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Shear Wave Propagation in Piezoelectric-Piezoelectric Composite layered structure

Abstract

The propagation behavior of shear wave in piezoelectric composite structure is investigated by two layer model presented in this approach. The composite structure comprises of piezoelectric layers of two different materials bonded alternatively. Dispersion equations are derived for propagation along the direction normal to the layering and in direction of layering. It has been revealed that thickness and elastic constants have significant influence on propagation behavior of shear wave. The phase velocity and wave number is numerically calculated for alternative layer of Polyvinylidene Difluoride (PVDF) and Lead Zirconate Titanate (PZT-5H) in composite layered structure. The analysis carried out in this paper evaluates the effect of volume fraction on the phase velocity of shear wave.

Keywords

Piezoelectric, Shear wave, dispersion equation, volume fraction, PVDF.

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1 INTRODUCTION

The phenomenon of piezoelectricity discovered by Perre and Jaques Quriee has proven to be limelight in development of potential material for new class of sensors and actuators. In recent years piezoelectric materials has drawn much attention towards application in surface acoustic wave (SAW) micro sensors, energy harvesting structure, health monitoring systems, transducers and actuators (Du et al. 2007). SAW devices based on piezoelectric composites have enhanced electromechanical response and high sensitivity in comparison to single material structure. Piezoelectric composites have found major application in sensing and measurement industries also invariably. The dynamic response of Saw sensor is evaluated by analyzing the wave propagation and vibration pattern in these piezoelectric based composite structures. Numerous researchers have investigated the propagation behavior of shear wave in piezoelectric composite due to its vast applicability in saw devices. Qin et al. (2004) investigated the propagation behavior of horizontally shear wave in piezoelectric polymer composite structure. The dispersive and attenuated characteristic of wave propagation in thin piezoelectric layer bounded to substrate is studied by Dua et al. 2009. They conclude that these characteristics are influenced by viscous dissipation considerably. Wang and Zhao (2013) obtained the dispersion relation for piezoelectric-elastic composite plates. Shear wave propagation in piezoelectric composite structure was discussed extensively by Qian et al. (2004) and Singh et al. (2013). Some researchers have discussed the effect of volume fraction on the stop band effect (Piliposian et al. 2012 and Pang et al. 2008). Shear horizontal acoustic Waves at the Boundary of Piezoelectric Crystals were investigated by Pyatakov (2001). Propagation properties of SH-waves in a piezo ceramic layered structure and effect of volume fraction on phase velocity was discussed by Vashisth et al. (2013 and 2009). Du et al. (2009), Zakharenko (2013) and Nie et al. (2012) studied the propagation properties of wave propagation in layered piezomagnetic -piezoelectric structure. The effect of imperfect bonding on interfacial waves in dissimilar piezoelectric composite has been studied by Huang, and Li extensively (2011). Recently some researchers have revealed the imperfect bonding is major cause in dispersion of shear wave propagation in PE-PM interface (Melkumyan et al. 2008, Huang et al. 2009, and Rahman et al. 2014). In past decade, research work focused much on shear wave propagation in piezoelectric–piezomagnetic interface, but no work was reported so far on shear wave propagation in piezoelectric-piezoelectric composites.

The objective of this paper is to investigate the propagation behavior of shear waves in piezoelectric composite structure. This study is focused to obtain dispersion equation for propagation of wave in direction normal to layering and in direction of layering. The influence of layer thickness and elastic constant on shear wave propagation has also been numerically evaluated by considering the interface of two material PVDF and PZT-5H. The effect of wave number and dimensional less frequency has been plotted to show the variation between the quantities. This work provides a theoretical framework for designing and development of PE-PE composite structure for sensor and transducer applications.

2 PROBLEM FORMULATION AND CONSTITUTIVE EQUATIONS

The piezoelectric layered (PE) structure is shown in Figure 1. The composite structure comprises of piezoelectric layers bonded perfectly alternatively of two different materials. These bonded layered have thickness of h_1 and h_2 respectively. Shear wave propagation is considered to be propagating either in direction normal to layering i.e. in positive direction of x axis or in direction of layering i.e. in positive direction of y axis with poling direction taken along the z axis.

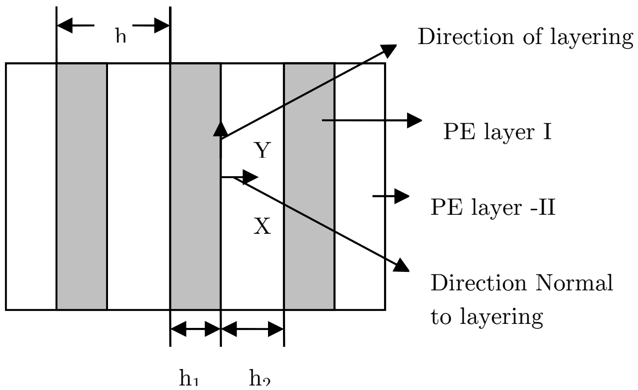

Figure 1: Schematic of periodic Piezoelectric -Piezoelectric (PE layer I, PE layer II) Layered Structure.

For shear wave propagating in x-y plane, the constitutive system of equations refereed to piezoelectric-piezoelectric interface can be written as (Qian et al. 2004):

$$\begin{Bmatrix} \sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k \\ D_j = e_{jkl}\varepsilon_{kl} + \epsilon_{jk}E_k \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \sigma_{ij} = c'_{ijkl}\varepsilon_{kl} - e'_{kij}E_k \\ D_j = e'_{jkl}\varepsilon_{kl} + \epsilon'_{jk}E_k \end{Bmatrix} \tag{2}$$

Where σ_{ij}, ε_{ij} are stress and strain sensor, D_j, *and* E_k are displacement and electric field intensity. c_{ijkl}, c'_{ijkl}, e_{kij}, e'_{kij}, ϵ_{jk}, ϵ'_{jk} are elastic, piezoelectric and dielectric constants for piezoelectric mediums respectively.

The motion equation for PE-I and PE-II can be represented in equation (3) as (Lee 2004 and Sun et al. 1968)

$$\begin{Bmatrix} \sigma_{ij,j} = \rho\ddot{u}_i \\ D_{i,j} = 0 \end{Bmatrix} \tag{3}$$

The strain tensor ε_{ij} and electric field intensity E_i can be represented as

$$\begin{Bmatrix} \varepsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j}) \\ E_i = -\frac{\partial\varphi}{\partial x_i} \end{Bmatrix} \tag{4}$$

Where u_i and ρ represents the mechanical displacement in i^{th} direction and mass density. Electrical potential function can be expressed as φ.

The constitutive system of equations for PE-I media are

$$\begin{aligned} \sigma_x &= c_{11}\varepsilon_x + c_{12}\varepsilon_y + c_{13}\varepsilon_z - e_{31}E_z \\ \sigma_y &= c_{12}\varepsilon_x + c_{11}\varepsilon_y + c_{12}\varepsilon_z - e_{31}E_y \\ \sigma_x &= c_{12}\varepsilon_x + c_{12}\varepsilon_y + c_{11}\varepsilon_z - e_{33}E_x \\ \tau_{zy} &= 2c_{44}\varepsilon_{zy} - e_{15}E_y \\ \tau_{zx} &= 2c_{44}\varepsilon_{zy} - e_{15}E_y \\ \tau_{xy} &= 2c_{44}\varepsilon_{xy} \\ D_x &= 2e_{15}\varepsilon_x + \epsilon_{11}E_x \\ D_y &= 2e_{15}\varepsilon_y + \epsilon_{11}E_y \\ D_z &= e_{31}\varepsilon_x + e_{31}\varepsilon_y + e_{33}\varepsilon_z + \epsilon_{33}E_z \end{aligned} \tag{5}$$

The constitutive system of equations for PE-II media are

$$
\begin{aligned}
\sigma_x &= c'_{11}\varepsilon_x + c'_{12}\varepsilon_y + c'_{13}\varepsilon_z - e'_{31}E_z \\
\sigma_y &= c'_{12}\varepsilon_x + c'_{11}\varepsilon_y + c'_{12}\varepsilon_z - e'_{31}E_y \\
\sigma_x &= c'_{12}\varepsilon_x + c'_{12}\varepsilon_y + c'_{11}\varepsilon_z - e'_{33}E_x \\
\tau_{zy} &= 2c'_{44}\varepsilon_{zy} - e'_{15}E_y \\
\tau_{zx} &= 2c'_{44}\varepsilon_{zy} - e'_{15}E_y \\
\tau_{xy} &= 2c'_{44}\varepsilon_{xy} \\
D_x &= 2e'_{15}\varepsilon_x + \epsilon'_{11}E_x \\
D_y &= 2e'_{15}\varepsilon_y + \epsilon'_{11}E_y \\
D_z &= e'_{31}\varepsilon_x + e'_{31}\varepsilon_y + e'_{33}\varepsilon_z + \epsilon'_{33}E_z
\end{aligned}
\tag{6}
$$

Where $c_{44} = \dfrac{(c_{11} - c_{12})}{2}, \qquad c'_{44} = \dfrac{(c'_{11} - c'_{12})}{2}$

For shear wave propagation, the mechanical displacement and electrical potential function components in x, y and z direction can be expressed as

$u = 0,\ v = 0,\ w = w(x,y,t),\ \varphi = \varphi(x,y,t)$.

Eliminating u and E from the equation (5) and (6), we get the system of equations (7) and (8)

$$
\left\{
\begin{aligned}
& c_{44}\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) + e_{15}\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2}\right) = \rho\frac{\partial^2 w_1}{\partial t^2} \\
& e_{15}\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) - \epsilon_{11}\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2}\right) = 0
\end{aligned}
\right\}
\tag{7}
$$

$$
\left\{
\begin{aligned}
& c'_{44}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) + e'_{15}\left(\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2}\right) = \rho'\frac{\partial^2 w_2}{\partial t^2} \\
& e'_{15}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) - \epsilon'_{11}\left(\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2}\right) = 0
\end{aligned}
\right\}
\tag{8}
$$

For interface at $x = 0$ between PE-PE layers, the following boundary conditions must satisfy as

$$
\begin{aligned}
& w_1(0,\mathrm{y}) = w_2(0,\mathrm{y}), \qquad \varphi_1(0,\mathrm{y}) = \varphi_2(0,\mathrm{y}) \\
& \tau_{zx1}(0,\mathrm{y}) = \tau_{zx2}(0,\mathrm{y}),\ \mathrm{D}_{x1}(0,\mathrm{y}) = \mathrm{D}_{x2}(0,\mathrm{y})
\end{aligned}
\tag{9}
$$

For all interfaces between PE-PE layers, the following boundary conditions must be satisfied as follow

$$
\begin{aligned}
& w_1(h_1,y) = w_2(-h_2,y), \qquad \varphi_1(h_1,y) = \varphi_2(-h_2,y) \\
& \tau_{zx1}(h_1,y) = \tau_{zx2}(-h_2,y),\ D_{x1}(h_1,y) = D_{x2}(-h_2,y)
\end{aligned}
\tag{10}
$$

3 SOLUTION

We will discuss the two cases for propagation of shear waves. The system of equations (7) and (8) is solved for these two cases of propagation behavior.

3.1 Propagation along the direction normal to the layering

For shear wave propagating along the positive direction of x axis, solution of system of equations (7) and (8) can be expressed in following form

$$\begin{aligned} w_1(x,t) &= W_1(x)\mathrm{e}^{ik(x-ct)} \\ \varphi_1(x,t) &= \Phi_1(x)\mathrm{e}^{ik(x-ct)} \end{aligned} \tag{11}$$

$$\begin{aligned} w_2(x,t) &= W_2(x)\mathrm{e}^{ik(x-ct)} \\ \varphi_2(x,t) &= \Phi_2(x)\mathrm{e}^{ik(x-ct)} \end{aligned} \tag{12}$$

Where k is wave number, c is propagation velocity of shear wave, $i=\sqrt{-1}$. $W_1(x), W_2(x), \Phi_1(x), \Phi_2(x)$ are some undetermined functions.

Substituting equation (11) and (12) in (7) and (8) we get

$$\begin{aligned} c_{44}\left(W_1'' + 2ikW_1' - k^2W_1\right) + e_{15}\left(\Phi_1'' + 2ik\Phi_1' - k^2\Phi_1\right) &= -\rho k^2c^2W_1 \\ e_{15}\left(W_1'' + 2ikW_1' - k^2W_1\right) - \epsilon_{11}\left(\Phi_1'' + 2ik\Phi_1' - k^2\Phi_1\right) &= 0 \end{aligned} \tag{13}$$

$$\begin{aligned} c'_{44}\left(W_2'' + 2ikW_2' - k^2W_2\right) + e'_{15}\left(\Phi_2'' + 2ik\Phi_2' - k^2\Phi_2\right) &= -\rho' k^2c^2W_2 \\ e'_{15}\left(W_2'' + 2ikW_2' - k^2W_2\right) - \epsilon'_{11}\left(\Phi_2'' + 2ik\Phi_2' - k^2\Phi_2\right) &= 0 \end{aligned} \tag{14}$$

The solution of equations (13) and (14) can be determined as

$$\begin{aligned} W_1 &= G_1\mathrm{e}^{(-1+c/c_{sh})ikx} + H_1\mathrm{e}^{(-1-c/c_{sh})ikx} \\ \Phi_1 &= \left(G_1' + H_1'x\right)\mathrm{e}^{-ikx} + \frac{e_{15}}{\epsilon_{11}}\left(G_1\mathrm{e}^{(-1+c/c_{sh})ikx} + H_1\mathrm{e}^{(-1-c/c_{sh})ikx}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} W_2 &= G_2\mathrm{e}^{(-1+c/c'_{sh})ikx} + H_2\mathrm{e}^{(-1-c/c'_{sh})ikx} \\ \Phi_2 &= \left(G_2' + H_2'x\right)\mathrm{e}^{-ikx} + \frac{e'_{15}}{\epsilon'_{11}}\left(G_2\mathrm{e}^{(-1+c/c'_{sh})ikx} + H_2\mathrm{e}^{(-1-c/c'_{sh})ikx}\right) \end{aligned} \tag{16}$$

$$c_{sh} = \sqrt{(\epsilon_{11}c_{44} + e^2{}_{15}) / \rho\epsilon_{11}}\ ,\ \ c'_{sh} = \sqrt{(\epsilon'_{11}c'_{44} + e'^2_{15}) / \rho'\epsilon'_{11}}$$

Where c_{sh} and c'_{sh} represents the bulk shear wave velocity in PE-I and PE-II media respectively.

The complete solution of mechanical displacement and electrical function in PE-PE interface can be obtained by substituting equation (15) and (16) in (11) and (12) which can be expressed in following form as

$$\begin{aligned} w_1(x,t) &= \left[G_1 e^{\left(-1+c/c_{sh}\right)ikx} + H_1 e^{\left(-1-c/c_{sh}\right)ikx}\right] e^{ik\left(x-ct\right)} \\ \varphi_1(x,t) &= \left[\left(G_1' + H_1' x\right) e^{-ikx} + \frac{e_{15}}{\epsilon_{11}}\left(G_1 e^{\left(-1+c/c_{sh}\right)ikx} + H_1 e^{\left(-1-c/c_{sh}\right)ikx}\right)\right] \\ & e^{ik\left(x-ct\right)} \end{aligned} \tag{17}$$

$$\begin{aligned} w_2\left(x,t\right) &= \left[G_2 e^{\left(-1+c/c'_{sh}\right)ikx} + H_2 e^{\left(-1-c/c'_{sh}\right)ikx}\right] e^{ik\left(x-ct\right)} \\ \varphi_2\left(x,t\right) &= \left[\left(G_2' + H_2' x\right) e^{-ikx} + \frac{e'_{15}}{\epsilon'_{11}}\left(G_2 e^{\left(-1+c/c'_{sh}\right)ikx} + H_2 e^{\left(-1-c/c'_{sh}\right)ikx}\right)\right] \\ & e^{ik\left(x-ct\right)} \end{aligned} \tag{18}$$

3.2 Propagation along the direction of the layering

For the wave propagation in positive direction of y axis, the mechanical displacement and electrical potential function represented as

$$\begin{aligned} w_1\left(x,y,t\right) &= W_1\left(x\right) e^{ik\left(y-ct\right)} \\ \varphi_1\left(x,y,t\right) &= \Phi_1\left(x\right) e^{ik\left(y-ct\right)} \end{aligned} \tag{19}$$

$$\begin{aligned} w_2\left(x,y,t\right) &= \mathrm{W}_2\left(\mathrm{x}\right) e^{ik\left(y-ct\right)} \\ \varphi_2\left(x,y,t\right) &= \Phi_2\left(\mathrm{x}\right) e^{ik\left(y-ct\right)} \end{aligned} \tag{20}$$

The complete solution of mechanical displacement and electrical function is obtained and represented as system of equations in (21) and (22)

$$\begin{aligned} w_1(x,y,t) &= \left[G_1 e^{-ib_1 x} + H_1 e^{ib_1 x}\right] e^{ik\left(y-ct\right)} \\ \varphi_1(x,y,t) &= \left[G_1' e^{-kx} + H_1' e^{kx} + \frac{e_{15}}{\epsilon_{11}}\left(G_1 e^{-ib_1 x} + H_1 e^{ib_1 x}\right)\right] e^{ik\left(y-ct\right)} \end{aligned} \tag{21}$$

$$\begin{aligned} w_2\left(x,y,t\right) &= \left[G_2 e^{-b_2 x} + H_2 e^{b_2 x}\right] e^{ik\left(y-ct\right)} \\ \varphi_2\left(x,y,t\right) &= \left[\left(G_2' e^{-kx} + H_2' e^{kx}\right) + \frac{e'_{15}}{\epsilon'_{11}}\left(G_2 e^{-b_2 x} + H_2 e^{b_2 x}\right)\right] e^{ik\left(y-ct\right)} \end{aligned} \tag{22}$$

Where $b_1 = k\sqrt{\frac{c^2}{c^2_{sh}} - 1}$, $b_2 = k\sqrt{1 - \frac{c^2}{c'^2_{sh}}}$

4 SHEAR WAVE PROPAGATION AND DISPERSION RELATION

4.1 Propagation along the direction normal to the layering

Substituting the equations (17) and (18) in equations (5) and (6) gives

$$\begin{aligned} \tau_{zx1} &= \left[e_{15} H_1' \mathrm{e}^{-ikx} + \frac{ikcP}{c_{sh}} \left(G_1 \mathrm{e}^{\left(-1+c/c_{sh}\right)ikx} - H_1 \mathrm{e}^{\left(-1-c/c_{sh}\right)ikx} \right) \right] \mathrm{e}^{ik\left(x-ct\right)} \\ D_{x1} &= -\epsilon_{11} H_1' \mathrm{e}^{-ikx}\, \mathrm{e}^{ik\left(x-ct\right)} \end{aligned} \tag{23}$$

$$\begin{aligned} \tau_{zx2} &= \left[e_{15}' H_2' \mathrm{e}^{-ikx} + \frac{ikcP_1}{c_{sh}'} \left(G_2 \mathrm{e}^{\left(-1+c/c_{sh}'\right)ikx} - H_2 \mathrm{e}^{\left(-1-c/c_{sh}'\right)ikx} \right) \right] \mathrm{e}^{ik\left(x-ct\right)} \\ D_{x2} &= -\epsilon_{11}' H_2' \mathrm{e}^{-ikx}\, \mathrm{e}^{ik\left(x-ct\right)} \end{aligned} \tag{24}$$

Where $P = \dfrac{\epsilon_{11} c_{44} + e^2_{15}}{\epsilon_{11}}$, $P_1 = \dfrac{\epsilon_{11}' c_{44}' + e_{15}'^2}{\epsilon_{11}'}$

Using the boundary conditions (9) in system of equations (17), (18), (23), and (24), provides the following linear algebraic equations with undetermined coefficient as G_1, G_2, G_1', G_2', H_1, H_2, H_1', and H_2'

$$\begin{aligned} & G_1 + H_1 - G_2 - H_2 = 0 \\ & G_1' - G_2' + \frac{e_{15}}{\epsilon_{11}} G_1 + \frac{e_{15}}{\epsilon_{11}} H_1 - \frac{e_{15}'}{\epsilon_{11}'} G_2 - \frac{e_{15}'}{\epsilon_{11}'} H_2 = 0 \\ & G_1 - H_1 + \frac{e_{15} c_{sh}}{ikcP} H_1' - \frac{e_{15} c_{sh}}{ikcP} H_2' - Q G_2 + Q H_2 = 0 \\ & -\epsilon_{11} H_1' + \epsilon_{11}' H_2' = 0 \\ & e^{i\alpha} G_1 + e^{-i\alpha} H_1 - e^{i\left(kh-\beta\right)} G_2 - e^{i\left(kh+\beta\right)} H_2 = 0 \\ & G_1' + h_1 H_1' + \frac{e_{15}}{\epsilon_{11}} e^{i\alpha} G_1 + \frac{e_{15}}{\epsilon_{11}} e^{-i\alpha} H_1 - e^{ikh} G_2' + h_2 e^{ikh} H_2' - \frac{e_{15}'}{\epsilon_{11}'} e^{i\left(kh-\beta\right)} G_2 - \frac{e_{15}'}{\epsilon_{11}'} e^{i\left(kh+\beta\right)} H_2 = 0 \\ & \frac{e_{15} c_{sh}}{ikcP} H_1' + e^{i\alpha} G_1 - e^{-i\alpha} H_1 - \frac{e_{15}' c_{sh}}{ikcP} e^{ikh} H_2' - Q e^{i\left(kh-\beta\right)} G_2 + Q e^{i\left(kh+\beta\right)} H_2 = 0 \\ & -\epsilon_{11} H_1' + \epsilon_{11}' e^{ikh} H_2' = 0 \end{aligned} \tag{25}$$

For solving the linear algebraic equations, we have introduced the following factors

$$\alpha = \frac{ckh_1}{c_{sh}}, \beta = \frac{ckh_2}{c_{sh}'}, Q = \frac{P_1 c_{sh}}{P c_{sh}'}, w = kc$$

For obtaining the solution of equations (25), the determinant of coefficients of 8x8 matrixes must be equal to zero. The determinant of system of equations (25) provides as (Christensen 1979 and Sun et al. 1968)

$$sin(\alpha)sin(\beta) + 2Qcos(hk) - 2Qcos(\alpha)cos(\beta) + Q^2 sin(\alpha)sin(\beta) = 0 \tag{26}$$

The above equation can be represented in much simpler form

$$\begin{aligned} &cos(hk) - cos\left(\frac{ckh_1}{c_{sh}}\right)cos\left(\frac{ckh_2}{c'_{sh}}\right) + \frac{\left(1+Q^2\right)}{2Q} sin\left(\frac{ckh_1}{c_{sh}}\right)sin\left(\frac{ckh_2}{c'_{sh}}\right) = 0 \\ &cos\left(\frac{wh}{c}\right) - cos\left(\frac{ckh_1}{c_{sh}}\right)cos\left(\frac{ckh_2}{c'_{sh}}\right) + \frac{(1+Q^2)}{2Q} sin\left(\frac{ckh_1}{c_{sh}}\right)sin\left(\frac{ckh_2}{c'_{sh}}\right) = 0 \end{aligned} \tag{27}$$

Where $h=h_1+h_2$ is total thickness of piezoelectric bonded layers. The equation (27) represents the dispersion relation for shear wave propagating in direction normal to layering.

4.2 Propagation along the direction of the layering

For Shear wave propagation along the direction of layering, substitution of equation (21) and (22) in equation (5) and (6) provides the stress and electrical displacement component as

$$\begin{aligned} \tau_{zx1} &= \left[ib_1P\left(-G_1e^{-ib_1x} + H_1e^{ib_1x}\right) + e_{15}k\left(-G'_1e^{-kx} + H'_1e^{kx}\right)\right] \\ &\quad e^{ik(y-ct)} \\ D_{x1} &= -\epsilon_{11}k\left(-G'_1e^{-kx} + H'_1e^{kx}\right)e^{ik(y-ct)} \end{aligned} \tag{28}$$

$$\begin{aligned} \tau_{zx2} &= \left[b_2P_1\left(-G_2e^{-b_2x} + H_2e^{b_2x}\right) + e'_{15}k\left(-G'_2e^{-kx} + H'_2e^{kx}\right)\right] \\ &\quad e^{ik(y-ct)} \\ D_{x2} &= -\epsilon'_{11}k\left(-G'_2e^{-kx} + H'_2e^{kx}\right)e^{ik(y-ct)} \end{aligned} \tag{29}$$

Using the boundary conditions (10) in system of equation (21), (22), (28) and (29), provides the following linear algebraic equation with undetermined coefficient as G_1, G_2, G'_1, G'_2, H_1, H_2, H'_1, and H'_2

$$\begin{aligned} &G_1 + H_1 - G_2 - H_2 = 0 \\ &\frac{e_{15}}{\epsilon_{11}}G_1 + \frac{e_{15}}{\epsilon_{11}}H_1 - G'_2 - H'_2 - \frac{e'_{15}}{\epsilon'_{11}}G_2 - \frac{e'_{15}}{\epsilon'_{11}}H_2 = 0 \\ &-G_1 + H_1 - \frac{e_{15}k}{ib_1P}G'_1 + \frac{e_{15}k}{ib_1P}H'_1 + QG_2 - QH_2 + \frac{e'_{15}k}{ib_1P}G'_2 - \frac{e'_{15}k}{ib_1P}H'_2 = 0 \\ &\epsilon_{11}G'_1 - \epsilon_{11}H'_1 - \epsilon'_{11}G'_2 + \epsilon'_{11}H'_2 = 0 \\ &e^{-ib_1h_1}G_1 + e^{ib_1h_1}H_1 - e^{b_2h_2}G_2 - e^{-b_2h_2}H_2 = 0 \\ &\frac{e_{15}}{\epsilon_{11}}e^{-ib_1h_1}G_1 + \frac{e_{15}}{\epsilon_{11}}e^{ib_1h_1}H_1 + e^{-kh_1}G'_1 + e^{kh_1}H'_1 - e^{-kh_2}G'_2 - e^{kh_2}H'_2 - \frac{e'_{15}}{\epsilon'_{11}}e^{-b_2h_2}G_2 - \frac{e'_{15}}{\epsilon'_{11}}e^{b_2h_2}H_2 = 0 \\ &-e^{-ib_1h_1}G_1 + e^{ib_1h_1}H_1 - \frac{e_{15}k}{ib_1P}e^{-kh_1}G'_1 + \frac{e_{15}k}{ib_1P}e^{kh_1}H'_1 + e^{-b_2h_2}QG_2 - e^{b_2h_2}QH_2 + \frac{e'_{15}k}{ib_1P}e^{-kh_2}G'_2 - \frac{e'_{15}k}{ib_1P}e^{kh_2}H'_2 = 0 \\ &\epsilon_{11}e^{-kh_1}G'_1 - \epsilon_{11}e^{kh_1}H'_1 - \epsilon'_{11}e^{-kh_2}G'_2 + \epsilon'_{11}e^{kh_2}H'_2 = 0 \end{aligned} \tag{30}$$

For obtaining the solution of equation (30) the determinant of coefficient matrix must be equated to zero, which provides the following dispersion relation (Qin et al. 2004)

$$
\begin{aligned}
&\left\{\left\{\left[e_{11} sinh\left(kh_1\right) cosh\left(kh_2\right) + e'_{11} sinh\left(kh_2\right) cosh\left(kh_1\right)\right]\left[r_1 c_{44} cos\left(b_1 h_1\right) sinh\left(b_2 h_2\right) + P1 r_2 sin\left(b_1 h_1\right) cosh\left(b_2 h_2\right)\right]\right\}\right. \\
&\left\{-\left[e_{11} sinh\left(kh_1\right) + e'_{11} sinh\left(kh_2\right)\right]\left[c_{44} r_1 sinh(b_2 h_2) + P_1 r_2 sin(b_1 h_1)\right]\right\}\Big\} \times 2e'^{2}_{15}\left(e_{11} / e'_{11}\right) \\
&-\left[e'^{4}_{15}\left(e_{11} / e'_{11}\right)^2 sinh\left(kh_1\right) sinh\left(kh_2\right) sinh\left(b_1 h_1\right) sinh\left(b_2 h_2\right)\right] \\
&-\left\{\left(P^2{}_1 r^2{}_2 - c^2{}_{44} r^2{}_1\right) sin\left(b_1 h_1\right) sinh\left(b_2 h_2\right) + 2c_{44} P_1 r_1 r_2\left[cos\left(b_1 h_1\right) cosh\left(b_2 h_2\right) - 1\right]\right\} \\
&\times\left\{\left(e^2{}_{11} + e'^{2}_{11}\right) sinh\left(kh_1\right) sinh\left(kh_2\right) + 2\epsilon_{11}\epsilon'_{III}\left[cosh\left(kh_1\right) cosh\left(kh_2\right) - 1\right]\right\} .. = 0
\end{aligned}
\qquad (31)
$$

$$
r_1 = \sqrt{\frac{c^2 - c^2{}_{sh}}{c^2{}_{sh}}},\ r_2 = \sqrt{\frac{c'^2_{sh} - c^2}{c'^2_{sh}}},\ Q = \frac{P_1 b_1}{P b_2}
$$

Equation (31) known as the dispersion equation for shear wave propagating along the direction of layering. The influence of layer thickness, volume fraction, existence of stop band and numerically analysis will be considered in preceding section.

5 NUMERICAL ANALYSIS AND DISCUSSION

The propagation characteristic of shear wave based on dispersion relation derived in equation (27) is investigated by numerically analysis carried out in this section for two different piezoelectric materials. The materials used in numerical calculation are PVDF and PZT-5H. Table 1 lists the properties of piezoelectric materials.

Properties → Materials ↓	Piezoelectric constant e_{15} (C/m^2)	Dielectric constant ϵ_{11} (E-10 F/m)	Elastic Constant c_{44}(E10 N/m^2)	Mass Density ρ(E3 kg/m^3)
PVDF	-0.16	1.062	0.91	1.78
PZT-5H	17	277	2.30	7.50

Table 1: Material Properties used in Numerical Calculation

Fig 2-7 are the dispersion curves showing the effect of variation in layer thickness on the circular frequency ω and wave number *k,* for propagation along the direction normal to layering. There exists a relationship between wave number and circular frequency which can be expressed as $k=\omega/c$, where c is propagation velocity. The value of c in numerical computation is taken as 1500 m/s. It can be observed from the figure 2 and 4 that the circular frequency decreases with increase in thickness of piezoelectric layers. The wave number found to be decreasing with increase in thickness of interface layer *h.* The variation of wave number with thickness is shown in figure 5, 6 and 7. The number of shear wave modes found to be increased, as the thickness of both piezoelectric layers become equal as evident from the figure 4 and 7.

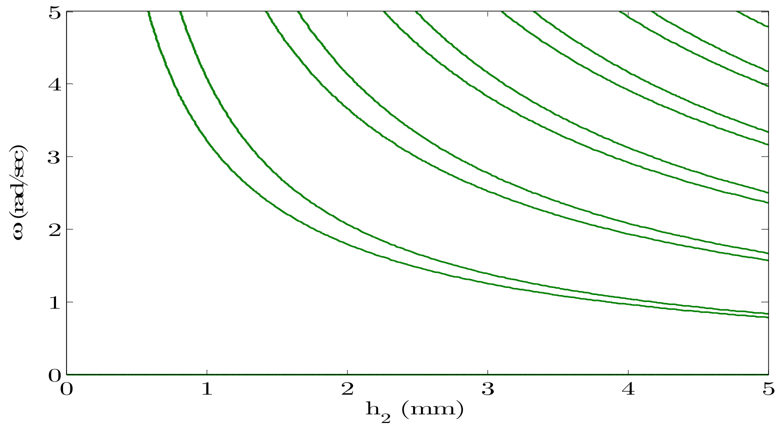

Figure 2: Circular frequency ω vs. h_2 for h_1=0.1mm.

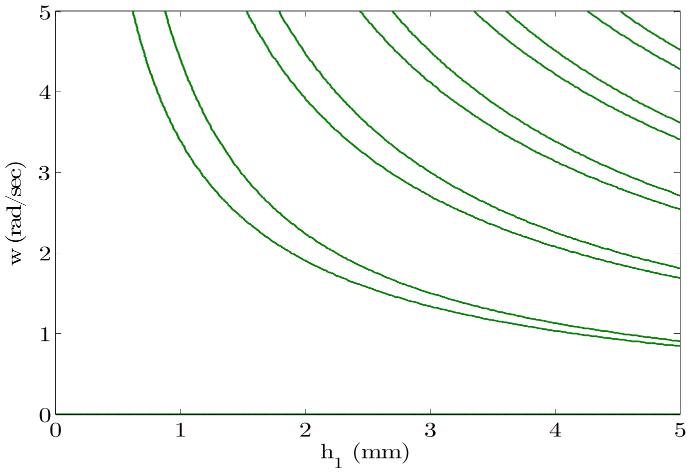

Figure 3: Circular frequency ω vs. h_1 for h_2=0.1mm.

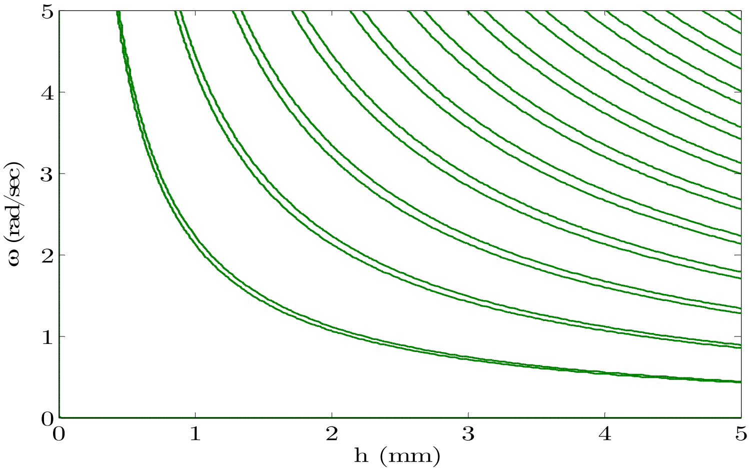

Figure 4: Circular frequency ω vs. total thickness h for h_1=h_2.

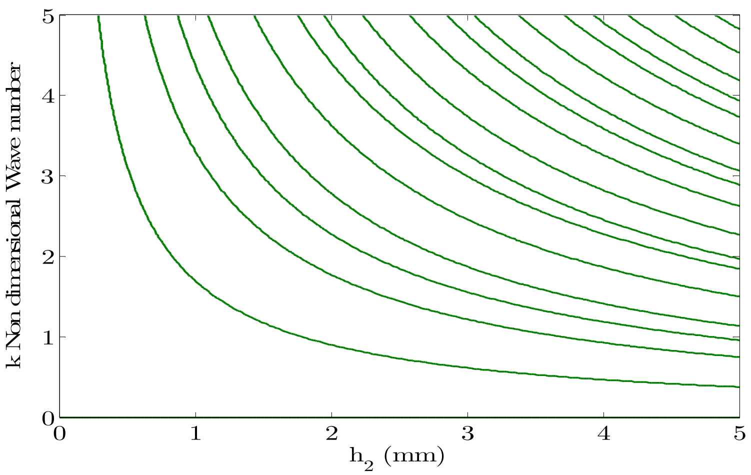

Figure 5: Wave number k vs. h_2 for h_1=0.1mm.

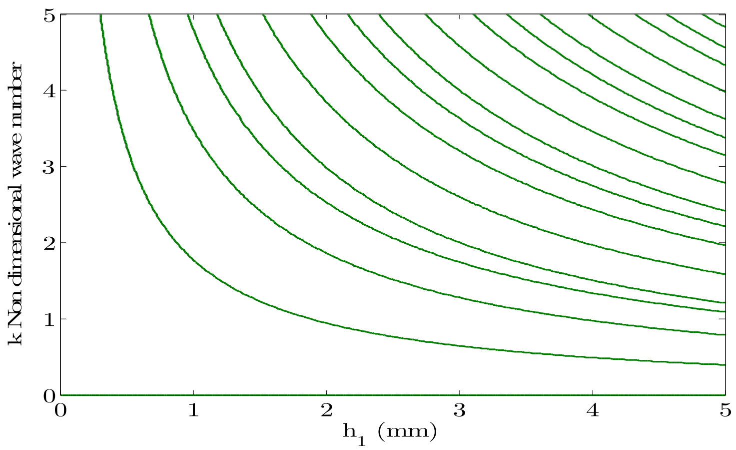

Figure 6: Wave number k vs. h_1 for h_2=0.1mm.

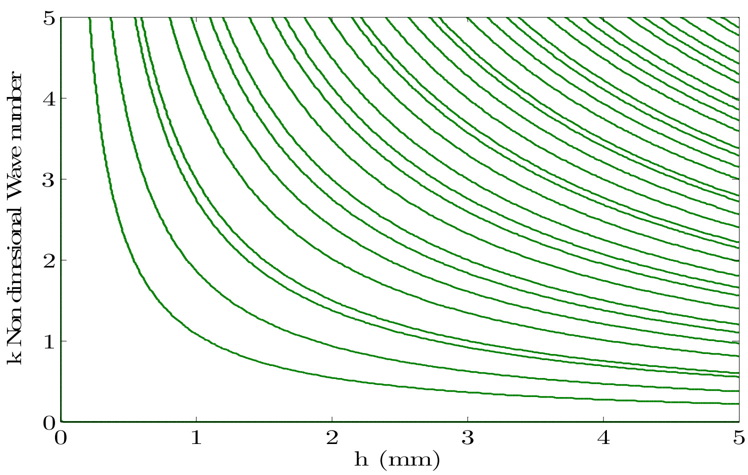

Figure 7: Wave number k vs. total thickness h for h_1=h_2.

To study the variation pattern of wave number kh_2 on $\omega h_2/c'_{sh}$, we assumed a new variable volume fraction, which is defined as $\gamma=h_1/(h_1+h_2)$, where h_1 and h_2 are thickness of PE-PE layers in composite structure. The curves are plotted for different value of γ ranging from 0.2 -0.8 as depicted in figure 8 and 9. For propagation along the direction normal to layering, it is observed from the curves that number of stop bands increases with increase in value of volume fraction. But as the volume fraction increases, the decrease in width of stop band is observed from the curves shown below.

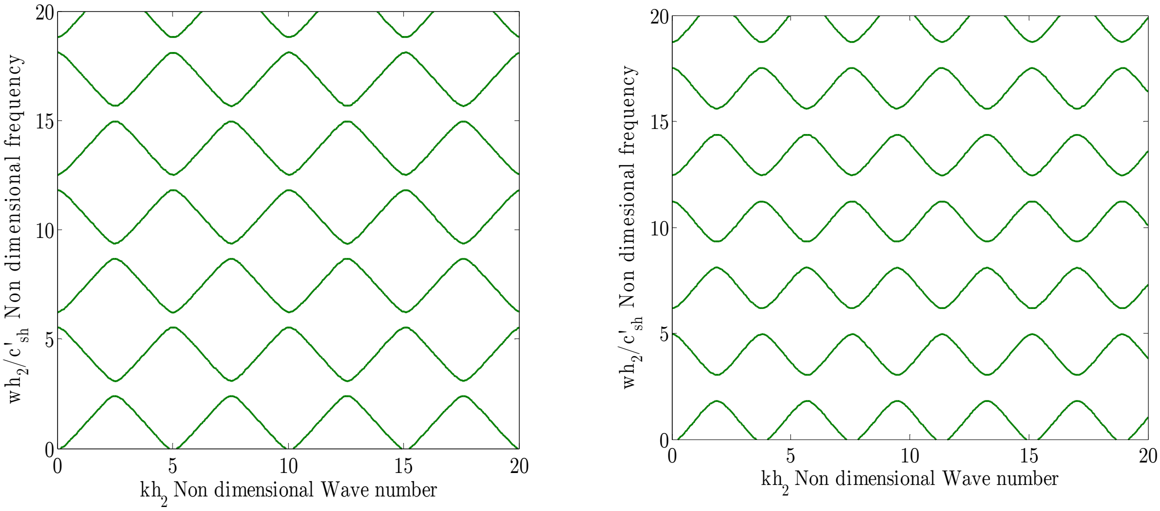

Figure 8: Stop band effect of propagation normal to layering for γ (a) 0.2 (b) 0.4.

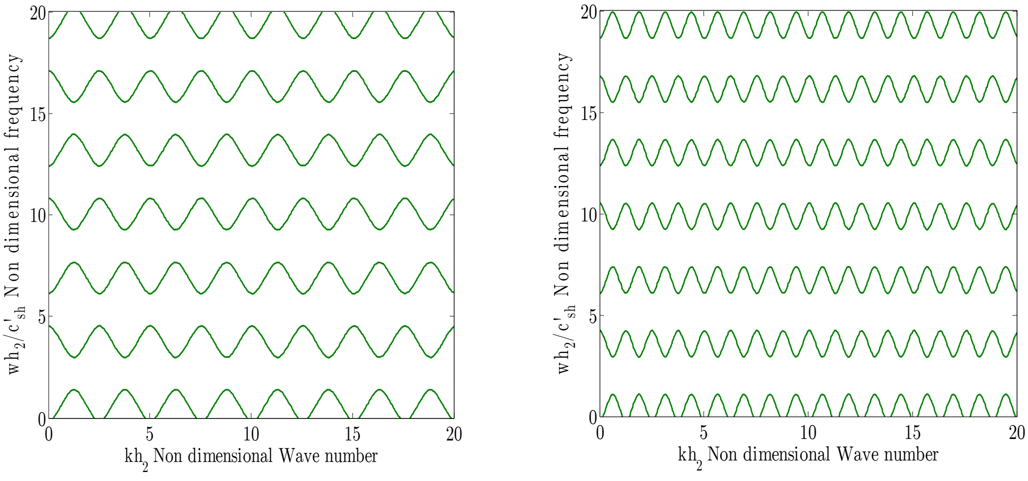

Figure 9: Stop band effect of propagation normal to layering for γ (a) 0.6 (b) 0.8.

For investigating the influence of volume fraction on phase velocity, figure 10 (a) and (b) plotted for two values of circular frequency 1000 Hz and 1500Hz with h_1 fixed at 1 mm. It is clearly observed from the curves, the phase velocity decrease gradually with increase in value of volume fraction γ. The curves are shown for wave propagation along the direction normal to layering.

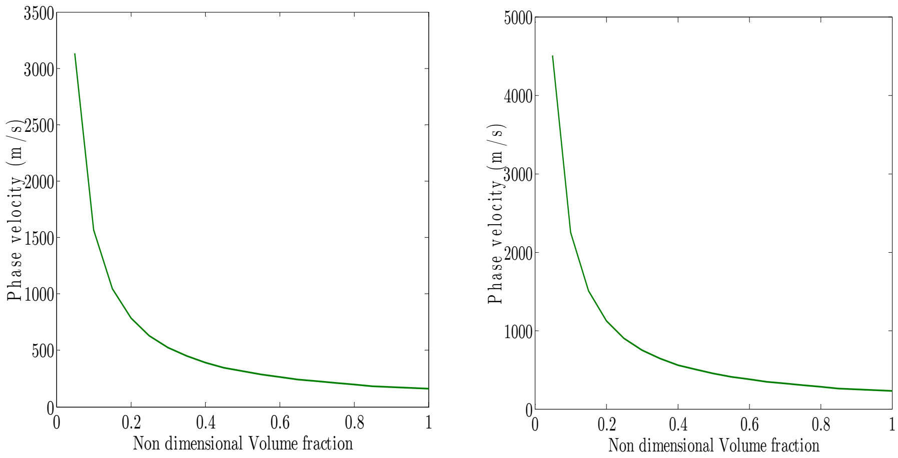

Figure 10: Phase velocity (c) vs. volume fraction (γ) for circular frequency (a) 1000 Hz (b) 1500 Hz.

6 CONCLUSIONS

In this paper, we have investigated Shear wave propagation in piezoelectric composite structure having two piezoelectric layers bonded together alternatively. The dispersion equations were obtained through analytical method for propagation along and in the direction of layering. The limitation of this investigation, we have assumed there was no initial stress present in either of the layers in piezoelectric composite structure. Future work can be done to study the propagation behavior in PE structures in the presence of initial stress. The numerical results obtained from this study, we draw following conclusions
(a) When Shear wave propagates in direction normal to layering, the stop band effect exists and number of stop band found to be increase with increase in volume fraction.
(b) For the case of wave propagation in direction of layering, no stop band exists.
(c) There is significant influence of volume fraction on the phase velocity. We found that, phase velocity decreases subsequently with increase in volume fraction. So we conclude that there exists a linear relationship between volume fraction and phase velocity.
The results found to be useful in developing a new class of SAW sensor based on PVDF –PZT composite with improved response and higher sensitivity. This can be achieved by obtaining the desired propagation of shear wave by selecting the appropriate material, thickness, elastic constants and other boundary conditions.

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