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Ship selection using a multiple-criteria synthesis approach

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Abstract A method is presented for selecting a preferred ship from a group of candidates as a reference ship for a new design. The method is based on a recently developed approach for multiple-criteria decision analysis under uncertainty, the evidential reasoning approach. Using this method, both quantitative and qualitative attributes of a complicated nature can be considered in the selection process. The method consists of three phases: identifying suitable candidate ships, evaluating them in terms of both conventional techno-economical and qualitative attributes, and aggregating all the attributes using the evidential reasoning approach. This three-phase procedure is illustrated by means of an oil tanker selection example. The results of this study show that the evidential reasoning approach can support multiple-criteria ship selection processes when both qualitative and quantitative information with or without uncertainties have to be taken into account. The outcomes generated by the method include the ranking of the candidate ships and indications of their strengths and weaknesses in the format of performance distributions over different assessment grades. Such information is vital in helping decision makers to make an informed

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Key words Reference ship selection · Shipping · Evidential reasoning · Multiple-criteria decision analysis

1 Introduction

Introducing new tonnage into shipping services is a frequent event in the maritime world. However, determining the best ship for an intended trade is a challenge facing many investors. There are many reasons for the existence of this challenge. For example, a modern ship is a very complex facility combining various types of equipment, technologies, and disciplines. The capital invested in a new merchant ship is huge and the life cycle of a ship could be 20 years or even more; the shipping market and environment keep changing and there are always many uncertain and stochastic factors involved. An easy, quick, and effective way to address this question for investors might be simply to compare their desired vessel with an existing ship or a reference ship.

This is also a question that designers frequently face at the preliminary stage of designing a new ship. A conventional and pragmatic method to start a new design is to find a suitable existing ship of a similar type as a reference for guiding and evaluating new designs; Tupper¹ has indicated that the starting point is usually a type ship. Designers are more likely to employ a reference ship than investors are. This is because designers need to be technically more precise in describing a new design and also they may have additional ship specifications at hand.

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A subsequent problem arises as to which existing ship would be the best reference for a new design with all relevant factors taken into account. If there are only a few candidates for selection based on a very limited number of attributes, it might not be difficult for an experienced designer to pick out a suitable one with confidence. However, with more and more new ships delivered each year and because of the progress of diverse modern technologies, there currently are many options, and each one may have its own advantages and individual features. Some candidates might be very successful, while others may not be appropriate for various reasons. It could be thought that the more candidates that are considered, the better the decision should be. On the other hand, it becomes increasingly difficult for human experts to select the best one intuitively from a large group of candidates. Therefore, mathematical approaches need to be used to handle this difficult ship selection problem systematically.

Some mathematical models, such as those used by Ji,² Kakamoukas,³ and Yang et al.,⁴ can rank a series or even a large number of ships in terms of techno-economic criteria of a quantitative nature, although problems might exist when multiple criteria need to be considered at the same time and contradictions exist among them. It is difficult to include any subjective expert knowledge that cannot be expressed numerically in a quantitative model. Non-numerical expert knowledge is widely applied in the evaluation of ship quality and has been proved to be of great help in many circumstances and very valuable in practice. It is highly desirable to include both quantitative and qualitative attributes in a comprehensive evaluation of ships. This is the goal of the research presented in this article and it is achieved by employing the evidential reasoning (ER) approach.

This article is organized as follows: the problem to be tackled is first defined. The methodology and mathematical model are then presented and an illustrative example is given to demonstrate their capability. This is followed by the conclusions.

2 Problem description

Finding the preferred ship from a number of options is a complicated decision involving not only quantitative evaluations but also qualitative judgments. The preferred ship, from the investors' viewpoint, is normally the one that can yield the highest profit or lowest transportation cost while complying with all mandatory regulations. Besides this ultimate desire, there are also many other important requirements for the successful operation of a ship. For example, the appearance of a ship may be considered to represent the image of the owner's company and may need to be attractive to the public eye; the standard of living facilities on board is an indication of modernization of the ship and an attraction to highquality seafarers. A ship preferred by one owner may not necessarily be the one preferred by another if the importance of various attributes may not be the same for different owners.

The reason that quantitative mathematical models for ship techno-economic evaluation are widely employed already is because the economy of transportation is normally the ultimate goal of owners or investors. However, such an approach does not always provide solutions that are completely satisfactory. The best ship selected by using mathematical approaches in terms of only quantitative attributes is sometimes rejected by owners based on other non-numerical attributes. After all, there are multiple attributes to be evaluated and balanced in the decision of ship-type selection, which is a multifaceted activity requiring the efficient utilization of all the information available. This means that the decision of selecting the best ship from a number of options is influenced by multiple factors and different viewpoints. Generally speaking, the decision-making process is a complicated and difficult one characterised by the following features:5

- Multiple criteria exist, which can be quantitative or qualitative in nature.
- Uncertainty and risk are involved.
- There are multiple decision makers.
- Some of the input data are vague, incomplete, or imprecise.

With these features in mind, the preferred ship is defined in this article as the ship that best meets the requirements of investors or designers according to their selection attributes.

The problem to be addressed here is further summarized as follows:

- There is a group of candidate ships that are not identical but are comparable in terms of the various characters and attributes concerned.
- All the candidate ships have been designed and delivered and meet all the technical requirements for the intended operation environment, i.e., the technical parameters of all the candidates are known and the technical feasibilities are guaranteed.
- The economic benefits make up the most important goal but other criteria need to be taken into account as well.
- Both quantitative and qualitative criteria are involved in the evaluation of each ship.

The purpose of this investigation is to rank all the candidate ships in the order of comprehensive preferences specified by investors or designers in order to select the best reference ship or ships.

3 Methodology

3.1 General procedure

To take into account both the valuable information of qualitative assessment and the accuracy of quantitative evaluation, a three-phase method is proposed in this article for tackling the preferred ship selection problem as described above. The three phases are listed as follows:

Phase 1. Investigate the historical data and the concurrent environment of the market the new ship is going to enter and all the restrictions and limitations on the ship's dimensions, predict the future trend of market demand and associated facility development, and list all the possible options that can be found for candidate ships.

Phase 2. Establish a mathematical model and appropriate software for conducting economic evaluation for each of the candidate ships. Multiple criteria may be used in the evaluation; however, all criteria are economic and must be quantitative to ensure a direct and accurate comparison. The result of this phase will give an initial ranking of the candidate ships in terms of economic criteria. It is also possible to reduce the number of candidates if some of them prove to be non-economical at the end of this phase.

Phase 3. Use the ER approach to grade the candidate ships based on both the quantitative and the qualitative criteria of concern to designers, investors, or any other parties in this regard; produce a comprehensive report on the ranking of the candidate ships to support decision making.

As most of the economic criteria in evaluating a ship are of a quantitative nature, a mathematical model can usually be established for evaluation of these criteria with high efficiency in Phase 2.⁶ Once a mathematical model is established, the calculation and comparison are normally straightforward with the help of available software such as Microsoft Excel. For this reason, a large number of candidate ships can be included for consideration in Phase 1 and compared in Phase 2.

In fact, Phase 1 is already common practice in most new ship design projects.^{7,8} Hence, it will not be discussed further here. To demonstrate the procedure for preferred ship selection, a few economic criteria with a high level of importance are listed in Sect. 3.2, together with their mathematical expressions. Some important qualitative criteria are identified and the method for assessing those criteria is discussed in Sect. 3.3. The quantitative criteria are then aggregated with qualitative criteria using the ER approach as shown in Sect. 3.4.

3.2 Evaluation of quantitative criteria: an economic evaluation model

There are many criteria for evaluating the economy of a merchant ship, such as the required freight rate, the net present value, and the average annual cost. Each criterion can normally be used to judge a ship from a specific angle. It is difficult to single out a unique index that can cover the economy of a ship in all senses. Therefore, multiple criteria are often employed in the evaluation of a ship. Because the required freight rate, net present value, payback period, and hire base are four indexes that need to be estimated frequently before an investment is made in a new merchant ship, they are selected as the economic evaluation criteria of ships, although others can also be added to the criteria list without difficulty if necessary.

The required freight rate (RFR) is calculated as follows:⁹

$$RFR = \frac{C_c - S_2 + P \cdot (A/P, i_0, Y) - RV \cdot (A/F, i_0, Y)}{Q}$$
(1)

where C_c is the total annual cost of a ship:

$$C_c = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8 + S_9$$

and S_1 is the annual salary and cost of welfare of the ship's crew, S_2 is the annual depreciation of the ship, S_3 is the annual repair cost of the ship, S_4 is the annual insurance cost of the ship, S_5 is the annual fuel oil cost of the ship, S_6 is the annual lubrication and related costs of the ship, S_7 is the annual cost of consumables for the ship, S_8 is the annual port costs of the ship, S_9 is the annual administrative and miscellaneous costs of the ship, RV is the residual value of the ship at the end of its depreciation period, i_0 is the discount rate, Y is the depreciation period of the ship, P is the initial investment in the ship, and Q is the annual transportation capacity of the ship.

$$(A/P, i_0, Y) = \frac{i_0 \cdot (1+i_0)^Y}{(1+i_0)^Y - 1}$$
 is the capital recovery factor and
$$(A/F, i_0, Y) = \frac{i_0}{(1+i_0)^Y - 1}$$
 is the sinking fund factor.

The net present value (*NPV*) is calculated as follows:

$$NPV = -P + \frac{Q \cdot P_r - C_c + S_2}{(A/P, i_0, Y)} + \frac{RV}{(1 + i_0)^Y}$$
(2)

where P_r is the freight rate.

The payback period (*PBP*) is calculated as follows:¹⁰

$$PBP = -\frac{\log\left(1 - \frac{i_0 \cdot P}{Q \cdot P_r - C_c + S_2}\right)}{\log(1 + i_0)}$$
(3)

The hire base (H/B) is calculated as follows:

$$H/B = \frac{P \cdot (A/P, i_0, Y) - RV \cdot (A/F, i_0, Y) + S_1 + S_3 + S_4 + S_6 + S_7 + S_9}{Cot \cdot Dwt}$$
(4)

where *Dwt* stands for the deadweight of the ship and *Cot* stands for the average number of months that the ship can be rented out per year.^{11,12}

The parameters used in Eqs. 1–4 are given or calculated for the individual cases in question. Investors or ship owners can determine these parameters based on their personal knowledge and the circumstances. Nevertheless, the principles of determining these parameters are basically the same. Taking crude oil transportation for example, the annual transportation capacity of a tanker is calculated as follows:

$$Q = \frac{V_c \cdot T_n}{T_1 + T_2 + T_3 + T_4}$$
(5)

where V_c is the average cargo capacity in a single voyage of the ship; T_n is the average operation time per annum of the ship; T_1 and T_2 are the sailing times for fully loaded and ballast voyages, respectively, in a round trip; and T_3 and T_4 are the times spent at loading and discharging ports, respectively, in a round trip.

The annual fuel oil cost, S_5 , is calculated using Eq. 6 based on two assumptions. The first assumption is that a tanker consumes both heavy fuel oil and marine diesel oil for the main engine and the electric generator, respectively. The second assumption is that the fuel consumption rates of fully loaded and ballast voyages may take different values, as is the case in practice.

$$S_{5} = \frac{T_{n}}{T_{1} + T_{2} + T_{3} + T_{4}} (T_{1} \cdot G_{1} \cdot P_{1} + T_{2} \cdot G_{2} \cdot P_{1}) + T_{n} \cdot G_{3} \cdot P_{2}$$
(6)

where G_1 and G_2 are the consumption rates of heavy fuel oil during fully loaded and ballast voyages, respectively; G_3 is the average consumption rate of marine diesel oil on a daily basis; and P_1 and P_2 are the prices for heavy fuel oil and marine diesel oil, respectively,

The annual administrative and miscellaneous cost, S_9 , is calculated assuming that S_9 accounts for 15% of the total annual cost for a running ship, as follows:

$$S_9 = 0.177 \cdot (S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8)$$
(7)

Other parameters can be determined in a similar way to that shown by Xie et al.⁹ Designers or investors can determine the values of such parameters using appropriate equations and this is usually based on market information and decision makers' experience in the relevant areas. It should also be noted that the influence on the economic performance of most, if not all, of the technical specifications of a ship is already included or reflected in the determination of these parameters.

3.3 Evaluation of qualitative criteria

Qualitative criteria used for assessing candidate ships may include some general features of the ships. For example, in the example discussed later, the following four qualitative criteria are considered (Fig. 1): performance, equipment, appearance, and automation.

The assessment of candidate ships based on qualitative attributes is not an easy task. On the one hand, qualitative attributes are subjective in nature and different people may make different judgments of the same ship. On the other hand, as different ships may have different performances for different attributes, tradeoffs among the performances on different attributes are difficult, but necessary, in order to compare the ships and make a selection. Taking the living facility aspect of a ship, for example, all the factors such as space, outfit, equipment, and position of the accommodation on board a ship play an important role in the assessment. It is difficult for one ship to have all these factors in the optimum state. In ship design, the final result is often a compromise between many requirements that need correct evaluation.¹³ Further adding to the complication, some information related to the assessment of qualitative attributes can be vague and incomplete with many uncertainties.

To overcome such difficulties, a number of methods and techniques developed in the field of multiple-criteria decision analysis can be applied. The first technique is to break down a more general or abstract qualitative criterion into less abstract and easier-to-assess subcriteria or even quantitative criteria. For example, the

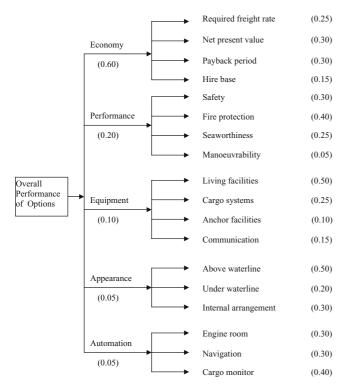


Fig. 1. Hierarchy of assessment criteria and their weights (*numerical values in parentheses*) for the ship selection problem

safety criterion can be measured using the transverse metacentric height (GM), the angle of vanishing stability, the floodable (permissible) length, the strength of the hull, and the safety standards of relevant facilities. Many of these attributes are quantitative and are required to be within certain ranges by statutory regulations or rules. For illustration purposes, the four qualitative attributes are decomposed to only one level of sub-criteria, as shown in Fig. 1. In practice, if necessary, sub-criteria may be further broken down until they can be easily assessed.

The assessment of candidate ships in terms of qualitative attributes is normally expressed using grades. For example, in this article, the same set of grades {Worst, Poor, Average, Good, Excellent} is used for assessing the ships in terms of all qualitative attributes. A different set of grades can be used for different attributes if necessary.

To increase consistency and reduce subjectivity in the assessment of qualitative attributes, the standard of each assessment grade need to be clearly defined; the definitions of the standards need expert knowledge in the field. The definitions also provide a formal platform for decision makers such as ship owners and designers to address their concerns. For example, the assessment grades of safety are defined in this work as follows:

- Excellent all the numerical measurements and subjective judgments relating to the safety attribute of a very large crude carrier (VLCC) fall in the most desirable regions.
- Good the numerical measurements and subjective judgments relating to the safety attribute of a VLCC are favourable but not excellent.
- Average the numerical measurements and subjective judgments relating to the safety attribute of a VLCC are at the average satisfactory level.
- Poor the numerical measurements and subjective judgments relating to the safety attribute of a VLCC are satisfactory but below average on the whole.
- Worst the numerical measurements and subjective judgments relating to the safety attribute of a VLCC satisfy only the relevant lowest standards.

If the metacentric height, the angle of vanishing stability, and all other safety standards of relevant facilities of a VLCC are favourable based on past experience, for instance, then it can be assessed to be good in terms of safety. If a ship satisfies the Good and the Average standards to certain degrees, such as 70% and 30%, respectively, then its performance on the attribute may be represented by a distribution $\{(Good, 0.7), (Average, 0.3)\}$.

To facilitate the cross comparison or trade-offs among different attributes, weights need to be assigned to attributes to reflect their relative importance and a utility function needs to be defined for the assessment grades of each attribute. A utility function is used to model a decision maker's preferences towards the values or the grades used to measure an attribute. It maps the whole range of an attribute's values or the set of grades to real numbers in the range [0, 1], with 0 representing the least preferred and 1 the most preferred values or grades, respectively. For example, a typical utility function for a set of grades {Worst, Poor, Average, Good, Excellent} may be that:

u(Worst) = 0, u(Poor) = 0.25, u(Average) = 0.5,u(Good) = 0.75, and u(Excellent) = 1.

Utility functions and weights of attributes jointly determine the trade-off relationships among attributes. For example, if a decision maker assigns 0.5 and 0.25 as weights to attributes Living Facility and Cargo System, respectively, and uses the above defined grades and utility function for assessing the two attributes, this means that the decision maker is willing to sacrifice the performance by one grade on Living Facility in order to obtain a performance increase by two grades on Cargo System, i.e., if all other attribute values are the same, a ship with good value of Living Facility and average value of Cargo System is equally preferred to a ship with average value of Living Facility and excellent value of Cargo System.

To model and manage uncertainties in qualitative assessment information, the concept of belief degree in the theory of evidence by Shafer¹⁴ is introduced into the ER approach for multiple-criteria decision analysis. Using belief degrees, the assessment of a ship on a criterion is expressed as a distribution instead of a single number. For example, if some of the safety features of a ship are not known, then its assessment on Safety may be modelled by a distribution such as $\{(Good, 0.7),$ (Average, 0.1), where 0.7 and 0.1 are belief degrees associated with good and average grades, indicating the extent to which the safety features of the ship conform with the standards of good and average grades. If the sum of the belief degrees in the distribution is 1, this means the assessment is complete. In the example, the total is 0.8, indicating that there is missing information in the assessment.

In addition to model uncertainties, the ER approach can also aggregate information with uncertainties, as discussed in the following section. This means that all available information, whether known or partially known, can be used to support the decision-making process.

3.4 Aggregation of criteria: the ER approach

3.4.1 The basic algorithm

Let E be a criterion to be assessed that is evaluated through L sub-criteria, denoted by

$$E = \left\{ e_1, e_2, \dots, e_i, \dots, e_L \right\}$$

A particular ship can be assessed on a criterion using a set of assessment grades $H = \{H_n, n = 1, ..., N\}$ with a set of associated belief degrees $B = \{\beta_n, n = 1, ..., N\}$.¹⁵ For example, in this article, the top criterion is assessed using the five grades: Worst, Poor, Average, Good, and Excellent, i.e., for this criterion, we have

$$N = 5, H_1 = \{Worst\}, H_2 = \{Poor\}, H_3 = \{Average\}, H_4 = \{Good\}, H_5 = \{Excellent\}.$$

Each belief degree in the set *B* is associated with a corresponding grade in the set *H*. For example, β_n is associated with grade H_n , representing that an alternative is assessed to grade H_n with a belief degree of β_n . Belief degrees are a type of subjective probability, and therefore they must satisfy the following relationship:

$$0 \le \beta_n \le 1$$
, $\sum_{n=1}^N \beta_n \le 1$, and $\beta_H = 1 - \sum_{n=1}^N \beta_n$

where β_{H} is the belief degree unassigned to any specific grade, representing the unknown or missing percentage of information in the assessment. If $B_i = \{\beta_{n,i}, n = 1, ..., N\}$ stands for the assessment of an alternative on subcriterion e_i , the following equations can be used for mapping B_i to B.

Let S(y) represent the assessment of a criterion y. Then, $S(E) = \{(H_n, \beta_n), n = 1, ..., N\}$ represents that a criterion E is assessed to grade H_n with degree of belief $\beta_n, n = 1, ..., N$. Therefore,

$$S(e_i) = \{ (H_n, \beta_{n,i}), n = 1, ..., N \}$$
 $i = 1, ..., L$ (8)

Let ω_i be the weight of criterion e_i to reflect its relative importance to its parent criterion E and $0 \le \omega_i \le 1$, $\sum_{i=1}^{L} \omega_i = 1$,

$$m_{n,i} = \omega_i \beta_{n,i}$$
 $n = 1, ..., N; i = 1, 2, ..., L$ (9)

$$m_{H,i} = 1 - \sum_{n=1}^{N} m_{n,i}$$
 $i = 1, 2, ..., L$ (10)

 $m_{n,i}$ is the basic probability mass representing the degree to which the *i*th sub-criterion e_i supports the hypothesis that the criterion *E* is assessed to the grade H_n . $m_{H,i}$ is the remaining probability mass unassigned to any individual grade and can be further broken down into two parts $\bar{m}_{H,i}$ and $\tilde{m}_{H,i}$ as shown in Eqs. 11 and 12, respectively:

$$\overline{m}_{H,i} = 1 - \omega_i \qquad \qquad i = 1, 2, \dots, L \tag{11}$$

$$\tilde{m}_{H,i} = \omega_i \left(1 - \sum_{n=1}^{\infty} \beta_{n,i} \right) \qquad i = 1, 2, \dots, L$$
(12)

To obtain the assessment of the parent criterion, $S(E) = \{(H_n, \beta_n), n = 1, ..., N\}$, the assessment of all the subcriteria are aggregated in the following recursive fashion. Firstly, $E_{I(i)}$ is defined as the subset of the first *i* subcriteria as follows:

$$E_{I(i)} = \{e_1, e_2, \dots, e_i\}$$

Let $m_{n,I(i)}$ be a probability mass defined as the degree to which all the *i* criteria in $E_{I(i)}$ support the hypothesis that the assessed alternative is assessed to grade H_n on E; let $m_{H,I(i)}$ be the remaining probability mass unassigned to individual grades after all the assessments on sub-criteria in $E_{I(i)}$ have been considered. The relationships shown in Eqs. 13 and 14 are obviously correct when i = 1.

$$m_{H,I(1)} = m_{H,1} \tag{14}$$

Then, based on Eqs. 13 and 14, the following iterative calculation can proceed for i = 1, 2, ..., L-1 to obtain the coefficients $m_{n,I(L)}$ and $\bar{m}_{H,I(L)}$, $\tilde{m}_{H,I(L)}$.

$$K_{I(i+1)} = \left[1 - \sum_{\substack{t=1\\j \neq t}}^{N} \sum_{\substack{j=1\\j \neq t}}^{N} m_{t,I(i)} m_{j,i+1}\right]^{-1}$$
(15)

where $K_{I(i+1)}$ is a normalization factor and:

$$m_{n,I(i+1)} = K_{I(i+1)} \left[m_{n,I(i)} m_{n,i+1} + m_{H,I(i)} m_{n,i+1} + m_{n,I(i)} m_{H,i+1} \right]$$

$$n = 1, 2, \dots, N$$
(16)

$$m_{H,I(i+1)} = K_{I(i+1)} \Big[\tilde{m}_{H,I(i)} \tilde{m}_{H,i+1} + \bar{m}_{H,I(i)} \tilde{m}_{H,i+1} + \tilde{m}_{H,I(i)} \bar{m}_{H,i+1} \Big]$$
(17)

(18)

$$\overline{m}_{H,I(i+1)} = K_{I(i+1)}\overline{m}_{H,I(i)}\overline{m}_{H,i+1}$$
(18)
$$m_{H,I(i)} = \widetilde{m}_{H,I(i)} + \overline{m}_{H,I(i)} \quad i = 1, 2, \dots, L$$
(19)

Finally, the combined degrees of belief in the assessment S(E) can be calculated as:

$$\beta_n = \frac{m_{n,I(L)}}{1 - \bar{m}_{H,I(L)}} \qquad n = 1, 2, \dots, N$$
(20)

$$\beta_{H} = \frac{m_{H,I(L)}}{1 - \bar{m}_{H,I(L)}}$$
(21)

In this way, the assessment of the parent criterion can be obtained by aggregating the assessments of all its sub-criteria.

3.4.2 The transformation between different sets of grades

If all criteria in question are qualitative and are assessed using the same set of grade H, then the above algorithm can be directly used to aggregate assessment information from sub-criteria to parent criteria up to the very top criterion. However, it is likely that a sub-criterion and its parent criterion have different assessment grades. This issue can be dealt with by the following transformation calculations.

For a sub-criterion with assessment grades differing from those of its parent criterion, the equivalent relationship between the two sets of grades needs to be established. Suppose a sub-criterion e_i has N_i grades, then $H_i = \{H_{l,i}, l = 1, ..., N_i\}, S(e_i) = \{(H_{l,i}, \gamma_{l,i}), l = 1, ..., \}$

 N_i and a grade H_{li} in H_i means a grade H_n in H to a degree of $\alpha_{n,l}$ (n = 1, ..., N). Then let

$$\beta_{n,i} = \sum_{l=1}^{N_i} \alpha_{n,l} \gamma_{l,i} \qquad n = 1, \dots, N$$
(22)

where γ_{li} is the degree of belief to which the criterion e_i is assessed to $H_{l,l}$, and $\alpha_{n,l}$ is determined by decision makers subjectively or by rules. It is necessary to keep

$$0 \le \alpha_{n,l} \le 1$$
 and $\sum_{n=1}^{N} \alpha_{n,l} = 1$ for any given *l*.

Based on Eq. 22, $S(e_i) = \{(H_{l,i}, \gamma_{l,i}), l = 1, ..., N_i\}$ can be transformed to $S(e_i) = \{(H_n, \beta_{n,i}), n = 1, \dots, N\}$ in terms of value and utility equivalence.¹⁷

3.4.3 The transformation between numeric assessment and grade assessment

It is also common that there may be numeric sub-criteria in question. In this case, a numeric value can be transformed to an equivalent assessment using the grades of its parent criterion in the following way.

Let $h_{N,i}$ be the largest and $h_{1,i}$ the smallest feasible values, respectively, that any assessed option can take on the sub-criterion. Suppose a value $h_{n,i}$ for a quantitative sub-criterion is judged to be equivalent to a grade H_n , $n = 1, \ldots, N$. Then, a value h on e_i is mapped to the grade set with degrees of belief by using Eqs. 23-25:

$$S(e_{i}(h)) = \{(h_{n,i}, \beta_{n,i}), n = 1, \dots, N\}$$
(23)

where
$$\beta_{n,i} = \frac{h_{n+1,i} - h}{h_{n+1,i} - h_{n,i}}, \beta_{n+1,i} = 1 - \beta_{n,i}$$
, if $h_{n,i} \le h \le h_{n+1,i}$
and $n = 1, \dots, N-1$ (24)

$$\beta_{k,i} = 0$$
 for $k = 1, ..., N$ and $k \neq n, n+1$ (25)

The assessment $S(e_i(h))$ transformed to the format of a belief structure as shown on the right hand side of Eq. 23 can be used directly in the ER aggregation algorithm.

3.4.4 Ranking the options

Theoretically, the ranking of options can be carried out after all the assessments of each option on the subcriteria are aggregated and its performance distributions on the top criterion T, denoted by $S(T) = \{(H_n, \beta_n), n =$ 1, ..., N}, become available. However, it is not straightforward in practice to rank options using their performance distributions in the format of $\{(H_n, \beta_n), n = 1, ..., n \}$ N}. In this case, a utility function u(x) can be defined for

the N assessment grades so that a utility score can be calculated for each performance distribution and a direct comparison based on the scores can be made.

The utility function $u(H_n)$ is defined as the utility of the grade H_n and $u(H_{n+1}) > u(H_n)$ if H_{n+1} is preferred to H_n . Taking the top criterion for instance, if $\beta_H = 0$, the utility of an option on the top criterion is then calculated by $u(T) = \sum_{n=1}^{N} \beta_n u(H_n)$. If $\beta_H \neq 0$, i.e., there is a degree of unknown which could be assigned to any grade, then the likelihood of an option being assessed to grade H_n on criterion T is a belief interval $[\beta_n, (\beta_n + \beta_H)]$ for $n = 1, \ldots, N$. Accordingly, a utility interval $[u_{\min}(T),$

 $u_{\max}(T)$] can be calculated for the assessment, where:

$$u_{\min}(T) = \sum_{n=2}^{N} \beta_n u(H_n) + (\beta_1 + \beta_H) u(H_1) \quad \text{and} \\ u_{\max}(T) = \sum_{n=1}^{N-1} \beta_n u(H_n) + (\beta_N + \beta_H) u(H_N).$$

The assessment based on a single scale of u(T) is obviously much easier and more intuitive for a decision maker to rank the options in question. To rank candidates on utility intervals, the simplest way is to use the middle point in each interval, i.e., $(u_{\min}(T) + u_{\max}(T)])/2$, as a performance indicator. For more detailed discussions of the above equations, readers are referred to Yang et al.,¹⁵ Yang and Du,¹⁶ Yang,¹⁷ and Yang and Singh.¹⁸

4 Illustration

To demonstrate the application of the above procedure, the selection of a preferred ship from a number of existing VLCC tankers for the purpose of design reference is taken as an example in this section. In fact, there exist a large number of different tankers suitable for various routes throughout the world that have evolved over 100 years of development. Each type of tanker has its own techno-economical features already confirmed in their practical operation. For example, 65 VLCCs built in the 1990s were identified, compared, and ranked by Xie et al.⁹ A suitable tanker can normally be chosen from among these existing tankers, based on the given criteria for an intended investor or for a new design subject to further improvement.

In this example, suppose that the following six existing VLCCs, named Tanker 1 to Tanker 6 as shown in Table 1, are identified as candidate tankers for comparison using the above methodology. Among these six VLCCs, Tanker 1, Tanker 4, Tanker 5, and Tanker 6 were built in the 1990s, whereas the other two have incomplete data. It is hence supposed that the four VLCCs built in 1990s have detailed technical specifications and drawings, whereas Tanker 2 and Tanker 3 have only limited technical data and no drawings to show their appearance and other features.

Five criteria, namely, Economy, Performance, Equipment, Appearance, and Automation, are chosen as the main criteria to make comparison among the candidate VLCCs. Each main criterion is assessed through several sub-criteria or basic criteria. Except for the four basic criteria of Economy, all the other basic criteria are nonnumeric. The hierarchical structure of the criteria for the ship selection problem is shown in Fig. 1, with the numerical values in the parentheses being the weights of the corresponding criteria or attributes.

4.1 The quantitative evaluation of economic criteria

To compare the economic criteria given by Eqs. 1–4, an identical trading environment is assumed to apply to all

Table 1. Technical parameters for the candidate very large crude carriers (VLCCs)

Parameter	Tanker 1ª	Tanker 2	Tanker 3	Tanker 4 ^a	Tanker 5 ^a	Tanker 6 ^a
Deadweight (t)	280491	291 640	293 000	238 500	258 080	307 000
Volume (m ³)	327 909			286 575	322815	350850
Gross tonnage	151127			137893	151 591	164371
Net tonnage	86409			67473	73875	100817
Length overall (m)	330			321.3	338	329.8
Moulded beam (m)	56			57	58	58
Moulded depth (m)	30.2			29.9	28.75	31.8
Design draught (m)	20.85			19.54	18.48	22.85
Service speed (kn)	14.2	14.50	15.40	15.5	15.8	16.17
Engine power (kW)	24860			24680	26850	34650
Fuel consumption (t/d)	63.7	67.90	88.60	62.9	71.3	80.3
Complement (people)	46	30	30	28	28	30

^a Source: Data collected from *Science of Ships* (1992.7, 1990.2, 1991.12, 1997.1)

 Table 2. Main trading environment parameters

$\mathbf{p}_{\text{contraction}}$	10.700
Route length (n mile)	10700
Freight rate (\$/t)	22
Fuel oil price (\$/t)	290
Diesel oil price (\$/t)	490
Expected rate of return (%)	12
Annual operation time (day)	330
Depreciation period (year)	20
Total time at ports per voyage (day)	5
Ship price (\$/Dwt)	400
Salvage value (%)	5

six VLCCs under consideration. The main parameters of the trading environment are listed in Table 2. The linear depreciation method is adopted in the calculation of the fixed costs of a VLCC. The fuel consumption rate for ballast voyages is assumed to be 75% of that in the fully loaded voyage for the same VLCC.

The four economic criteria of the candidate tankers were calculated using Microsoft Excel based on Eqs. 1–4 and are listed in rows 2 to 5 of Table 3. Tanker 5 is the best in terms of the three economic criteria *NPV*, *RFR*, and *PBP*. A careful analysis can reveal two pieces of economic evidence that can support this result. One is that its cargo volume is large enough to make good use of its deadweight. The other is that it is a type of shallow draught ship with a large beam to draught ratio, i.e., *B/T* = 3.14, which makes it suitable for ports with draught restrictions. The *H/B* of Tanker 5 ranks fourth of the six candidates. The main reason for this is that it has a slightly smaller deadweight, especially compared with that of Tanker 6 and Tanker 2.

It is not surprising to find that the six tankers are ranked in the same order of favour on the first three economic criteria because the four economic criteria are correlated to a certain extent. It is also possible for them to be ranked in the same order on the fourth criterion at the same time, though this outcome is not guaranteed in general.

If the economic criteria were the only concerns or the other qualitative criteria, such as Automation and Appearance, were unimportant in the decision-making process, there would be no need for further analysis. Otherwise, if subjective attributes need to be considered in the comparison, the third phase is implemented and the ER approach is used for this purpose, as shown in the following section.

4.2 The transformation of the quantitative basic attributes

To aggregate all the initial information for a candidate using the basic ER algorithm, the assessments on quan-

Table 3. Assessment of the basic attributes of the selected VI	ic attributes of the selected V	LCCs				
Attribute	Tanker 1.	Tanker 2	Tanker 3	Tanker 4	Tanker 5	Tanker 6
Required freight rate (\$/t)	23.13	21.56	21.86	21.46	20.99	21.42
Net present value (\$k)	-10746	4635	1522	4878	10180	6624
Payback period (year)	41.84	17.48	19.07	16.90	14.93	16.77
Hire base (\$/Dwt·month)	6.92	6.79	6.97	6.91	6.92	6.84
Safety	0.0, 0.0, 0.0, 0.0, 0.6, 0.2			0.0, 0.2, 0.8, 0.0, 0.0	0.0, 0.7, 0.3, 0.0, 0.0	0.0, 0.0, 0.6, 0.3, 0.0
Fire protection	0.0, 0.0, 0.0, 0.0, 0.9, 0.0			0.6,	0,0	0.0, 0.0, 0.0, 0.0, 0.6, 0.2
Seaworthiness	0.0, 0.0, 0.0, 0.8, 0.0			0.0,	0.3, 0.6, 0.0, 0.0, 0.0	0.0, 0.0, 0.0, 0.5, 0.2
Manoeuvrability	0.0, 0.0, 0.0, 0.0, 1.0			0.5,	0.0,	0.0, 0.7, 0.2, 0.0, 0.0
Living facilities	0.0, 0.0, 0.0, 0.1, 0.8			0.5,	0.0,	0.0, 0.1, 0.8, 0.0, 0.0
Cargo systems	0.0, 0.0, 0.0, 0.0, 0.7, 0.2			0.5,).6,	0.0, 0.0, 1.0, 0.0, 0.0
Anchor facilities	0.0, 0.0, 0.0, 1.0, 0.0			0.0, 0.0,	0.0, 0.0,	0.0, 0.0, 0.0, 1.0, 0.0
Communication	0.0, 0.0, 0.0, 1.0, 0.0			0.0, 1.0, 0.0, 0.0, 0.0	0.0, 1.0, 0.0, 0.0, 0.0	0.0, 0.0, 0.0, 1.0, 0.0
Above waterline	0.0, 0.0, 0.0, 0.0, 1.0			0.6,	0.0,	0.0, 0.0, 0.0, 0.1, 0.1
Under waterline	0.0, 1.0, 0.0, 0.0, 0.0			0.5,	.8,	0.0, 0.0, 0.4, 0.2, 0.0
Internal arrangement	0.0, 0.0, 0.0, 1.0, 0.0			0.5,	.3,	1.0,
Engine room	0.3, 0.5, 0.0, 0.0, 0.0			0.0,	.5,	0.0,
Navigation	0.0, 1.0, 0.0, 0.0, 0.0			0.0,	0.0	0.0,
Cargo monitor	0.2,0.6,0.0,0.0,0.0			0.0, 0.0, 0.0, 1.0, 0.0	0.0, 0.0, 0.0, 0.0, 0.6	0.0, 0.0, 0.4, 0.6, 0.0
Belief degrees correspond to grades Worst to Excellent from	ades Worst to Excellent from		left to right for qualitative attributes			

Table 4.	Definition of	the worst a	and best of	the numeric criteria
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Grade	<i>RFR</i>	<i>NPV</i>	PBP	<i>H/B</i>
	(\$/t)	(\$ k)	(Year)	(\$/Dwt·month)
Excellent	20	$11000 \\ -11000$	14	6.7
Worst	24		42	7.0

RFR, required freight rate; *NPV*, net present value; *PBP*, payback period; *H/B*, hire base

titative attributes need to be transformed into assessments using a common set of grades in the format of belief structures. To do this, a region or a pair of worst and best values for each of the quantitative criteria needs to be specified initially. The worst and best values should be selected in such a way that the values of this attribute for all considered candidates are in the specified range. Then the best value is normally regarded to be equivalent to the most preferred grade and the worst corresponds to the least preferred. In the case of the example here, they are Excellent and Worst, respectively. Based on the results of Phase 2 discussed in Sect. 4.1, the Worst and Excellent values for the four numeric sub-criteria of Economy are set as shown in Table 4.

For each of the other grades between Worst and Excellent, an equivalent value also needs to be identified by decision makers according to their judgment. The grades between the two extreme values are assumed to be distributed evenly in this example. For instance, the value of NPV equivalent to Worst is -11000 and to Excellent is 11000; the values equivalent to Poor, Average, and Good are calculated as follows:

$$Poor = h_{2,2} = Worst + \frac{Excellent - Worst}{4}$$

= -11000 + $\frac{11000 + 11000}{4}$ = -5500
Average = $h_{3,2} = Worst + \frac{2 \times (Excellent - Worst)}{4}$
= -11000 + $\frac{2 \times (11000 + 11000)}{4}$ = 0
Good = $h_{4,2} = Worst + \frac{3 \times (Excellent - Worst)}{4}$
= -11000 + $\frac{3 \times (11000 + 11000)}{4}$ = 5500

The assessment grades for the other three quantitative criteria are determined in a similar way. Of course, decision makers may use other appropriate values to represent the grades if they prefer.

After the relationships between grades and numeric values are established, Eqs. 23–25 can directly be used

to transform the numerical assessments into the assessments represented by a set of grades and an associated set of belief degrees.

4.3 Assessment of qualitative criteria

To carry out a comprehensive comparison in the third phase, the performance of each candidate VLCC in terms of each qualitative sub-attribute needs to be assessed first. The assessment can be conducted by comparing available knowledge about the performances of each candidate VLCC with the standards of each assessment grade.

For example, for Tanker 1 on Safety, there is evidence showing that it has some Excellent safety features and quite a lot of Good features. However, for some other features of concern to decision makers, there is no information available. Therefore, the performance of Tanker 1 in terms of Safety is judged to be Good to a degree of 0.6 and Excellent to a degree of 0.2. Note that the total degree is less than 1, which implies that the degree of incompleteness in the assessment is 1 - 0.8 = 0.2. The assessment is represented by the set of belief degrees (0.0, 0.0, 0.0, 0.6, 0.2) in Table 3. The order of the belief degrees in the set is arranged to correspond to the order of grades in the set (Worst, Poor, Average, Good, Excellent).

In addition, to accommodate the variations in information about the performance of an option, the variations in expert opinions can also be modelled and taken into account by using the belief degree set. For example, if among the five experts responsible for making the selection, four of them judge the performance of Tanker 4 on Safety to be Average and one judges it to be Poor, then the assessment may be represented by the set of belief degrees (0.0, 0.2, 0.8, 0.0, 0.0). The assessments of the six VLCCs for each of the qualitative sub-attributes are given in Table 3.

4.4 Aggregated assessment results

The calculation and aggregation process is conducted using the Intelligent Decision System software—IDS.¹⁹ The utility function is defined as a linear function with

$$u(H_1) = 0, u(H_2) = 0.25, u(H_3) = 0.5, u(H_4) = 0.75,$$

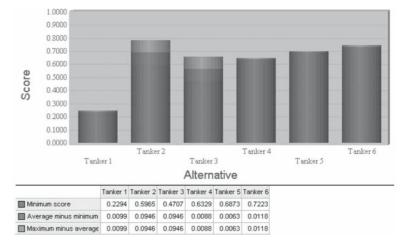
and $u(H_5) = 1.$

This is an evenly distributed function between 0 and 1. Other patterns of distribution can also be employed in a similar way if necessary.

The results produced by IDS are shown in Fig. 2 and Table 5 for the six candidate tankers. Figure 2 shows the

	Grade name for top criterion							
Ship	Worst	Poor	Average	Good	Excellent	Unsure	Average utility	Ranking order
Tanker 1	0.5010	0.2843	0.0054	0.1359	0.0538	0.0197	0.2393	6
Tanker 2	0	0	0.1496	0.5576	0.1035	0.1893	0.6912	3
Tanker 3	0.0567	0.0378	0.3670	0.2860	0.0633	0.1893	0.5653	5
Tanker 4	0	0.1529	0.2439	0.4509	0.1346	0.0177	0.6418	4
Tanker 5	0.0450	0.2391	0.0265	0.2504	0.4264	0.0126	0.6936	2
Tanker 6	0	0.0053	0.2268	0.5470	0.1973	0.0236	0.7341	1

Fig. 2. Rank of options for the top attribute (overall performance of options, see Fig. 1)



utility of each of the six candidates. Normally, candidates are ranked on their utilities; the higher the utility is, the better the option. As there is incompleteness in the assessments, the grey areas in the graph represent the possible utility variation ranges of the candidates, i.e., $[u_{\min}(T), u_{\max}(T)]$. To rank the candidates on utility intervals, the average utility of each option is calculated by $(u_{\min}(T) + u_{\max}(T))/2$ and used for ranking purposes. The ranking of all the options is listed in Table 5.

According to the average utility, Tanker 6 is ranked as the best among the six candidates, rather than Tanker 5, which seems to have the best economic evaluation as shown in Sect. 4.1. The reason for this is that Tanker 6 performs better on the qualitative attributes than Tanker 5 does. This fact can be observed by direct comparison of the data in columns 6 and 7 of Table 3. For example, except for the four basic qualitative attributes of Cargo System, Under Waterline, Navigation, and Cargo Monitor, on which the performances of Tanker 5 and Tanker 6 are more or less comparable, the performances of Tanker 6 are all preferred to those of Tanker 5 for the other ten basic qualitative attributes.

From Fig. 2, it can be seen that if all the missing information for Tankers 1, 3, 4, and 5 turns out to be in their favour and they achieve their maximum possible

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utility values, the values are still below the minimum utility of Tanker 6, i.e., it is impossible for those four tankers to outperform Tanker 6. The only tanker that could potentially outperform Tanker 6 is Tanker 2. However, it would be very risky to select Tanker 2 as it has the largest uncertain utility interval and its minimum utility value is apparently lower than that of Tankers 4, 5, and 6. If it is too costly or difficult to collect more information about Tanker 2, then the recommendation would be to select Tanker 6 as a reference tanker. Otherwise, more information about Tanker 2 should be collected before a selection is made.

Some phenomena and points have been observed during this study concerning the features of the ER approach. These are summarised as follows:

— There are occasions in various tests when the performance of an option is distributed towards the two ends of the set of grades, i.e., towards the worst and the best. This pattern of distribution on the top criterion results from the fact that the candidate tanker is simultaneously assessed to the best grade on some basic criteria and the worst grade on some other basic criteria. The final distribution of the belief degrees reflects the nature of the candidate being

extremely favourable in some aspects but extremely unfavourable in some others.

— It is also found that the changes of the worst and the best values assigned for a quantitative criterion could lead to a change in the ranking order of some candidates. The reason for this phenomenon is that the worst and the best values of a criterion are defined to have the lowest and the highest utilities of the criterion, usually normalised to 0 and 1, respectively. Therefore, the changes of the worst and the best values will lead to a change in the utility function of the criterion. In this article, a linear utility function is assumed for each quantitative criterion. If the interval from the worst to the best values is larger after the change, then the difference of the utilities of the two values on the criterion will become smaller and vice versa. As such, a candidate performing well on such a criterion may get less (or more) credit from the criterion than before the change. If such a change is large enough, the ranking position of the candidate could be changed. Therefore, when the worst and the best values for each of the basic quantitative criteria have to be changed, the utility function for this attribute needs to be adjusted accordingly to reflect the decision makers' preferences on the attribute values. More details on how to elicit decision makers' utility can be found in references on utility theory such as work by Keeney and Raiffa²⁰ and French.²¹

There are some other points that are not presented here in detail. For example, instead of building a threelevel hierarchical structure like the one in this research, it would not be difficult to build assessment structures with four, five, or even more levels for other problems. The number of candidates included in a problem can be increased if necessary. The effect of changing the distribution of a utility function and sensitivity analysis can be carried out easily with the help of IDS software. In addition, IDS is capable of producing various types of tables and figures to help users understand and analyse the results of assessment aggregation from basic criteria to the top criterion.

5 Conclusions

It has always been difficult to evaluate ship types with both quantitative and qualitative attributes by using a mathematical model. The main barrier lies in the difficulties of aggregating the two types of criteria in a credible and consistent way. The situation may become worse when there are uncertainties in the performance information of candidate ships such as vague subjective judgments, incomplete data, imprecise data, and probability distributions. In this article, the ER approach is introduced to handle such a difficult evaluation problem. A three-phase methodology is proposed for preferred ship selection problems in which the first two phases are similar to the existing practice, but the last phase is based on the ER approach to deal with subjective criteria effectively. In this way, the complex assessment process is logically decomposed and structured.

Implementation of the three-phase methodology is in fact an interactive process in which decision analysts provide support to decision makers or information providers, such as owners, investors, and designers concerned in modelling their preferences, structuring the selection problems, and assessing candidate ships. In this way, the concerns of the decision makers can be addressed and the final selection decision will represent the best value option according to their preferences. A tanker selection example is used to demonstrate the application of the methodology. The results show that the ER approach can support the multiple-criteria ship selection process when both qualitative and quantitative data have to be considered. In addition, IDS software provided a useful support tool for problem structuring, data collection, information aggregation, and decision outcome presentation.

From the tanker selection example, it is easy to see that the proposed method is flexible and can be used for other selection problems. The attribute hierarchy can be easily redesigned for other cases. Although it is challenging, especially for decision makers, to manually carry out the numeric calculation and assessment aggregation process of the methodology, with the help of Microsoft Excel and the IDS software it is relatively easy to carry out the data collection, calculation, and presentation tasks. Indeed, the ER approach has been applied to many other multiple-criteria selection problems, such as business performance assessment, in which both objective and subjective judgments exist simultaneously.

Finally, it is important to bear in mind that the methodology introduced in this article is used to select the best ship from among existing ships. Obviously, the performance of the best ship depends completely on the sample of existing ships; however, many categories of modern ships, such as oil tankers, general cargo ships, bulk carriers, and container ships, have already been developed over many years. There are plenty of similar options for consideration in most cases. Therefore, this methodology will be useful for owners, investors, and designers in the selection of a reference ship as the start of a new design.

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