrivial correction from the nontrivial dilaton coupling.

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