Progress of Theoretical Physics, Vol. 66, No. 2, August 1981

Short Range Part of Baryon-Baryon Interaction in a Quark Model. I

—— *Formulation* ——

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(Received December 19, 1980)

The short range part of the interaction between non-strange baryons (N and Δ) is studied in a nonrelativistic quark model. The mass of a quark is assumed to be about one-third of the nucleon mass and the quark-quark interaction consists of a confinement term and the one gluon exchange potential. Baryons are described as clusters of three quarks and the resonating group method, which has been extensively developed in the nuclear cluster model, is used to treat the bound state and scattering problems of two baryons. This paper discusses the formal aspects of the present approach, while the numerical results will be given in the subsequent paper.

§1. Introduction

One of the most promising model for hadrons is the quantum chromodynamics (QCD), which has been especially successful in explaining high energy phenomena. (For references see the reviews of Refs. 1) and 2).) According to the model, hadrons consist of colored quarks and antiquarks, which interact with each other through the gauge field of the color $SU(3)$, i.e., gluon field. In the low energy phenomena, the "color singlet" hypothesis seems to be hopeful for the problem of the quark confinement and some phenomenological models,[3)~5)] constructed on the basis of the QCD, are seen to give quite satisfactory explanations of the low lying hadron spectra. The problems of the strong interactions between hadrons, however, have not been well clarified on the basis of their quark structure. The nuclear force, which is a typical one, has been phenomenologically described by a potential which has a repulsive (hard or soft) core at short distance and an attractive part in the intermediate range. The latter has been explained by several different meson exchange contributions, while the former has either been introduced phenomenologically or been attributed to the vector meson exchange.[6)~7)] The radius of the core is, however, of the order of or smaller than the size of the nucleon in which quarks are confined, and therefore it would be more natural to understand it on the basis of the quark structure of nucleons. Some recent attempts in this direction[8)~11)] have suggested that the short range repulsive force might be explained as the effect of quark exchange induced by the antisymmetrization.

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Recent observations of dibaryon renonances[12),13)] also require a more fundamental approach to the interaction between two baryons. Two possible interpretations of the dibaryon resonance are suggested, that is (1) a potential resonance of the two baryons and (2) a six-quark state which cannot be regarded as (1). The former picture has been employed in the case of the lowest "dibaryon bound state", that is, deuteron, while the latter might be a better picture for the dibaryon resonances recently found. These two distinct pictures can be compared by considering the quark structures and dynamics. It is noteworthy that a similar situation was seen in the relation between the nuclear cluster model and the shell model. We will be able to study the two-baryon problems with the help of techniques developed in the nuclear cluster model.

In this series of papers, we will apply a simple nonrelativistic quark model to the bound state and the low energy scattering problems of two baryons. Our main concern is the role of the quark exchange interaction induced by the antisymmetrization. A very simple dynamical model is used, which has been adopted by several authors[4),5)] in their studies of the low lying spectra of mesons and baryons. We assume that quarks in a color singlet system have a finite nonzero (effective) mass and can be treated nonrelativistically. There is a favorable feature of using nonrelativistic kinematics that there is no ambiguity in separating the center of mass motion from the internal motion, which is especially important in scattering problems. It is also assumed that quarks interact with each other by a two-body local potential, which confines the quark in a color singlet system and also has the effect of one gluon exchange between the quarks. The confinement of quarks by a two-body potential is known to have the difficulty concerning the long range van der Waals force.[17)] Here, we consider the confinement part of the potential only as an agent providing with localized wave functions for quarks and formulate the problem in such a way as to make its dynamical effects as small as possible.

The interaction between two baryons in the present model is the exchange effect of two quarks induced by the antisymmetrization. This is certainly an oversimplified picture of baryon-baryon interaction. An actual baryon is considered to have meson cloud surrounding the cluster of three quarks and it will induce the meson exchange interaction between two baryons, which cannot be described by the quark exchange in a nonrelativistic model. In this series of papers, we are mainly concerned with the short range part of the baryon-baryon interaction for which the quark exchange interaction is expected to dominate. The effect of the meson exchange will be discussed only in a phenomenological way.

This paper discusses the formal aspects of the present work, while the numerical results on the S wave baryon-baryon interaction are given in the subsequent paper.[22)] In § 2, the symmetry structures of two baryon states are discussed. Our model of the quark dynamics is given in § 3, where the origins of the interaction between two baryons are also discussed. Bound state and the scattering problems for two baryons are formulated by the resonating group method in §§ 4 and 5, respectively, and some comments upon the special features for the present case appear in § 6. A brief summary is given in § 7.

§ 2. Symmetry properties of two-baryon states

In this paper, only the lowest non-strange baryons N and Δ are considered and therefore up (u)- and down (d)-quarks with three color states are necessary. We can then classify multi-quark states by the irreducible representations of the spin-isospin $SU(4)$ ($\supset SU(2)_{\rm isospin} \otimes SU(2)_{\rm spin}$) group. Orbital and $SU(4)$ wave functions of N and Δ are known to be both totally symmetric (denoted by the partition number [3]), while their color states are totally antisymmetric (denoted by [111]). The other non-strange baryons are considered to belong to excited orbital configurations.

The symmetry properties of two-baryon states have already been studied in Ref. 8). The color singlet six quark state has a definite symmetry $[222]=\widetilde{[33]}$ in the color space (tilde denotes the conjugate representation), while four choices are possible for the orbital and the spin-isospin $SU(4)$ symmetry, that is

$$[3]\times[3]=[6]+[42]+[51]+[33].$$

Among the four, [6] and [42] ([51] and [33]) representations are symmetric (antisymmetric) with respect to an exchange of the two baryons, which means a simultaneous exchange of the three quarks. Total six-body states must be totally antisymmetric and possible combinations of the orbital and the $SU(4)$ symmetries are limited as given in Table I.

It should be noted that the [51] symmetric $SU(4)$ state cannot couple to the [6] symmetric orbital state, which contains the $(0s)^6$ configuration in the quark shell model. This implies that the $0s$ state is forbidden for the relative motion of the two baryons in the [51] symmetric $SU(4)$ channel. Such a forbidden state is familiar in the nuclear cluster model (for instance, α-α scattering) and is known

Table I. Possible combinations of the orbital and the spin-isospin $SU(4)$ symmetries for two-baryon states.

L	orbital	$SU(4)$
even	[6] or [42]	[33]
	[42]	[51]
odd	[51] or [33]	[42]
	[33]	[6]

Table II. Classification of two-baryon states in terms of the $SU(4)$ symmetry for L=even(a) and odd(b). Unitary transformation between the particle representation and the symmetry representation is also given.

Table II(a)

	(S, T, β)
[33]	$(3, 0, \Delta\Delta)$ $(0, 3, \Delta\Delta)$
[51]	$(3, 2, \Delta\Delta)$ $(2, 3, \Delta\Delta)$ $(2, 2, N\Delta_a)$ $(1, 1, N\Delta_a)$
[33]+[51]	$(2, 1, \Delta\Delta+N\Delta_a)$ $(1, 2, \Delta\Delta+N\Delta_a)$ $(1, 0, NN+\Delta\Delta)$ $(0, 1, NN+\Delta\Delta)$
	$\begin{pmatrix}[51]\\ [33]\end{pmatrix}=\begin{pmatrix}2/3 & \sqrt{5}/3\\ \sqrt{5}/3 & -2/3\end{pmatrix}\begin{pmatrix}\Delta\Delta\\ N\Delta_a\end{pmatrix}$ $\begin{pmatrix}2/3 & \sqrt{5}/3\\ \sqrt{5}/3 & -2/3\end{pmatrix}\begin{pmatrix}NN\\ \Delta\Delta\end{pmatrix}$

Table II(b)

[6]	$(3, 3, \Delta\Delta)$
[42]	$(3, 1, \Delta\Delta)$ $(1, 3, \Delta\Delta)$ $(2, 0, \Delta\Delta)$ $(0, 2, \Delta\Delta)$ $(2, 1, N\Delta_s)$ $(1, 2, N\Delta_s)$
[6]+[42]	$(2, 2, N\Delta_s+\Delta\Delta)$ $(0, 0, NN+\Delta\Delta)$
	$\begin{pmatrix}[6]\\ [42]\end{pmatrix}=\begin{pmatrix}2/\sqrt{5} & 1/\sqrt{5}\\ 1/\sqrt{5} & -2/\sqrt{5}\end{pmatrix}\begin{pmatrix}N\Delta_s\\ \Delta\Delta\end{pmatrix}$ or $\begin{pmatrix}NN\\ \Delta\Delta\end{pmatrix}$
$[6]+[42]^2$	$(1, 1, \Delta\Delta+N\Delta_s+NN)$
	$\begin{pmatrix}[6]\\ [42]_1\\ [42]_2\end{pmatrix}=\begin{pmatrix}1/3 & 2/3 & 2/3\\ 4/\sqrt{18} & -1/\sqrt{18} & -1/\sqrt{18}\\ 0 & 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix}\begin{pmatrix}\Delta\Delta\\ N\Delta_s\\ NN\end{pmatrix}$

to be responsible for a short range repulsive core.[15] In our problem, it will produce a repulsive core in the *pure* [51] symmetric $SU(4)$ channel. The spin-isospin $SU(4)$ symmetry, however, is actually broken and therefore [51] and [33] symmetries mix with each other in several channels including those of the ^3S and ^1S two nucleons. Later, we will find another cause of the repulsive core between two nucleons. Table IIa (IIb) gives the relations between the particle representation (NN, $N\Delta$ and $\Delta\Delta$) and the symmetry representation for L=even (odd) states.

§ 3. Dynamics

The following total hamiltonian for quarks is used in the present calculation:

$$H=K+V\,, \qquad (3\cdot1)$$

where

$$K=\sum_i \frac{\boldsymbol{p}_i^2}{2m_q}-K_G \tag{3·2}$$

and

$$V=\sum_{i<j} V_{ij}=\sum_{i<j}(V_{ij}^{\mathrm{CONF}}+ V_{ij}^{\mathrm{OGEP}}). \tag{3·3}$$

In Eq. (3·2), u and d quarks are assumed to have the mass $m_q=300\ \mathrm{MeV}/c^2$ and K_G is the kinetic energy for the center of mass motion of the total system. The first term V_{ij}^{CONF} of the two-body potential (3·3) is responsible for confining the quarks in a color singlet hadron and is assumed to be either

$$V_{ij}^{\mathrm{CONF}}=-a(\lambda_i\cdot\lambda_j)r_{ij} \tag{3·4}$$

or

$$=-a'(\lambda_i\cdot\lambda_j)r_{ij}^2 , \tag{3·5}$$

where

$$(\lambda_i\cdot\lambda_j)\equiv\sum_a \lambda_i^a\lambda_j^a \tag{3·6}$$

and λ_i^a is the a-th generator of the color $SU(3)$ for the i-th quark.

The second term in Eq. (3·3) has its origin in the exchange of a single gluon which belongs to an octet representation of the color $SU(3)$. According to De Rujula et al.,[4] we will employ the following potential:

$$V_{ij}^{\mathrm{OGEP}}=(\lambda_i\cdot\lambda_j)\frac{\alpha_s}{4}\left[\frac{1}{r_{ij}}-\frac{2\pi}{3}\frac{1}{m_q{}^2}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\delta(\boldsymbol{r}_{ij})\right]+(\text{tensor term}), \tag{3·7}$$

where α_s is the quark-gluon coupling constant and $\boldsymbol{\sigma}_i$ is the Pauli spin operator for the i-th quark. The tensor term is omitted in the present treatment. The second term of V^{OGEP} in Eq. (3·7), i.e., spin-spin contact interaction, is not invariant under the spin-isospin $SU(4)$ group, while the other terms of the present hamiltonian are invariant. Only the $SU(4)$ breaking interaction can give the mass difference between N and Δ. Thus, we can determine α_s from the observed mass difference. We neglect the Coulomb interaction and the other electromagnetic interactions in the present calculations.

Many studies[4),5)] have already been done on the spectra of hadrons by using this kind of hamiltonian. Those analyses have concluded that the excitation spectra of low lying mesons and baryons can be explained fairly well by a suitable choice of parameters, although another constant is necessary to fit the observed masses of hadrons. Many of them have assumed the harmonic oscillator wave functions for the orbital part of the quark wave functions. In the present calculation, we also assume that the orbital part is the $0s$ harmonic oscillator wave function. This enables us to separate the center of mass motion in a clear way and to use many techniques which have been developed in the nuclear cluster model calculations.

Now we consider the interaction between two baryons on the basis of the above quark hamiltonian. The potential V_{ij} does not contribute to the interaction between two *isolated* baryons in the first order, since the matrix element of $(\lambda_i \cdot \lambda_j)$ vanishes when the i-th quark is in a baryon and the j-th in the other. This can be easily understood by an analogy with the system of two neutral atoms, where no Coulomb force exists. However, once the electron wave functions of the two atoms overlap with each other, the well-known exchange force appears due to the antisymmetrization between the electrons. The situation is the same for the quarks in the two baryons and as the results we expect a quark exchange interaction, the range of which is given by the extension of the quark wave function in a baryon.

The hamiltonian (3·1) is known to give a long range force which is the analogue of the van der Waals force between two neutral atoms. This is due to the simultaneous excitations of the color octet dipole states in both of the two baryons, and the corresponding potential behaves like $1/R^3$ $(1/R^2)$ at long distance for the linear (quadratic) confinement. With the estimated strength, such a long range force clearly contradicts with the nucleon-nucleon data.[17] This difficulty is a common feature of the potential model for confinement and has not been overcome. We will see, however, that the baryon-baryon interaction, as given by the present treatment, is almost independent of the confinement potential V^{CONF}. We can thus expect that the results of this work will remain essentially unchanged, even if the mechanism of confinement is quite different from that of the potential model.

In the present work, we will mainly concentrate on the quark exchange interaction by using the resonating group method. The effect of the meson exchange will be briefly discussed by simulating it with an additional local potential between two *baryons,* which we call "effective meson exchange potential" (EMEP).

§ 4. Bound state problem

In this section, we present the formulation of a method for solving the bound state problems of two composite particles (we denote them as the cluster A and B). The method used here is the resonating group method (RGM), which was first proposed by Wheeler[14] and has been developed in the nuclear cluster model calculations. In the present application, all constituents are quarks instead of nucleons and a baryon is treated as a single cluster which is made of three quarks. In order to avoid unnecessary complexities, we will give here the expressions in which spin, isospin and color degrees of freedom are dropped. Besides the straightforward generalization due to these degrees of freedom, the possible coupling between particle channels (NN, $N\Delta$ and $\Delta\Delta$) is taken into account in the actual calculation.

We take the following choice of the coordinates to construct the total wave function of the system:

$$\boldsymbol{\xi}_1 = \boldsymbol{r}_1 - \boldsymbol{r}_2\,, \qquad \boldsymbol{\xi}_2 = \boldsymbol{r}_3 - \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}\,, \quad \boldsymbol{R}_A = \frac{1}{3}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3),$$

$$\boldsymbol{\xi}_3 = \boldsymbol{r}_4 - \boldsymbol{r}_5\,, \qquad \boldsymbol{\xi}_4 = \boldsymbol{r}_6 - \frac{\boldsymbol{r}_4 + \boldsymbol{r}_5}{2}\,, \quad \boldsymbol{R}_B = \frac{1}{3}(\boldsymbol{r}_4 + \boldsymbol{r}_5 + \boldsymbol{r}_6),$$

$$\boldsymbol{R}_{AB} = \boldsymbol{R}_A - \boldsymbol{R}_B\,,\ \boldsymbol{R}_G = \frac{1}{2}(\boldsymbol{R}_A + \boldsymbol{R}_B). \tag{4·1}$$

Here, $\boldsymbol{r}_i$ is the coordinate of the i-th quark, $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ ($\boldsymbol{\xi}_3$ and $\boldsymbol{\xi}_4$) are the internal coordinates for the cluster $A(B)$, $\boldsymbol{R}_A$ ($\boldsymbol{R}_B$) is the center of mass coordinate of the cluster $A(B)$, $\boldsymbol{R}_{AB}$ is the relative coordinate between A and B, and $\boldsymbol{R}_G$ is the center of mass coordinate of the total system.

Following the cluster model calculation, the RGM wave function is written as

$$\Psi(\boldsymbol{\xi}_A, \boldsymbol{\xi}_B, \boldsymbol{R}_{AB}) = \mathcal{A}[\phi_A(\boldsymbol{\xi}_A)\phi_B(\boldsymbol{\xi}_B)\chi(\boldsymbol{R}_{AB})]. \tag{4·2}$$

Here $\phi_A(\boldsymbol{\xi}_A)(\phi_B(\boldsymbol{\xi}_B))$ is the internal wave function of the cluster $A(B)$, $\boldsymbol{\xi}_A = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$, $\boldsymbol{\xi}_B = (\boldsymbol{\xi}_3, \boldsymbol{\xi}_4)$, $\chi(\boldsymbol{R}_{AB})$ is the relative wave function between A and B and $\mathcal{A}$ is the antisymmetrizing operator defined as

$$\mathcal{A} \equiv 1 - \mathcal{A}' \equiv 1 - \sum_{\substack{i\in A\\ j\in B}} P_{ij}\,. \tag{4·3}$$

Note that exchanges of more than one particle are redundant in the present case since the exchange of three particles such as $P_{14}P_{25}P_{36}$ are interpreted as the exchange of two baryons, i.e., P_{AB}, and the exchanges of two particles can be expressed as the one particle exchanges times P_{AB}.

When one knows the internal wave functions ϕ_A and ϕ_B, the equation of motion for $\chi(\boldsymbol{R}_{AB})$ can be obtained as follows:

$$\int \phi_A^+(\boldsymbol{\xi}_A)\phi_B^+(\boldsymbol{\xi}_B)(H - E)\,\Psi(\boldsymbol{\xi}_A, \boldsymbol{\xi}_B, \boldsymbol{R}_{AB})\,d\boldsymbol{\xi}_A d\boldsymbol{\xi}_B = 0\,. \tag{4·4}$$

In order to rewrite Eq. (4·4) into a more convenient form, let us define RGM hamiltonian and normalization kernels as follows:

$$H(\boldsymbol{R}', \boldsymbol{R}) \equiv K(\boldsymbol{R}', \boldsymbol{R}) + V(\boldsymbol{R}', \boldsymbol{R}), \tag{4·5}$$

$$K(\boldsymbol{R}', \boldsymbol{R}) \equiv K^{(D)}(\boldsymbol{R})\delta(\boldsymbol{R}' - \boldsymbol{R}) - K^{(EX)}(\boldsymbol{R}', \boldsymbol{R}), \tag{4·6a}$$

$$V(\boldsymbol{R}', \boldsymbol{R}) \equiv V^{(D)}(\boldsymbol{R})\delta(\boldsymbol{R}'-\boldsymbol{R}) - V^{(EX)}(\boldsymbol{R}', \boldsymbol{R}), \tag{4·6b}$$

$$N(\boldsymbol{R}', \boldsymbol{R}) \equiv N^{(D)}(\boldsymbol{R})\delta(\boldsymbol{R}'-\boldsymbol{R}) - N^{(EX)}(\boldsymbol{R}', \boldsymbol{R}), \tag{4·7}$$

where

$$\begin{aligned}\begin{pmatrix} N^{(D)} \\ K^{(D)} \\ V^{(D)} \end{pmatrix}(\boldsymbol{R}) &\equiv \int \phi_A^+(\boldsymbol{\xi}_A)\phi_B^+(\boldsymbol{\xi}_B) \\ &\quad \times \begin{pmatrix} 1 \\ K \\ V \end{pmatrix} \delta(\boldsymbol{R}-\boldsymbol{R}_{AB})\phi_A(\boldsymbol{\xi}_A)\phi_B(\boldsymbol{\xi}_B)\,d\boldsymbol{\xi}_A d\boldsymbol{\xi}_B d\boldsymbol{R}_{AB} \\ &= \begin{bmatrix} 1 \\ -\dfrac{1}{2\mu}\nabla_R{}^2 + K_{\text{int}} \\ V_{\text{rel}}^{(D)}(\boldsymbol{R}) + V_{\text{int}} \end{bmatrix}, \end{aligned} \tag{4·8}$$

$$\begin{aligned}\begin{pmatrix} N^{(EX)} \\ K^{(EX)} \\ V^{(EX)} \end{pmatrix}(\boldsymbol{R}', \boldsymbol{R}) &\equiv \int \phi_A^+(\boldsymbol{\xi}_A)\phi_B^+(\boldsymbol{\xi}_B)\delta(\boldsymbol{R}'-\boldsymbol{R}_{AB}) \begin{pmatrix} 1 \\ K \\ V \end{pmatrix} \\ &\quad \times \mathcal{A}'[\phi_A(\boldsymbol{\xi}_A)\phi_B(\boldsymbol{\xi}_B)\delta(\boldsymbol{R}-\boldsymbol{R}_{AB})]\,d\boldsymbol{\xi}_A d\boldsymbol{\xi}_B d\boldsymbol{R}_{AB}\,. \end{aligned} \tag{4·9}$$

In Eq. (4·8), $\mu(=\frac{3}{2}m_q)$ is the reduced mass of A and B, K_{int} is the internal kinetic energy

$$K_{\text{int}} \equiv \int \phi_A^+(\boldsymbol{\xi}_A)\left(\frac{1}{2m_q}\sum_{i\in A}\boldsymbol{p}_i{}^2 - \frac{1}{6m_q}(\sum_{i\in A}\boldsymbol{p}_i)^2\right)\phi_A(\boldsymbol{\xi}_A)\,d\boldsymbol{\xi}_A + (A\to B) \tag{4·10}$$

and V_{int} and $V_{\text{rel}}^{(D)}(\boldsymbol{R})$ are, respectively, the internal potential energy and the direct interaction between A and B defined as

$$V_{\text{int}} \equiv \int \phi_A^+(\boldsymbol{\xi}_A)\sum_{i<j\in A} V_{ij}\phi_A(\boldsymbol{\xi}_A)\,d\boldsymbol{\xi}_A + (A\to B), \tag{4·11}$$

$$\begin{aligned} V_{\text{rel}}^{(D)}(R) &\equiv \int \phi_A^+(\boldsymbol{\xi}_A)\phi_B^+(\boldsymbol{\xi}_B)\sum_{\substack{i\in A \\ j\in B}} V_{ij}\delta(\boldsymbol{R}-\boldsymbol{R}_{AB}) \\ &\quad \times \phi_A(\boldsymbol{\xi}_A)\phi_B(\boldsymbol{\xi}_B)\,d\boldsymbol{\xi}_A d\boldsymbol{\xi}_B d\boldsymbol{R}_{AB}\,. \end{aligned} \tag{4·12}$$

Using these kernels, we obtain the following integro-differential equation (called RGM equation):

$$\int \mathcal{L}(\boldsymbol{R}', \boldsymbol{R})\chi(\boldsymbol{R})\,d\boldsymbol{R} = 0 \tag{4·13}$$

with

$$\mathcal{L}(\boldsymbol{R}',\boldsymbol{R})\equiv H(\boldsymbol{R}',\boldsymbol{R})-EN(\boldsymbol{R}',\boldsymbol{R})$$
$$=\left[\frac{-1}{2\mu}\nabla_R{}^2+V_{\rm rel}^{(D)}(\boldsymbol{R})-E_{\rm rel}\right]\delta(\boldsymbol{R}-\boldsymbol{R}')$$
$$-[K^{(EX)}(\boldsymbol{R}',\boldsymbol{R})+V^{(EX)}(\boldsymbol{R}',\boldsymbol{R})-EN^{(EX)}(\boldsymbol{R}',\boldsymbol{R})], \tag{4·14}$$

where $E_{\rm rel}=E-E_{\rm int}=E-(K_{\rm int}+V_{\rm int})$ is the energy of the relative motion.

In the actual calculation, we take the internal wave functions ϕ_A and ϕ_B as Gaussian, i.e.,

$$\phi_A(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)=\left(\frac{1}{2\pi b^2}\right)^{3/4}\left(\frac{2}{3\pi b^2}\right)^{3/4}\exp\left\{-\left(\frac{\xi_1{}^2}{4b^2}+\frac{\xi_2{}^2}{3b^2}\right)\right\}, \tag{4·15a}$$

$$\phi_B(\boldsymbol{\xi}_3,\boldsymbol{\xi}_4)=\left(\frac{1}{2\pi b^2}\right)^{3/4}\left(\frac{2}{3\pi b^2}\right)^{3/4}\exp\left\{-\left(\frac{\xi_3{}^2}{4b^2}+\frac{\xi_4{}^2}{3b^2}\right)\right\}. \tag{4·15b}$$

One can then solve the RGM equation (4·13) for the bound state problem by a method familiar in the nuclear cluster model calculation, in which the wave function of the relative motion $\chi(\boldsymbol{R})$ is expanded by similar Gaussian functions with their peaks at $\boldsymbol{R}=\boldsymbol{R}_i$ $(i=1,2,\cdots,N)$, i.e.,

$$\chi_i(\boldsymbol{R})=\left(\frac{3}{2\pi b^2}\right)^{3/4}\exp\left\{-\frac{3}{4b^2}(\boldsymbol{R}-\boldsymbol{R}_i)^2\right\}. \tag{4·16}$$

Expanding it into partial waves, one obtains

$$\chi(\boldsymbol{R})\equiv\sum_L\frac{1}{R}\chi^L(R)\,Y_{LM}(\hat{\boldsymbol{R}}), \tag{4·17}$$

$$\chi^L(R)=\sum_{i=1}^N c_i\chi_i{}^L(R), \tag{4·18a}$$

$$\chi_i{}^L(R)=4\pi R\left(\frac{3}{2\pi b^2}\right)^{3/4}\exp\left\{-\frac{3}{4b^2}(R^2+R_i{}^2)\right\}i_L\left(\frac{3}{2b^2}R_iR\right), \tag{4·18b}$$

where i_L is the L-th modified spherical Bessel function. For $L=0$, one has $i_0(x)=(\sinh x/x)$. Minimizing the energy with respect to the coefficients c_i's, one obtains a linear eigenvalue problem,

$$\sum_j\mathcal{L}_{ij}^L c_j=0\,, \tag{4·19}$$

where $\mathcal{L}_{ij}^L$ is defined by

$$\mathcal{L}_{ij}^L\equiv\int\chi_i{}^L(R')\mathcal{L}^L(R',R)\chi_j{}^L(R)\,dR'\,dR \tag{4·20}$$

with

$$\mathcal{L}^L(R',R)\equiv\int Y_{LM}^*(\hat{\boldsymbol{R}}')\mathcal{L}(\boldsymbol{R}',\boldsymbol{R})\,Y_{LM}(\hat{\boldsymbol{R}})\,d\hat{\boldsymbol{R}}'\,d\hat{\boldsymbol{R}}\,. \tag{4·21}$$

The calculation of $\mathcal{L}_{ij}^{L}$ becomes easier by introducing another Gaussian function of the center of mass coordinate $\boldsymbol{R}_G$,

$$\phi_G(\boldsymbol{R}_G)\equiv\left(\frac{6}{\pi b^2}\right)^{3/4}\exp\left(-\frac{3}{b^2}\boldsymbol{R}_G{}^2\right). \tag{4·22}$$

Since our hamiltonian H is independent of $\boldsymbol{R}_G$, one has

$$\begin{aligned}\mathcal{L}_{ij}^{L}=\int&\left[\phi_A{}^{+}(\boldsymbol{\xi}_A)\phi_B{}^{+}(\boldsymbol{\xi}_B)\frac{\chi_i{}^L(R_{AB})}{R_{AB}}Y_{LM}^{*}(\hat{\boldsymbol{R}}_{AB})\right]\phi_G(\boldsymbol{R}_G)(H-E)\\&\times\mathcal{A}\left[\phi_A(\boldsymbol{\xi}_A)\phi_B(\boldsymbol{\xi}_B)\frac{\chi_j{}^L(R_{AB})}{R_{AB}}Y_{LM}(\hat{\boldsymbol{R}}_{AB})\right]\\&\times\phi_G(\boldsymbol{R}_G)d\boldsymbol{\xi}_A d\boldsymbol{\xi}_B d\boldsymbol{R}_{AB}d\boldsymbol{R}_G\,.\end{aligned} \tag{4·23}$$

Noting that the product $\chi_i(\boldsymbol{R}_{AB})\phi_G(\boldsymbol{R}_G)$ can be written as

$$\chi_i(\boldsymbol{R}_{AB})\phi_G(\boldsymbol{R}_G)=\left(\frac{3}{\pi b^2}\right)^{3/2}\exp\left\{-\frac{3}{2b^2}\left[\left(\boldsymbol{R}_A-\frac{\boldsymbol{R}_i}{2}\right)^2+\left(\boldsymbol{R}_B-\frac{\boldsymbol{R}_i}{2}\right)^2\right]\right\} \tag{4·24}$$

and that

$$\phi_A^{SM}\left(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\frac{\boldsymbol{R}_i}{2}\right)\equiv\phi_A(\boldsymbol{\xi}_A)\left(\frac{3}{\pi b^2}\right)^{3/2}\exp\left\{-\frac{3}{2b^2}\left(\boldsymbol{R}_A-\frac{\boldsymbol{R}_i}{2}\right)^2\right\} \tag{4·25}$$

is nothing but the shell model wave function with $(0s)^3$ configuration in the harmonic oscillator potential with its origin at $\boldsymbol{R}_i/2$, the expression (4·23) for $\mathcal{L}_{ij}^{L}$ can be written as

$$\begin{aligned}\mathcal{L}_{ij}^{L}=\mathcal{N}\int&\Psi^{SM\,\dagger}(\boldsymbol{r}_1\cdots\boldsymbol{r}_6;\boldsymbol{R}_i)\,Y_{LM}^{*}(\hat{\boldsymbol{R}}_i)(H-E)\\&\times\Psi^{SM}(\boldsymbol{r}_1\cdots\boldsymbol{r}_6;\boldsymbol{R}_j)\,Y_{LM}(\hat{\boldsymbol{R}}_j)\prod_{k=1}^{6}d\boldsymbol{r}_k d\hat{\boldsymbol{R}}_i d\hat{\boldsymbol{R}}_j\,,\end{aligned} \tag{4·26}$$

where Ψ^{SM} is an antisymmetrized wave function for two clusters A and B situated at $\boldsymbol{R}_i/2$ and $-\boldsymbol{R}_i/2$, respectively, i.e.,

$$\Psi^{SM}(\boldsymbol{r}_1\cdots\boldsymbol{r}_6;\boldsymbol{R}_i)\equiv\mathcal{A}\left[\phi_A^{SM}\left(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\frac{\boldsymbol{R}_i}{2}\right)\phi_B^{SM}\left(\boldsymbol{r}_4,\boldsymbol{r}_5,\boldsymbol{r}_6;-\frac{\boldsymbol{R}_i}{2}\right)\right], \tag{4·27}$$

and $\mathcal{N}$ is the normalization factor. The expression (4·26) is more convenient for the actual calculation, since the antisymmetrization operator $\mathcal{A}$, which gives main cause of difficulty in the calculation of the RGM kernel, is much easier to handle in this form. $\mathcal{L}_{ij}^{L}$ with arbitrary values of R_i and R_j is called the kernel of the generator coordinate method (GCM).[18]

§ 5. Scattering problem

The scattering problems can be solved by a variational method formulated by Kamimura et al.,[19] which is analogous to that for the bound state problem stated in the previous section. The wave function of the relative motion $\chi^L(R)$ is now expanded as

$$\chi^L(R)=\sum_{i=1}^{N} c_i\tilde{\chi}_i{}^L(R), \tag{5·1}$$

$$\tilde{\chi}_i{}^L(R)\equiv\begin{cases}a_i\chi_i{}^L(R) & R\leq R_c\\ (h_L^{(-)}(kR)-S_i h_L^{(+)}(kR))R & R\geq R_c\end{cases} \tag{5·2}$$

with the condition $\sum_{i=1}^N c_i=1$. Here $\chi_i{}^L(R)$ is defined in Eq. (4·18b), $h_L^{(\pm)}$ is the L-th spherical Hankel function, $k\equiv\sqrt{2\mu E_{\rm rel}}$ and a_i and S_i are parameters determined by the smooth continuity condition at $R=R_c$. The matching point R_c must be chosen so that the interaction between the clusters can be safely neglected for $R\geq R_c$. (Recall that we are neglecting the Coulomb interaction.) We define a functional J by[20]

$$J[\chi_L]\equiv S_L+i\frac{\mu}{k}\int\chi^L(R')\mathcal{L}^L(R',R)\chi^L(R)dRdR' \tag{5·3}$$

and

$$S_L\equiv\sum_i c_iS_i\ . \tag{5·4}$$

Making J stationary with respect to the variation of c_i's ($i=1,2,\cdots,N-1$; c_N being eliminated by the condition $\sum_{i=1}^N c_i=1$), one obtains

$$\sum_{j=1}^{N-1}\tilde{\mathcal{L}}^L_{ij}c_j=\mathcal{M}_i{}^L \tag{5·5}$$

with

$$\tilde{\mathcal{L}}^L_{ij}\equiv\mathcal{K}^L_{ij}-\mathcal{K}^L_{iN}-\mathcal{K}^L_{Nj}+\mathcal{K}^L_{NN} \tag{5·6a}$$

and

$$\mathcal{M}_i{}^L\equiv\mathcal{K}^L_{NN}-\mathcal{K}^L_{iN}\ , \tag{5·6b}$$

where

$$\mathcal{K}^L_{ij}\equiv\int\tilde{\chi}_i{}^L(R')\mathcal{L}_L(R',R)\tilde{\chi}_j{}^L(R)dR'dR \tag{5·7}$$

can be easily calculated by using Eq. (4·26). Note that we must modify the outer part of the integral ($R>R_c$) because of the difference between $\tilde{\chi}_i{}^L$ and $\chi_i{}^L$ in this region, which is fairly simple since no exchange term contributes there. With c_i given, S-matrix is given by Eq. (5·4).

Extension to the case of the coupled channels is quite easy. We regard all the kernels as matrices such as $N_{\beta'\beta}(\boldsymbol{R}', \boldsymbol{R})$, where $\beta(\beta')$ indicates the initial (final) channel. The boundary condition (5·2) for the scattering problem must be slightly modified.

The function $\chi(R)$ obtained by the present variational method cannot be interpreted as a usual relative wave function of two baryons, since it has an ambiguity of including the forbidden states, which are eigenstates of the normalization kernel N associated with vanishing eigenvalues. No physical observables can be altered by adding the forbidden states with an arbitrary amplitude to $\chi(R)$. This ambiguity can be eliminated by using the following *renormalized* wave function:

$$\chi_R(\boldsymbol{R}) \equiv \int N^{1/2}(\boldsymbol{R}, \boldsymbol{R}')\chi(\boldsymbol{R}')d\boldsymbol{R}' , \tag{5·8}$$

where $N^{1/2}$ is the square root of the RGM normalization kernel. It can be easily seen that χ_R becomes orthogonal to any forbidden states and satisfies the following ordinary non-local Schrödinger type equation:

$$\int \mathcal{H}(\boldsymbol{R}, \boldsymbol{R}')\chi_R(\boldsymbol{R}')d\boldsymbol{R}' = E\chi_R(\boldsymbol{R}), \tag{5·9}$$

where $\mathcal{H}$ is the renormalized effective hamiltonian defined by

$$\mathcal{H}(\boldsymbol{R}, \boldsymbol{R}') \equiv \int N^{-1/2}(\boldsymbol{R}, \boldsymbol{R}'')H(\boldsymbol{R}'', \boldsymbol{R}''')N^{-1/2}(\boldsymbol{R}''', \boldsymbol{R}')d\boldsymbol{R}''d\boldsymbol{R}''' . \tag{5·10}$$

We may then interpret χ_R as a relative wave function of two baryons.

In order to understand the results of calculation intuitively, let us define the equivalent local potential (EQLP) $V^{EQ}(R)$ as follows:

$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2} + \frac{L(L+1)}{2\mu R^2} + V^{EQ}(R)\right]\chi_R{}^L(R) = E_{\rm rel}\chi_R{}^L(R) \tag{5·11}$$

or

$$V^{EQ}(R) = \frac{1}{2\mu}\left[k^2 - \frac{L(L+1)}{R^2} + \frac{\chi_R^{L''}(R)}{\chi_R{}^L(R)}\right]. \tag{5·12}$$

Note that this local potential, in general, depends on the energy $E_{\rm rel}$.

As has been described in § 3, the quark exchange interaction does not include the effect of the meson exchanges, which are attributed to the interaction of meson cloud in the present model. In order to simulate it, we will add an effective local potential and investigate its influence upon the quark exchange interaction. It will be most appropriate to assume an additional local potential $\mathcal{V}^{\mathrm{EMEP}}(\boldsymbol{R})$, which we call "effective meson exchange potential" (EMEP), in the renormalized effective hamiltonian $\mathcal{H}(\boldsymbol{R}, \boldsymbol{R}')$. The corresponding resonating group method interaction kernel V^{EMEP} is obtained as

$$V^{\mathrm{EMEP}}(\boldsymbol{R}, \boldsymbol{R}') = \int N^{1/2}(\boldsymbol{R}, \boldsymbol{R}'') \mathcal{V}^{\mathrm{EMEP}}(\boldsymbol{R}'') N^{1/2}(\boldsymbol{R}'', \boldsymbol{R}') d\boldsymbol{R}'' . \qquad (5\cdot 13)$$

§ 6. Some comments on the kernels

We discuss here some features of the RGM integral kernels. First, we can express the direct and exchange kernels in terms of diagrams as illustrated in Figs. 1~3. Figures 1 and 2 show the normalization and kinetic energy kernels, respectively. Interaction parts, $V^{(D)}$ and $V^{(EX)}$, are somewhat more complicated and are listed in Figs. 3(a) and 3(b), where wavy lines denote the two-body potential. The second diagram of Fig. 3(a) does not contribute in our case, because the direct matrix element of $(\lambda_i \cdot \lambda_j)$ vanishes when the i-th and the j-th quarks belong to different color singlet baryons. In other words, $V_{\mathrm{rel}}^{(D)}(\boldsymbol{R}) = 0$ in Eq. (4·8) and it corresponds to the fact that isolated hadrons cannot exchange a single gluon from the field theoretical aspects. Therefore, the only interaction is the exchange interaction of two quarks induced by the antisymmetrization.

The second point is that the forbidden state can be explicitly seen as an eigenstate associated with a vanishing eigenvalue of the normalization kernel. A Gaussian function (that is, the $0s$ harmonic oscillator wave function) $\chi_0(\boldsymbol{R})$ given by

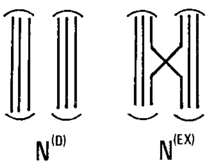

Fig. 1. The normalization kernel.

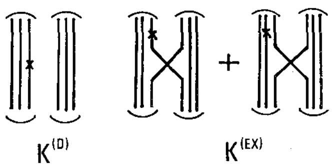

Fig. 2. The kinetic energy kernel. The symbol × denotes the kinetic energy insertion.

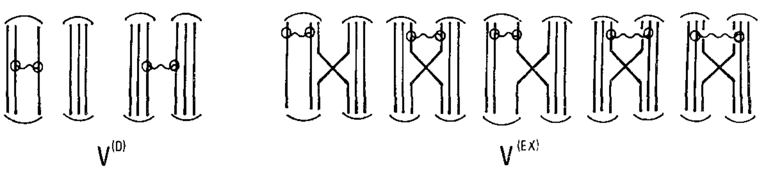

Fig. 3. The direct (a) and the exchage (b) parts of the interaction kernel. The wavy line denotes the two-body potential.

$$\chi_0(\boldsymbol{R})=\left(\frac{3}{2\pi b^2}\right)^{3/4}\exp\left(-\frac{3}{4b^2}\boldsymbol{R}^2\right), \tag{6·1}$$

actually satisfies

$$\int N_{[51]}(\boldsymbol{R}', \boldsymbol{R})\chi_0(\boldsymbol{R})d\boldsymbol{R}=0\,, \tag{6·2}$$

where $N_{[51]}$ denotes the normalization kernel for the [51] symmetric $SU(4)$ channel. It is consistent with the fact that the [6] symmetric $(0s)^6$ orbital configuration in the quark shell model is forbidden for the [51] spin-isospin symmetry as discussed in § 2.

A special simplification appears for the quadratic confinement potential. For this potential, the interaction kernel V is found to be proportional to the normalization kernel N,[21] i.e.,

$$V=V^{(D)}+V^{(EX)}=V_{\rm int}(N^{(D)}+N^{(EX)}). \tag{6·3}$$

For the direct term, the relation holds in general with the potential of the form (3·3), since $V_{\rm rel}^{(D)}=0$ in Eq. (4·8) as mentioned previously. The relation (6·3) for the exchange term implies

$$\begin{aligned}V^{(EX)}&\propto\langle\phi_A\phi_B\delta(\boldsymbol{R}'-\boldsymbol{R}_{AB})|\sum_{i<j}V_{ij}P_{14}|\phi_A\phi_B\delta(\boldsymbol{R}-\boldsymbol{R}_{AB})\rangle\\ &=V_{\rm int}\cdot\langle\phi_A\phi_B\delta(\boldsymbol{R}'-\boldsymbol{R}_{AB})|P_{14}|\phi_A\phi_B\delta(\boldsymbol{R}-\boldsymbol{R}_{AB})\rangle\,,\end{aligned} \tag{6·4}$$

where the bracket denotes the integration over the internal variables and the relative coordinate $\boldsymbol{R}_{AB}$. This can be shown by the observation that the quadratic confinement potential does not affect the relative motion between two color singlet clusters. In fact, the relevant part of the potential can be written as

$$\sum_{i=1}^{3}\sum_{j=4}^{6}(\lambda_i\cdot\lambda_j)(\boldsymbol{r}_i-\boldsymbol{r}_j)^2=\sum_{i=1}^{3}\sum_{j=4}^{6}(\lambda_i\cdot\lambda_j)(\boldsymbol{r}_i-\boldsymbol{R}_A-\boldsymbol{r}_j+\boldsymbol{R}_B+\boldsymbol{R}_{AB})^2 \tag{6·5}$$

and the terms depending on $\boldsymbol{R}_{AB}$ have either the factor $\sum_{i=1}^{3}\lambda_i$ or the factor $\sum_{j=4}^{6}\lambda_j$ or both, which annihilates a color singlet state. Since ϕ_A and ϕ_B are the lowest harmonic oscillator states, the only possible terms in $V^{(EX)}$ other than the term $V_{\rm int}\cdot N^{(EX)}$ are due to that part of the potential $\sum_{i<j}V_{ij}$ which increases the oscillator quanta by 2, when multiplied to the left in Eq. (6·4), which however gives zero when multiplied to the right because it commutes with P_{14}. This fact is remarkable because the part of the interaction kernel which is proportional to the normalization kernel cannot contribute to any scattering processes. This is strictly true only for the quadratic confinement potential, but we shall find a very similar situation for the linear confinement potential through numerical calculations. The special feature of the interaction kernel is very favorable since we

want to get the results least dependent on the effect of the confinement term V^{CONF}.

§ 7. Summary

We have formulated the interaction between non-strange baryons (N and Δ) in a nonrelativistic quark model, using the resonating group method (RGM) which has been developed in the nuclear cluster model calculation. The symmetry properties of six-quark systems, in the spin-isospin $SU(4)$ space and the orbital space are discussed and the relation between the symmetry classification and the particle classification of the two-baryon states are studied.

The norm kernels of the RGM reflect the symmetry properties and, with the harmonic oscillator model for the baryon internal wave functions, the $0s$ state with [51] $SU(4)$ symmetry and the $0p$ state with [6] $SU(4)$ symmetry are identified as the Pauli forbidden states in the relative motion.

The interaction kernels are found to have a remarkable feature of being almost independent of the confinement term V^{CONF} in the quark-quark interaction. This implies that the confinement term contributes very little to the baryon-baryon interaction in the present RGM, which is quite favorable in the light of the remark on potential models of the confinement mentioned in § 1. Numerical results for the relative S state will be given in the subsequent paper.[22]

Acknowledgements

We would like to thank A. Arima, H. Hyuga, M. Ichimura, T. Kamae, L. Kisslinger, T. Matsuse, H. Miyazawa, N. Onishi, R. Tamagaki, T. Terasawa and L. Wilets for useful discussions and comments. This paper is dedicated to the celebration of the sixtieth birthday of Professor H. Narumi.

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