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SHIFT-OPERATOR FDTD METHOD FOR ANISOTROPIC PLASMA IN *kDB* COORDINATES SYSTEM

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Abstract—Electromagnetic (EM) problem model for anisotropic plasma in *kDB* coordinates system is set up. And the model includes almost all the respects of EM-problems for anisotropic plasma. Based on shift-operator finite difference time-domain (SO-FDTD) method, Maxwell equations and EM-field constitutive equations are solved and discrete difference scheme of each EM-field component is obtained. Then the propagation characteristics of eigen wave are expressed by the two components of electric displacement vector as well. Lastly, three typical examples are calculated by SO-FDTD method, and the results verify the effectiveness and exactness of SO-FDTD method in *kDB* coordinates system.

1. INTRODUCTION

When cold plasma is placed in bias magnetic field, the plasma becomes dielectric anisotropy, and the equivalent permittivity of it is a tensor [1–4]. Therefore, the solutions of plane electromagnetic (EM) wave in

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anisotropic plasma could not be obtained easily in xyz Cartesian coordinates system. However, there are only two components of electric placement vector D and magnetic induction B respectively in kDB coordinates system, which is much easier for solving the problem above. Some scholars have paid their attention to the interaction between EM-wave and anisotropic medium in kDB coordinates system [5–7]. For instance, Guo et al. discussed the relations between EM-wave propagation characteristics and medium parameters in kDB coordinates system [5]. Using the kDB system, Wei et al. calculated the reflection coefficient when the layered uniaxial anisotropic medium is illuminated by a plane EM-wave [6]. Zhan et al. studied the plane wave propagation in chiral plasma and chiral ferrite media in kDB coordinate system [7]. However, there are no numerical results in the references mentioned above, so we want to find an effective numerical method to solve the EM-problems in anisotropic medium.

Shift-operator FDTD (SO-FDTD) method is a kind of effective numerical method for dealing with EM-problems in dispersive and anisotropic mediums, such as magnetized plasma. In 2003, Ge et al. presented the concept of the SO-FDTD method and applied it to calculating EM scattering problems of un-magnetized plasma [8]. Yang et al. [9, 10] and Wang et al. [11] expanded the SO-FDTD method to calculate the reflection coefficient and transmission coefficient of magnetized plasma respectively. However, only a case of EM-wave propagation direction paralleling to bias magnetic is concerned in [9] and [11]. In [10], the propagation characteristic of EM-wave propagating through magnetized plasma is analyzed when wave vector is perpendicular to the bias magnetic field. Nevertheless, before discussion, the numerical results are obtained by using the complex permittivity of the extraordinary wave, which breaks up the integrity of anisotropic plasma EM-problem. Therefore, the way in [10] is unadvisable. In [12], piecewise linear recursive convolution FDTD is applied to the numerical analysis of magnetized plasma with arbitrary magnetic declination, whereas, the derivation of EM-field iterative schemes are very complicated, and the numerical results are not analyzed comprehensively.

In this paper, EM-problem model for anisotropic plasma in kDB coordinates system is set up. The model nearly covers all the respects of EM-problem for anisotropic plasma. Then, the SO-FDTD method is introduced into kDB coordinates system. By using this method, Maxwell equations and EM-field constitutive equations are solved numerically. Lastly, three kinds of EM-model are calculated by the SO-FDTD method. Excellent agreement between the numerical results and exact analytical solutions is demonstrated, which validates the availability of SO-FDTD method in kDB coordinates system.

2. CONSTITUTIVE RELATIONS FOR ANISOTROPIC PLASMA IN kDB COORDINATES SYSTEM

kDB coordinates system consists of wave vector k and DB-plane, in which three unit vector are e_1, e_2 and e_3, respectively. Let e_3 parallel to k, e_2 be on the intersection line between koz- and DB-planes, and e_1 is on the intersection line between xoy- and DB-planes. Unit vector e_1, e_2 and e_3 are perpendicular to each other and constitute a right-handed coordinate system, shown in Figure 1.

From Figure 1, we get the conversion matrix between xyz Cartesian coordinates system and kDB coordinates system [1, 2]

$$\hat{\mathbf{T}} = \begin{bmatrix} \sin\varphi & -\cos\varphi & 0 \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{bmatrix} \tag{1}$$

The inverse matrix is

$$\hat{\mathbf{T}}^{-1} = \begin{bmatrix} \sin\varphi & \cos\theta\cos\varphi & \sin\theta\cos\varphi \\ -\cos\varphi & \cos\theta\sin\varphi & \sin\theta\sin\varphi \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}. \tag{2}$$

The conversion matrix is acceptable for all the vector fields including E, D, B and H.

In constitutive equations, E and H can be expressed by D and B

$$\begin{bmatrix} E \\ H \end{bmatrix} = \begin{bmatrix} \hat{\kappa} & \hat{\chi} \\ \hat{\gamma} & \hat{\nu} \end{bmatrix} \cdot \begin{bmatrix} D \\ B \end{bmatrix} \tag{3}$$

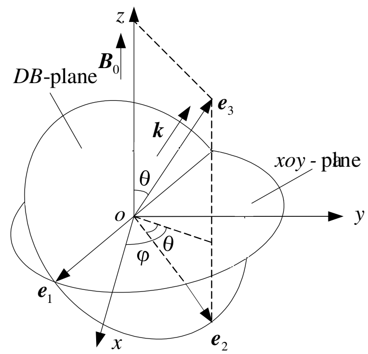


Figure 1. kDB coordinates system.

where $\hat{\kappa}$, $\hat{\chi}$, $\hat{\gamma}$ and $\hat{\nu}$ are constitutive parameter matrixes, which are defined as follows:

$$\hat{\kappa} = \begin{bmatrix} \kappa & -j\kappa_g & 0 \\ j\kappa_g & \kappa & 0 \\ \kappa_g & 0 & \kappa_z \end{bmatrix} \tag{4a}$$

$$\hat{\nu} = \nu\hat{\mathbf{I}} \tag{4b}$$

$$\hat{\chi} = \hat{\gamma} = 0 \tag{4c}$$

In Equation (4), $v = \frac{1}{\mu}$ is reluctivity. The components of $\hat{\kappa}$ are listed as follow [2]

$$\kappa = \frac{\varepsilon_{xx}}{\varepsilon_{xx}^2 + \varepsilon_{xy}^2} = \frac{1}{\varepsilon_0} \frac{\left(1 - \frac{jv_{en}}{\omega}\right)\left(1 - \frac{jv_{en}}{\omega} - \frac{\omega_p^2}{\omega^2}\right) - \frac{\omega_b^2}{\omega^2}}{\left(1 - \frac{jv_{en}}{\omega} - \frac{\omega_p^2}{\omega^2}\right)^2 - \frac{\omega_b^2}{\omega^2}} \tag{5a}$$

$$\kappa_g = \frac{j\varepsilon_{xy}}{\varepsilon_{xx}^2 + \varepsilon_{xy}^2} = \frac{1}{\varepsilon_0} \frac{\frac{\omega_p^2}{\omega^2}\frac{\omega_b}{\omega}}{\left(1 - \frac{jv_{en}}{\omega} - \frac{\omega_p^2}{\omega^2}\right)^2 - \frac{\omega_b^2}{\omega^2}} \tag{5b}$$

$$\kappa_z = \frac{1}{\varepsilon_z} = \frac{1}{\varepsilon_0} \frac{1 - \frac{jv_{en}}{\omega}}{1 - \frac{jv_{en}}{\omega} - \frac{\omega_p^2}{\omega^2}}. \tag{5c}$$

The constitutive relation in kDB coordinates system is

$$\begin{bmatrix} E_k \\ H_k \end{bmatrix} = \begin{bmatrix} \hat{\kappa}_k & \hat{\chi}_k \\ \hat{\gamma}_k & \hat{\nu}_k \end{bmatrix} \cdot \begin{bmatrix} D_k \\ B_k \end{bmatrix} \tag{6}$$

where

$$E_k = \hat{\mathbf{T}} \cdot E \tag{7a}$$

$$H_k = \hat{\mathbf{T}} \cdot H \tag{7b}$$

$$\hat{\kappa}_k = \hat{\mathbf{T}} \cdot \hat{\kappa} \cdot \hat{\mathbf{T}}^{-1} \tag{7c}$$

$$\hat{\chi}_k = \hat{\mathbf{T}} \cdot \hat{\chi} \cdot \hat{\mathbf{T}}^{-1} \tag{7d}$$

$$\hat{\gamma}_k = \hat{\mathbf{T}} \cdot \hat{\gamma} \cdot \hat{\mathbf{T}}^{-1} \tag{7e}$$

$$\hat{\nu}_k = \hat{\mathbf{T}} \cdot \hat{\nu} \cdot \hat{\mathbf{T}}^{-1}. \tag{7f}$$

To illustrate the problem clearly and simply, suppose bias magnetic field B_0 parallel to z axis and EM-wave propagation direction, which is wave vector k, declining an angle θ from z axis. Let DB-plane rotate an angle around e_3, and then e_1 is coincident with x axis. In kDB coordinates system, the constitutive parameter matrixes are

$$\hat{\kappa}_k = \begin{bmatrix} \kappa & -j\kappa_g \cos\theta & -j\kappa_g \sin\theta \\ j\kappa_g \cos\theta & \kappa\cos^2\theta + \kappa_2 \sin^2\theta & (\kappa - \kappa_z)\sin\theta\cos\theta \\ j\kappa_g \sin\theta & (\kappa - \kappa_z)\sin\theta\cos\theta & \kappa\sin^2\theta + \kappa_2\cos^2\theta \end{bmatrix} \quad (8a)$$

$$\hat{\nu}_k = \nu\hat{\mathbf{I}} \quad (8b)$$

$$\hat{\chi}_k = \hat{\gamma}_k = 0 \quad (8c)$$

From constitutive relations (6) and constitutive parameters (8) and (5), we get the components of E_k, H_k expressed by D_k, B_k

$$E_1 = \kappa D_1 - j\kappa_g \cos\theta D_2 \quad (9a)$$

$$E_2 = j\kappa_g \cos\theta D_1 + \left(\kappa \cos^2\theta + \kappa_z \sin^2\theta\right) D_2 \quad (9b)$$

$$E_3 = j\kappa_g \sin\theta D_1 + (\kappa - \kappa_z)\sin\theta\cos\theta D_2 \quad (9c)$$

$$H_1 = \frac{1}{\mu} B_1 \quad (9d)$$

$$H_2 = \frac{1}{\mu} B_2. \quad (9e)$$

As for k parallel to B_0, the constitutive relations between E and D can be written as

$$E_1 = \kappa D_1 - j\kappa_g D_2 \quad (10a)$$

$$E_2 = j\kappa_g D_1 + \kappa D_2 \quad (10b)$$

$$E_3 = 0. \quad (10c)$$

As for k perpendicular to B_0, the constitutive relations between E and D can be denoted as

$$E_1 = \kappa D_1 \quad (11a)$$

$$E_2 = \kappa_z D_2 \quad (11b)$$

$$E_3 = j\kappa_g D_1. \quad (11c)$$

3. SO-FDTD METHOD IN *kDB* COORDINATES SYSTEM

Maxwell equations in kDB coordinates system are

$$\nabla \times H_k = \frac{\partial D_k}{\partial t} \quad (12a)$$

$$\nabla \times E_k = -\frac{\partial B_k}{\partial t}. \quad (12b)$$

Take a component for example, the discrete difference schemes of (12) in one dimensional are

$$D_1^{n+1} = D_1^n - \Delta t \frac{H_2^{n+1/2}(k+1/2) - H_2^{n+1/2}(k-1/2)}{\Delta l} \quad (13a)$$

$$H_1^{n+1/2} = H_1^{n-1/2} + \frac{\Delta t}{\mu} \frac{E_2^n(k+1) - E_2^n(k)}{\Delta l}. \quad (13b)$$

where Δl is the space step size.

Substitute constitutive parameters (5) into (10), for application of the SO-FDTD method. The coefficients in two sides of the equation are written as rational fractional function

$$\sum_{n=0}^{N} h_n (j\omega)^n E_1 = \varepsilon_0 \sum_{n=0}^{N} g_n (j\omega)^n D_1 - \varepsilon_0 \sum_{n=0}^{N} r_n (j\omega)^n D_2 \quad (14a)$$

$$\sum_{n=0}^{N} h_n (j\omega)^n E_2 = \varepsilon_0 \sum_{n=0}^{N} r_n (j\omega)^n D_1 + \varepsilon_0 \sum_{n=0}^{N} g_n (j\omega)^n D_2 \quad (14b)$$

where N is 4, and h_n, g_n and r_n $(n = 0, 1, \ldots, N)$ are

$$\begin{cases} h_0 = \omega_p^4,\ h_1 = 2\omega_p^2 v_{en},\ h_2 = 2\omega_p^2 + \omega_b^2 + v_{en}^2,\ h_3 = 2v_{en},\ h_4 = 1; \\ g_0 = 0,\ g_1 = \omega_p^2 v_{en},\ g_2 = \omega_p^2 + \omega_b^2 + v_{en}^2,\ g_3 = 2v_{en},\ g_4 = 1; \\ r_0 = r_2 = r_3 = r_4 = 0,\ r_1 = \omega_b \omega_p^2. \end{cases} \quad (15)$$

Transform Equation (14) to time domain and introduce shift operator into the formula by using the SO-FDTD method, and then the discrete difference schemes of E_1 and E_2 are arranged as

$$E_1^{n+1} = \frac{1}{a_0} \left[\frac{1}{\varepsilon_0} \left(\sum_{i=0}^{4} b_i D_1^{n+1-i} - \sum_{i=0}^{4} c_i D_2^{n+1-i} \right) - \sum_{i=1}^{4} a_i E_1^{n+1-i} \right] \quad (16a)$$

$$E_2^{n+1} = \frac{1}{a_0} \left[\frac{1}{\varepsilon_0} \left(\sum_{i=0}^{4} c_i D_1^{n+1-i} + \sum_{i=0}^{4} b_i D_2^{n+1-i} \right) - \sum_{i=1}^{4} a_i E_2^{n+1-i} \right] \quad (16b)$$

where

$$\begin{cases} \delta = \left(\frac{2}{\Delta t}\right),\ a_0 = \sum\limits_{i=0}^{4} h_i \cdot \delta^i, \\ a_1 = 4h_0 + 2h_1\delta - 2h_3\delta^3 - 4h_4\delta^4,\ a_2 = 6h_0 - 2h_2\delta^2 + 6h_4\delta^4, \\ a_3 = 4h_0 - 2h_1\delta + 2h_3\delta^3 - 4h_4\delta^4,\ a_4 = \sum\limits_{i=0}^{4} (-1)^i \cdot h_i \cdot \delta^i; \\ b_0 = \sum\limits_{i=1}^{4} g_i \cdot \delta^i,\ b_1 = +2g_1\delta - 2g_3\delta^3 - 4g_4\delta^4,\ b_2 = -2g_2\delta^2 + 6g_4\delta^4, \\ b_3 = -2g_1\delta + 2g_3\delta^3 - 4g_4\delta^4,\ b_4 = \sum\limits_{i=1}^{4} (-1)^i \cdot g_i \cdot \delta^i; \\ c_0 = r_1\delta,\ c_1 = 2r_1\delta,\ c_2 = 0,\ c_3 = -2r_1\delta,\ c_4 = -r_1\delta. \end{cases} \quad (17)$$

Substitute constitutive parameters (5) into (11), and the coefficients in two sides of the equation are written as rational fractional function

$$\sum_{n=0}^{N} h_n(j\omega)^n E_1 = \varepsilon_0 \sum_{n=0}^{N} g_n(j\omega)^n D_1 \tag{18a}$$

$$\sum_{m=0}^{M} q_m(j\omega)^m E_2 = \varepsilon_0 \sum_{m=0}^{M} p_m(j\omega)^m D_2 \tag{18b}$$

$$\sum_{n=0}^{N} h_n(j\omega)^n E_3 = \varepsilon_0 \sum_{n=0}^{N} r_n(j\omega)^n D_1 \tag{18c}$$

where, $N = 4$ and $M = 2$; h_n, g_n and r_n are the same as in (15); q_m and p_m $(m = 0, 1, \ldots, M)$ are as follow

$$\begin{cases} q_0 = \omega_p^2, & q_1 = v_{en}, \quad q_2 = 1; \\ p_0 = 0, & p_1 = v_{en}, \quad p_2 = 1 \end{cases} \tag{19}$$

Similarly, the discrete difference schemes of E_1, E_2 and E_3 are

$$E_1^{n+1} = \frac{1}{a_0}\left(\frac{1}{\varepsilon_0}\sum_{i=0}^{4} b_i D_1^{n+1-i} - \sum_{i=1}^{4} a_i E_1^{n+1-i}\right) \tag{20a}$$

$$E_2^{n+1} = \frac{1}{d_0}\left(\frac{1}{\varepsilon_0}\sum_{i=0}^{2} f_i D_2^{n+1-i} - \sum_{i=1}^{2} d_i E_2^{n+1-i}\right) \tag{20b}$$

$$E_3^{n+1} = \frac{1}{a_0}\left[\frac{1}{\varepsilon_0}\left(\sum_{i=0}^{4} c_i D_1^{n+1-i} - \sum_{i=1}^{4} a_i E_3^{n+1-i}\right)\right] \tag{20c}$$

where a_i, b_i and c_i $(i = 0, 1, \ldots, N)$ are the same as in (17), and d_j and f_j $(j = 0, 1, \ldots, M)$ are as follow

$$\begin{cases} d_0 = q_0 + q_1\delta + q_2\delta^2, & d_1 = 2q_0 - 2q_2\delta^2, \quad q_2 = q_0 - q_1\delta + q_2\delta^2; \\ f_0 = p_1\delta + p_2\delta^2, & f_1 = -2p_2\delta^2, \quad f_2 = -p_1\delta + p_2\delta^2. \end{cases} \tag{21}$$

Substitute constitutive parameters (5) into (8), and the coefficients in two sides of the equation are written as rational fractional function

$$\sum_{n=0}^{N} h_n(j\omega)^n E_1 = \varepsilon_0 \sum_{n=0}^{N} g_n(j\omega)^n D_1 - \varepsilon_0 \sum_{n=0}^{N} r_n(j\omega)^n \cdot \cos\theta \cdot D_2 \tag{22a}$$

$$\sum_{l=0}^{L} v_l(j\omega)^l E_2 = \varepsilon_0 \sum_{l=0}^{L} w_l(j\omega)^l \cdot \cos\theta \cdot D_1 + \varepsilon_0 \sum_{l=0}^{L} y_l(j\omega)^l D_2 \tag{22b}$$

$$\sum_{l=0}^{L} v_l(j\omega)^l E_2 = \varepsilon_0 \sum_{l=0}^{L} w_l(j\omega)^l \cdot \sin\theta \cdot D_1 + \varepsilon_0 \sum_{l=0}^{L} z_l(j\omega)^l D_2 \tag{22c}$$

where, $N = 4$ and $L = 6$, and h_n, g_n and r_n are the same as in (15). v_l, p_l, y_l and z_l are as follows

$$\begin{cases} v_0 = \omega_p^6, v_1 = 3\omega_p^4 v_{en}, v_2 = 3\omega_p^4 + 3\omega_p^4 v_{en}^2 + v_{en}^2\omega_b^2, v_3 = v_{en}^3 + 6\omega_p^2 v_{en} + \omega_b^2 v_{en}, \\ v_4 = 3\omega_p^2 + 2v_{en}^2 + \omega_b^2, v_5 = 3v_{en}, v_6 = 1; \\ w_0 = 0, w_1 = \omega_p^4\omega_b, w_2 = \omega_p^2\omega_b v_{en}, w_3 = \omega_p^2\omega_b, w_4 = w_5 = w_6 = 0; \\ y_0 = 0, y_1 = \omega_p^4 v_{en}, y_2 = \omega_p^4 + 2\omega_p^4 v_{en}^2 + v_{en}^2\omega_b^2 \cdot \cos^2\theta, y_3 = v_{en}^3 + 4\omega_p^2 v_{en} + \omega_b^2 v_{en}, \\ y_4 = 2\omega_p^2 + 2v_{en}^2 + \omega_b^2, y_5 = 3v_{en}, y_6 = 1; \\ z_0 = z_1 = 0, z_2 = \omega_p^2\omega_b^2 \cdot \sin\theta \cdot \cos\theta, z_3 = z_4 = z_5 = z_6 = 0. \end{cases} \tag{23}$$

Similarly, the discrete difference schemes of E_1, E_2 and E_3 are

$$E_1^{n+1} = \frac{1}{a_0}\left[\frac{1}{\varepsilon_0}\left(\sum_{i=0}^{4} b_i D_1^{n+1-i} - \cos\theta \cdot \sum_{i=0}^{4} c_i D_2^{n+1-i}\right) - \sum_{i=1}^{4} a_i E_1^{n+1-i}\right] \tag{24a}$$

$$E_2^{n+1} = \frac{1}{A_0}\left[\frac{1}{\varepsilon_0}\left(\cos\theta \cdot \sum_{i=0}^{6} C_i D_1^{n+1-i} + \sum_{i=0}^{6} E_i D_2^{n+1-i}\right) - \sum_{i=1}^{6} A_i E_2^{n+1-i}\right] \tag{24b}$$

$$E_3^{n+1} = \frac{1}{A_0}\left[\frac{1}{\varepsilon_0}\left(\sin\theta \cdot \sum_{i=0}^{6} C_i D_1^{n+1-i} + \sum_{i=0}^{6} F_i D_2^{n+1-i}\right) - \sum_{i=1}^{6} A_i E_3^{n+1-i}\right] \tag{24c}$$

where a_i, b_i and c_i $(i = 0, 1, \ldots, N)$ are the same as in (17), and A_k, C_k, E_k and F_k $(k = 0, 1, \ldots, L)$ are as follows

$$\begin{cases} A_0 = \sum_{i=0}^{6} v_i \cdot \delta^i, A_1 = 6v_0 + 4v_1\delta + 2v_2\delta^2 - 2v_4\delta^4 - 4v_5\delta^5 - 6v_6\delta^6, \\ A_2 = 15v_0 + 5v_1\delta - v_2\delta^2 - 3v_3\delta^3 - v_4\delta^4 + 5v_5\delta^5 6 + 15v_6\delta^6, \\ A_3 = 20v_0 - 4h_2\delta^2 + 4h_4\delta^4 - 20v_6\delta^6, \\ A_4 = 15v_0 - 5v_1\delta - v_2\delta^2 + 3v_3\delta^3 - v_4\delta^4 - 5v_5\delta^5 6 + 15v_6\delta^6, \\ A_5 = 6v_0 - 4v_1\delta + 2v_2\delta^2 - 2v_4\delta^4 + 4v_5\delta^5 - 6v_6\delta^6, A_6 = \sum_{i=0}^{6} (-1)^i \cdot v_i \cdot \delta^i; \\ C_0 = \sum_{i=1}^{6} w_i \cdot \delta^i, C_1 = 4w_1\delta + 2w_2\delta^2 - 2w_4\delta^4 - 4w_5\delta^5 - 6w_6\delta^6, \\ C_2 = 5w_1\delta - w_2\delta^2 - 3w_3\delta^3 - w_4\delta^4 + 5w_5\delta^5 6 + 15w_6\delta^6, C_3 = -4w_2\delta^2 + 4w_4\delta^4 - 20w_6\delta^6, \\ C_4 = -5w_1\delta - w_2\delta^2 + 3w_3\delta^3 - w_4\delta^4 - 5w_5\delta^5 6 + 15w_6\delta^6, \\ C_5 = -4w_1\delta + 2w_2\delta^2 - 2w_4\delta^4 + 4w_5\delta^5 - 6w_6\delta^6, C_6 = \sum_{i=1}^{6} (-1)^i \cdot w_i \cdot \delta^i; \\ E_0 = \sum_{i=1}^{3} y_i \cdot \delta^i, E_1 = 4y_1\delta + 2y_2\delta^2, E_2 = 5y_1\delta - y_2\delta^2 - 3y_3\delta^3, E_3 = -4y_2\delta^2, \\ E_4 = -5y_1\delta - y_2\delta^2 + 3y_3\delta^3, E_5 = -4y_1\delta + 2y_2\delta^2, E_6 = \sum_{i=1}^{3} (-1)^i \cdot y_i \cdot \delta^i; \\ F_0 = z_2\delta^2, F_1 = 2z_2\delta^2, F_2 = -z_2\delta^2, F_3 = -4z_2\delta^2, F_4 = -z_2\delta^2, F_5 = 2z_2\delta^2, F_6 = z_2\delta^2. \end{cases} \tag{25}$$

From formulae mentioned above, there comes a conclusion of the computation processes by the SO-FDTD:

(a) From formula (13a), H is yielded from E;

(b) From formulae (13b), D is yielded from H;
(c) From formula (16) or (20) or (24), E is yielded from D;
(d) According to the sequences of (a), (b) and (c), a recursion is completed, and next recursion begins.

4. NUMERICAL EXAMPLES AND DISCUSSION

4.1. Example 1

When EM-waves propagates through uniform magnetized plasma plate with wave vector k paralleling to bias magnetic field B_0, the reflection and transmission coefficients of which are calculated.

It is known that the eigen wave in this case becomes two kinds of circularly polarized wave [1–3], and there are only two components for E and D along direction e_1 and e_2. Using the above SO-FDTD method, only four formulae (13a), (13b), (16a) and (16b) work. The FDTD problem space consists of 450 cells, the magnetized plasma occupying cells 200–320. Each cell is 75 μm long, so the plasma layer is 9.0 mm thick. Each end of the cell space is set up with Mur's absorbing boundary condition [13]. For these simulations, the plasma parameters are: $\omega_p = 28.7\times 2\pi\times 10^9$ rad/s, $\omega_b = 8.8\times 10^{10}$ rad/s, $v_{en} = 2\times 10^{10}$. To satisfy courant stability condition, the time step Δt is set as 0.125 ps. The incidence excite source is a Gaussian-derivative pulsed plane wave: $E_{inc} = (t-t_0)/\tau\times\exp[-4\times\pi(t-t_0)^2/\tau^2]$, where $t_0 = 70\Delta t$, $\tau = 140\Delta t$.

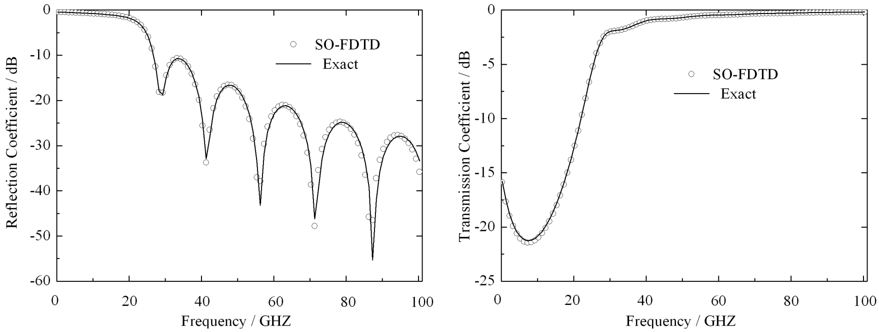
The electric displacement vector data are recorded at cell 199 and cell 321, then are transformed to frequency domain through discrete Fourier transform. From the FDTD data, the reflection and transmission coefficients of left-handed circularly polarized wave (LCP) and right-handed circularly wave (RCP) are achieved by

$$R_{LCP,RCP} = D_{1r}(\omega) \mp jD_{2r}(\omega) \tag{26a}$$
$$T_{LCP,RCP} = D_{1t}(\omega) \mp jD_{2t}(\omega) \tag{26b}$$

where the subscripts r and t correspond to the reflected wave and the transmitted one, respectively.

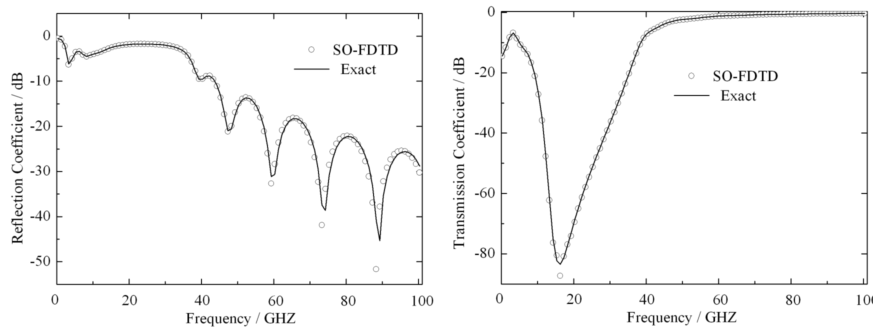
Figures 2 and 3 show the comparisons of the reflection coefficient and transmission coefficient vs. frequency obtained from the SO-FDTD method and analytical method [14] for LCP wave and RCP wave, respectively.

From Figures 2 and 3, it is shown that the SO-FDTD method in kDB coordinates system is quite accurate and coincident with the analytical method very well. So it is successful to use the SO-FDTD method to calculate the EM-problem in anisotropic plasma.



(a) (b)

Figure 2. LCP wave reflection and transmission coefficients in anisotropic plasma. (a) Reflection coefficient. (b) Transmission coefficient.



(a) (b)

Figure 3. RCP wave reflection and transmission coefficients in anisotropic plasma. (a) Reflection coefficient. (b) Transmission coefficient.

It also can be seen that the propagation characteristic of LCP and RCP waves is of great difference. It can be explained that the working principle of them are different. For LCP wave, there only exists a cut-off frequency [15] (in this example it is 22.54 GHz). As EM-wave frequency is below this frequency, the reflection coefficient is big, and the transmission coefficient is small. For RCP wave, there are two pass-bands separated by a stop-band [12, 15, 16]. The stop-band is from 14.01 GHz to 36.55 GHz. However, the propagation performance of RCP wave in the left pass-band is much worse than that in right pass-band. Trough the whole stop-band, especially near the resonant frequency, the reflection coefficient is above $-3\,\mathrm{dB}$, and the transmission coefficient is bellow $-30\,\mathrm{dB}$, which is mainly caused by plasma resonant absorbing. This phenomenon only occurs in RCP wave because the electric circumnutation in magnetized plasma is right-hand rotating towards bias magnetic field B_0 [3].

4.2. Example 2

When EM-waves propagates through uniform magnetized plasma plate with wave vector k perpendicular to bias magnetic field B_0, the reflection and transmission coefficients of which are calculated.

It is known that there are two kinds of eigen waves in this case. One is ordinary wave (O-wave), and the other is extraordinary wave (X-wave) [1–4]. Both of them are linearly polarized waves. Using the SO-FDTD method above, formulae (13a), (13b), (20a), (20b) and (20c) are selected to implement the different iterations.

Calculation parameters and plasma parameters are the same as those in Example 1. Because these two linearly polarized waves are not coupling with each other, the reflection and transmission coefficients of X-wave and O-wave are obtained by D_1 and D_2 respectively. Figure 4 and Figure 5 show the comparisons of the reflection coefficient and transmission coefficient vs. frequency obtained from the SO-FDTD method and the analytical method for X-wave and O-wave, respectively.

It can be seen from Figure 4 that the propagation characteristic of O-wave in anisotropic plasma is similar to that in un-magnetized plasma [8]. However, the propagation characteristic of X-wave is much

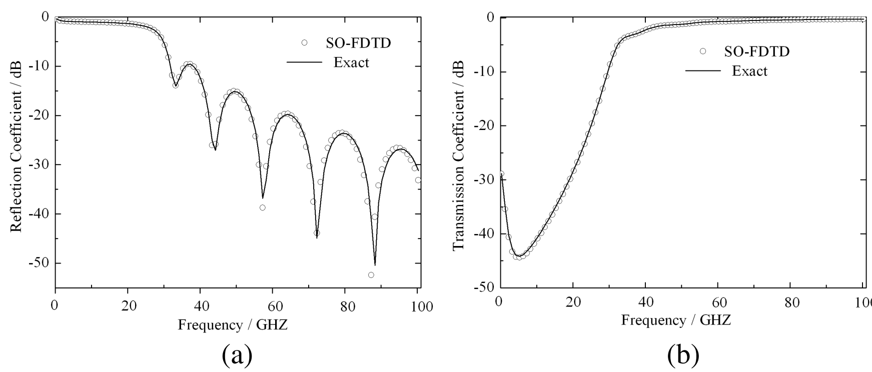


Figure 4. O-wave reflection and transmission coefficients in anisotropic plasma. (a) Reflection coefficient. (b) Transmission coefficient.

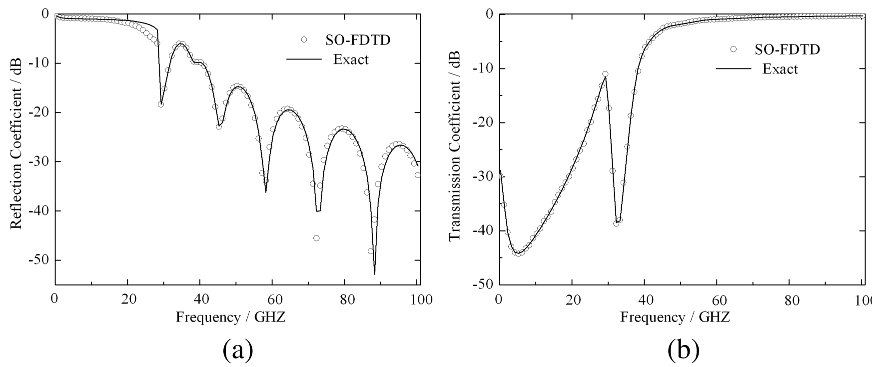


Figure 5. X-wave reflection and transmission coefficients in anisotropic plasma. (a) Reflection coefficient. (b) Transmission coefficient.

different from O-wave, as seen from Figure 5. There are two pass-bands and one resonant frequency for X-wave in anisotropic plasma [15]. One pass-band is from 22.54 GHz to 31.94 GHz, and the other is from 31.94 GHz to 36.55 GHz. It is also can be seen that there is a sudden change at the point of plasma frequency. It can be explained that the plasma frequency is an inherently turning point in the pass-band.

4.3. Example 3

When EM-waves propagates through uniform magnetized plasma plate with an angle 45° between wave vector k and bias magnetic field B_0, the reflection and transmission coefficients of which are calculated.

It is known that there are two kinds of eigen wave in this case, which are both elliptically polarized wave. One is called first type wave (I-wave), and the other is second type wave (II-wave) [1–3]. Using the above SO-FDTD method, formulae (13a), (13b), (24a), (24b) and (24c) are applied to implement the different iterations. The two components of electric displacement vector satisfy such a relation as follows [1, 2]:

$$\frac{D_2}{D_1} = \frac{2j\kappa_g \cos\theta}{(\kappa - \kappa_z)\sin^2\theta \pm \sqrt{(\kappa - \kappa_z)^2 \sin^4\theta + 4(\kappa_g \cos\theta)^2}} \tag{27}$$

where, "+" denotes I-wave, and "−" denotes II-wave. When θ is 0°, it becomes the case as Example 1. When θ is 90°, k is perpendicular to B_0, and it is the case as Example 2.

The simulation parameters related to plasma medium are selected as: $\omega_p = 50 \times 2\pi \times 10^9$ rad/s, $v_{en} = 2 \times 10^{10}$ Hz, $\omega_b = 3 \times 10^{11}$ Hz.

The other simulation parameters are the same as those in Example 1. Figure 6 and Figure 7 are the comparisons of the reflection coefficient and transmission coefficient vs. frequency obtained from the SO-FDTD method and the analytical method for I-wave and II-wave, respectively.

From (27), it is known that when $\kappa > \kappa_z$, I-wave is left-handed elliptically polarized wave, and II-wave is right-handed elliptically polarized wave. So I-wave possesses the property similar to LCP wave and II-wave alike that of RCP wave. These can be seen from Figure 5 and Figure 6. However, there is a sudden change in the point of plasma frequency. It can be explained that the plasma frequency is inherently one of the reflection points [12]. It also can be seen that the SO-FDTD

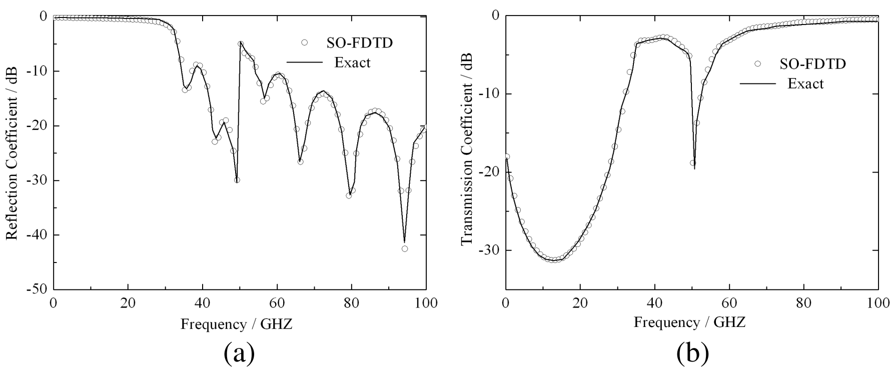


Figure 6. I-wave reflection and transmission coefficients in anisotropic plasma. (a) Reflection coefficient. (b) Transmission coefficient.

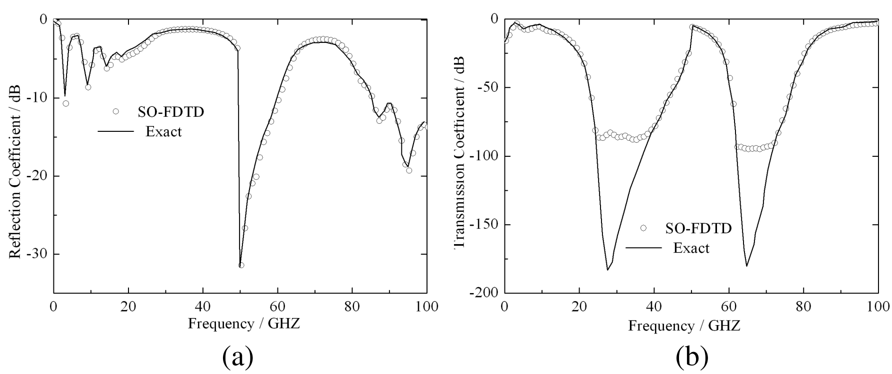


Figure 7. II-wave reflection and transmission coefficients in anisotropic plasma. (a) Reflection coefficient. (b) Transmission coefficient.

method coincides with the analytical well except some points, such as in Figure 6(b). The reason is that there exists the stop-band in the frequency spectrum.

5. CONCLUSION

In this paper, EM-problem model for anisotropic plasma in *kDB* coordinates system is set up. The model nearly covers all the respects of EM-problem for anisotropic plasma. Then, the SO-FDTD method is introduced. Its application is expanded, and the discrete different schemes of each EM-field component are deduced in *kDB* coordinates system. By three typical one-dimensional examples, excellent agreement between the numerical results and analytical solutions is demonstrated, which validates the availability of the SO-FDTD method in *kDB* coordinates system. Furthermore, an effective method dealing with EM-problem of anisotropic plasma is presented too. Though only one-dimensional examples are discussed in the paper, the SO-FDTD method in *kDB* coordinates system can also be applied to calculating EM-wave scattering for anisotropic plasma in two- or three-dimension.

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