Short-term and medium-term reliability evaluation for power systems with high penetration of wind power

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Short-term and Medium-term Reliability Evaluation for Power Systems with High Penetration of Wind Power

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Abstract — **The expanding share of the fluctuating and less predictable wind power generation can introduce complexities in power system reliability evaluation and management. This entails a need for the system operator to assess the system status more accurately for securing real time balancing. The existing reliability evaluation techniques for power systems are well developed. These techniques are more focused on steady state (time-independent) reliability evaluation and have been successfully applied in power system planning and expansion. In the operational phase, however, they may be too rough an approximation of the time varying behavior of power systems with high penetration of wind power. This paper proposes a time varying reliability assessment technique. Time varying reliability models for wind farms, conventional generating units and rapid start-up generating units are developed and represented as the corresponding universal generating functions (UGF), respectively. A multi-state model for a hybrid generation and reserve provider is also proposed based on the developed UGF representations of wind farms, conventional generating units and rapid start-up generating units. The proposed technique provides a useful tool for the system operator to evaluate the reliability and arrange reserve for maintaining secure system operation in the short as well as medium-terms.**

Index Terms —**Reliability, short-term, medium-term, wind power, random process, universal generating function**

NOMENCLATURE

WTG	wind turbine generator
UGF	universal generating function
MWF	multi-state wind farm
MCGP	multi-state conventional generation provider
MRRP	multi-state rapid start-up reserve provider
MHGR	multi-state hybrid generation and reserve provider
MTP	multi-state transmission provider

I. INTRODUCTION

With the rising costs of conventional energy and public environmental concerns about greenhouse effects, the utilization of renewable energy is rapidly expanding. Unlike other forms of renewable sources, wind farms are able to compete with conventional power plants in terms of both

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capacity ratings and electricity production cost. In the last decade, the global installed wind capacity has increased about 30% per annum. Among developed countries, Denmark is a pioneer in developing wind power generation, where wind power provided 30.0% of electricity production in 2012, as compared to 28.2% in 2011 [1]. In developing countries such as China, wind power accounted for 75 GW of electricity generating capacity in 2012 [2].

The outputs of wind turbine generators (WTG) are determined by wind velocity and availability of WTGs. The fast fluctuation and unpredictable characteristics of wind velocity and random nature of failures of WTGs make the generation output of wind farm stochastic and totally different from that of the conventional generating units [3]. The incorporation of a large number of WTGs into existing electric power systems can therefore bring complexities in reliability evaluation and management.

Several methods for the reliability evaluation of power systems with wind power generation have been developed [3]–[9], in the last two decades. These methods can be classified into the two categories, Monte Carlo simulation approaches [4, 8] and analytical techniques [3, 5, 6, 7, 9]. Monte Carlo simulation approaches are based on accurately modeling the chronological characteristics of wind velocity and time-varying load, hence they provide detailed information on the reliability indices of wind power systems. Monte Carlo simulation approaches usually require longer computational time for reliability evaluation of large electric power systems. On the other hand, analytical techniques are more efficient for evaluating reliability indices and are usually used for power system planning.

The previously cited research works mainly focus on the long term reliability evaluation - they have been successfully applied in power system planning and expansion. The state probabilities of system components, e.g. generating units, vary with time and approaches static (steady-state) values after a long time [10, 11]. Reliability indices such as loss of load probability (*LOLP*) based on the steady-state probabilities of components are constant values (time-independent), which represent system reliability over a long period or as an average over a long period. The high fluctuations of wind power generation are mainly caused by real time changes in weather conditions. The share of the fluctuating and less predictable wind power generation will increase significantly at the global level, and for a country like Denmark this share is expected to reach 50% of the total electricity consumption by 2025. This entails a need for the

system operator to assess the system status accurately and arrange sufficient reserve for securing real time balancing.

Time-independent reliability evaluation techniques may not accurately include the time varying behavior of power systems with high penetration of wind power. The term "time varying" is used here to denote "as a function of time". In operation of power systems, therefore, using these techniques the system reliabilities cannot be evaluated accurately, which is necessary for maintaining secure operation of power systems.

The "short-term" and "medium-term" denoted here are relative compared with the "long term" when the state probabilities of system components approaches static values. The "short-term" in reliability evaluation refers to a few hours (from hour-ahead up to day-ahead), which also correspond to the timeframe of short-term wind power forecast [12] [13]. The system lead time ranging from one hour to a few hours can be considered. However, response time for correcting the frequency error has not been considered and is out of scope of the paper. The "medium-term" here refers to a few days, before the state probabilities of system approach steady-state values [11, 14]. The inaccurate reliability indices could lead to over-scheduling which although more reliable is uneconomic, or to under-scheduling which although less costly to operate can be unreliable [15].

Techniques based on stochastic processes can be used to evaluate the system reliabilities in the short-term or the medium-term. In order to model different wind speeds and component outages, a multi-state system model is used to represent the performance behavior of a power system. However, even for a small power system consisting of a few WTGs and conventional generating units, the number of system states can be relatively large. This number can increase exponentially as the number of components increases [16]. Enormous effort will have to be spent to develop a stochastic process model for a power system with high wind power penetration, and solve it. It is usually a difficult process for the state-space diagram building or the model construction [16], which may cause numerous mistakes even for a relatively small power system. Determining all the system states and transitions correctly is an impractical task which can challenge the available computing resources. If the stochastic process is identified as a K state Markov process, the state probabilities can be obtained only by solving K differential equations either analytically or numerically by matrix multiplication [17].

In order to evaluate the time varying reliabilities of power systems with high wind power penetration, a special technique has been proposed in this paper. The technique is based on the combination of universal generating functions (UGF) and stochastic process methods, which provides a practical solution for the time varying behavior of large system reliabilities [16].The UGF techniques, which have been widely used for multi-state system reliability and performance evaluation [16, 18], can also be used to represent the various reliability models and their correlation in a power system with high wind power penetration [9].

In the proposed technique, only the differential equations for representing Markov models of WTG, conventional generating units and rapid start-up generating units have to be solved in order to obtain the corresponding time varying state probabilities. Then the UGF for wind farms, conventional generating units and rapid start-up generating units are developed to represent their time varying behaviors of reliabilities. A multi-state model for a hybrid generation and reserve provider is also developed by utilizing composition operator over developed UGF representations. Furthermore, customers' time varying reliabilities at each bulk load point are evaluated by considering the impact of the transmission network. The technique is applied to the IEEE Reliability Test System (RTS) [19] to illustrate the validity and benefits of the proposed approach.

II. RELIABILITY MODEL OF GENERATION SYSTEM

A. RELIABILITY MODEL OF A WIND FARM

Electric power output from a WTG is determined by the wind speed, which has great uncertainty due to the random nature of the weather. In the proposed technique, the UGF approach can be combined with different wind speed models such as the well-known autoregressive moving average (ARMA) model and the Markov process model.

The ARMA wind speed model can effectively predict the time series of wind speed from past values alone [12, 20, 21], which can be described as:

$$V^w(t) = \sum_{m=1}^{p} a_m V^w(t-m) + \sum_{n=1}^{q} b_m \varepsilon(t-m) + \varepsilon(t) \tag{1}$$

where $V^w(t)$ is the forecasted wind speed at time t, $V^w(t-m)$ is the wind speed at previous time $(t-m)$, p and a_m are the order and coefficient of the autoregressive part, respectively, q and b_m are the order and coefficient of the moving average part, respectively, $\varepsilon(t)$ is the normal white noise with zero mean and constant variance [20, 21].

For example the ARMA (3, 2) wind speed model for the Toronto location can be described as [12]:

$$V^w(t) = 1.7901 \cdot V^w(t-1) - 0.9087 \cdot V^w(t-2) + 0.0948 \cdot V^w(t-3)$$
$$+ \varepsilon(t) - 1.0929 \cdot \varepsilon(t-1) + 0.2892 \cdot \varepsilon(t-2), \ \varepsilon(t) \in NID(0, 0.474762^2) \tag{2}$$

The corresponding UGF utilizing the ARMA wind speed model of WTG l at time t in the wind farm at bus i can be represented as a simplified form:

$$u_{il}^w(z,t) = z^{w\hat{p}_l(t)} \tag{3}$$

where $w\hat{p}_l(t)$ is the expected power output of WTG l at bus i for time t corresponding to the wind speed prediction at time t utilizing the ARMA model.

The Markov process model of wind speed can also be integrated into the UGF approach, which has been widely used in the reliability evaluation of power systems with wind power penetration [3, 9, 22]. However, the previously research works mainly focus on the steady-state reliability evaluation - they have been successfully applied in power system long-term planning and expansion. In the steady-state reliability evaluation, reliability indices such as loss of load probability (*LOLP*) are independent of the initial state [16] and constant values (time-independent), which represent system reliability over a long-term period. The Markov process model can also

be used to predict the probability distribution of a time varying behavior after considering the initial state [16].

The Markov ~~process~~ model is usually stationary and lacks the memory. However, the Markov process model for predicting the probability distribution of a time varying behavior can provide a better approximation of wind speed than the widely used steady-state analysis in the operational phase. Moreover in some recent research, a second order Markov process model has been used in [13] for short-term wind power forecast, which can consider the current state but also the preceding value.

In the Markov process model for the operational phase, probabilities of future wind states are strongly dependent on transition rates among the current wind state and possible future wind states. The transition rates are determined by the statistical analysis based on the available data for the operation period. For a given operation period, e.g., in the night (wind may increase), if the transition rates from the current state to the high wind speed states are larger than the transition rates from the current state to the low wind speed states, the probability of a wind increase is higher compared to a situation with decreasing winds. Suppose that the current wind speed is in the state j^w and transition rates $\lambda_{j^w,j^w+1} > \lambda_{j^w,j^w-1}$, which represents that the probability of wind speed increase is higher than the wind speed decrease. Similarly if transition rates $\lambda_{j^w,j^w+1} < \lambda_{j^w,j^w-1}$, it is more probable that wind speed will decrease.

In the Markov process model, the wind speed $V^w(t)$ at any time t is assumed as a random variable, which takes values from the set of possible wind speeds $\{v_1,...,v_{K^w}\}$. The state space diagram for the wind speed model is shown in Fig. 1.
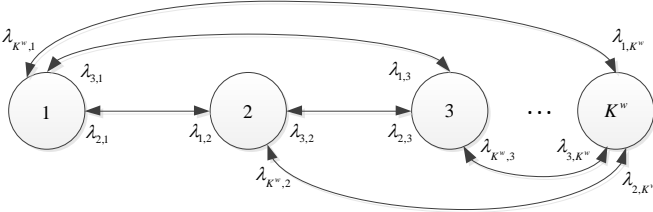

Fig. 1 State space diagram for wind speed model

The Markov process model assumes that the state transition depends only on its present state and is independent of its previous changes [16]. Therefore the state probabilities ($p_{j^w}(t)$, $j^w = 1,...,K^w$) of the wind speed process $V^w(t)$ at future time t following the Markov property can be represented as:

$$p_{j^w}(t) = \Pr\left\{V^w(t) = v_{j^w}\middle|V^w(t_0)\right\}, \qquad j^w = 1,...,K^w \qquad (4)$$

where t_0 indicates the current time, $t_{-1},t_{-2},...$ represent the past time series. It is noticed that $t > t_0 > t_{-1} > t_{-2}...$.

For a Markov process model model, the system of differential equations are used to solve the state probabilities $p_{j^w}(t)$ at time t, $j^w = 1,...,K^w$, given the current state [16]:

$$\frac{dp_{j^w}(t)}{dt} = \left[\sum_{\substack{s^w=1 \\ s^w \neq j^w}}^{K^w} p_{s^w}(t) \cdot \lambda_{s^w,j^w}\right] - p_{j^w}(t)\sum_{\substack{j^w=1 \\ s^w \neq j^w}}^{K^w} \lambda_{j^w,s^w}, \; j^w = 1,...,K^w \; (5)$$

where λ_{s^w,j^w} and λ_{j^w,s^w} are the transition rates from state s^w to state j^w, and from state j^w to state s^w, respectively. By solving the system of differential equations (4) under initial conditions, the numerical solution of $p_{j^w}(t)$, $j^w = 1,...,K^w$ can be obtained. Suppose the present state is s^w, then the initial conditions of differential equations:

$$p_{s^w}(t_0) = 1, \; p_{j^w}(t_0) = 0, \quad j^w = 1,...,K^w, j^w \neq s^w \qquad (6)$$

The following example can be used to illustrate the Markov process model for the operational phase.

Example A. Suppose a simple wind model has 4 states with the wind speed of 3 m/s, 6 m/s, 10 m/s and 15m/s, respectively, as shown in Fig. 2. The state transition rates (occurrences/h) are: $\lambda_{3,4} = 2.959$, $\lambda_{4,3} = 0.067$, $\lambda_{3,2} = 0.093$, $\lambda_{2,3} = 0.045$, $\lambda_{2,1} = 0.365$, and $\lambda_{1,2} = 0.039$.
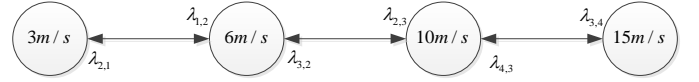

Fig. 2. State space diagram for the simple wind speed model of example A

Suppose the current wind speed is 10 m/s (state 3). After solving the system of differential equations (4) given the initial conditions, it can be found that the probability transferring to high wind speed (15 m/s, state 4) after one hour is 75.1%, which is much higher than the probabilities transferring to low wind speeds: 0.25% in state 1 (3 m/s) and 2.1% in state 2 (6 m/s), and staying in the same wind speed: 22.5% in state 3. Therefore wind speed will probably increase more.

The power output $wp_l(t)$ of WTG l at time t for wind speed $V^w(t)$ is calculated using the following equation [23]:

$$wp_l(t) = \begin{cases} 0 & 0 \leq V^w(t) \leq V_{ci,l}^w \\ \left(A_l + B_l \times V^w(t) + C_l \times V^w(t)^2\right) \cdot P_{r,l} & V_{ci,l}^w \leq V^w(t) \leq V_{r,l}^w \\ P_{r,l} & V_{r,l}^w \leq V^w(t) \leq V_{co,l}^w \\ 0 & V^w(t) \geq V_{co,l}^w \end{cases} \quad (7)$$

where $V_{ci,l}^w$, $V_{r,l}^w$, $V_{co,l}^w$ and $P_{r,l}$ are the cut-in speed, the rated speed, the cut-out speed and the rated power of the WTG unit l respectively. The parameters A_l , B_l and C_l are presented in [23]. $wp_l(t)$ is a random variable at time t, which takes values from $\{wp_{l,1},...wp_{l,j^w}...,wp_{l,K^w}\}$. wp_{l,j^w} is the power output of WTG l for wind state j^w, which can be evaluated by corresponding wind speed v_{j^w} using (7).

The stochastic performance behavior of a WTG in the short-term or the medium-term can be evaluated using the UGF technique. The corresponding UGF to represent the power output distribution of WTG l at time t in the wind farm at bus i can be defined as a polynomial:

$$u_{il}^w(z,t) = \sum_{j_i^w=1}^{K_i^w} p_{j_i^w}(t) \cdot z^{wp_{l,j_i^w}} \qquad (8)$$

where $p_{j_i^w}(t)$ and wp_{l,j_i^w} are the probability at time t and power output of WTG l at bus i for wind speed state j_i^w respectively, and K_i^w is the number of wind speed states of the wind farm at bus i.

A wind farm consists of many installed WTGs. The power output model of a wind farm can be represented as a multi-state wind farm at bus i for time t ($MWF_i(t)$) using UGF equivalent techniques. The power output of the $MWF_i(t)$ can be obtained by summing up the power output of all WTGs at the time:

$$WP_i(t) = \sum_{l=1}^{n_i^w} wp_{li}(t) \qquad (9)$$

where n_i^w is the number of WTGs installed in the farm.

The random variable $WP_i(t)$ takes value from the set $\{WP_{i,1},...WP_{i,j_i^w}...,WP_{i,K_i^w}\}$. WP_{i,j_i^w} is power output of $MWF_i(t)$ corresponding to wind state j_i^w, which can be evaluated as:

$$WP_{i,j_i^w} = \sum_{l=1}^{n_i^w} wp_{l,j_i^w} \qquad (10)$$

The corresponding UGF to represent the power output distribution of $MWF_i(t)$ can be defined as the following polynomial:

$$u_i^W(z,t) = \sum_{j_i^w=1}^{K_i^w} p_{j_i^w}(t) \cdot z^{WP_{i,j_i^w}} = \sum_{j_i^w=1}^{K_i^w} p_{j_i^w}(t) \cdot z^{\sum_{l=1}^{n_i^w} wp_{l,j_i^w}} \qquad (11)$$

The failure of WTGs will also affect the power output of the wind farm. In most research works [3, 9, 20, 22], the binary-state model is used to represent the reliability of a WTG. However some minor failures of the WTG, e.g. wind vane, may lead to situations where the WTG continues to operate, but at reduced performance. Multi-state reliability models for renewable generating units have been introduced in [24].

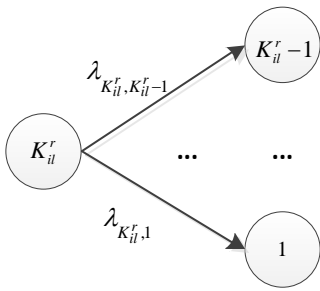The multi-state reliability model for WTG can be illustrated in Fig. 3.

Fig. 3 State space diagram for multi-state reliability model of WTG

In general, the multi-state reliability model of WTG l can have K_{il}^r states, $K_{il}^r \geq 2$. If $K_{il}^r = 2$, the reliability model can be reduced to the classical binary-state model. In this model, repair can be neglected for a relatively short operating period.

The state probabilities $p_{l,K_{il}^r}(t)$, $j_{il}^r = 2,...,K_{il}^r - 1$ can be evaluated by solving the differential equations [16] under the initial conditions, e.g. $p_{l,K_{il}^r}(0) = 1$, $p_{l,j_{il}^r}(0) = 0$, $j_{il}^r = 2,...,K_{il}^r - 1$, which can be represented as:

$$\begin{cases} \dfrac{dp_{l,K_{il}^r}(t)}{dt} = -p_{l,K_{il}^r}(t) \cdot \sum_{j_{il}^r=1}^{K_{il}^r-1} \lambda_{K_{il}^r, j_{il}^r} \\ \dfrac{dp_{l,j_{il}^r}(t)}{dt} = p_{l,K_{il}^r}(t) \cdot \lambda_{K_{il}^r, j_{il}^r}, \quad j_{il}^r = 2,...,K_{il}^r-1 \end{cases} \qquad (12)$$

where $\lambda_{K_{il}^r, j_{il}^r}$ is the failure rate between the state K_{il}^r and the state j_{il}^r.

The UGF of WTG l at time t considering random failures of the unit is defined as:

$$u_{il}^r(z,t) = \sum_{j_{il}^r=1}^{K_{il}^r} p_{l,j_{il}^r}(t) \cdot z^{ws_{l,j_{il}^r}} \qquad (13)$$

ws_{l,j_{il}^r} represents performance indicator of the reliability state j_{il}^r of WTG l at bus i, where $0 \leq ws_{l,j_{il}^r} \leq 1$. $ws_{l,j_{il}^r} = 1$ represents the normal functioning, $ws_{l,j_{il}^r} = 0$ represents the total failure of WTG l and performance is reduced to zero.

The UGF of $MWF_i(t)$ considering the random failures of the WTGs can be obtained using the parallel composition operator $\Omega_{\phi p}$ [16]:

$$u_i^r(z,t) = \Omega_{\phi p}\left\{u_{i1}^r(z,t),...,u_{in_i^w}^r(z,t)\right\}$$
$$= \Omega_{\phi p}\left\{\sum_{j_{i1}^r=1}^{K_{i1}^r} p_{1,j_{i1}^r}(t) \cdot z^{ws_{1,j_{i1}^r}},...,\sum_{j_{in_i^w}^r=1}^{K_{in_i^w}^r} p_{n_i^w, j_{in_i^w}^r}(t) \cdot z^{ws_{n_i^w, j_{in_i^w}^r}}\right\}$$
$$= \sum_{j_{i1}^r=1}^{K_{i1}^r}...\sum_{j_{in_i^w}^r=1}^{K_{in_i^w}^r}\left(\prod_{l=1}^{n_i^w} p_{l,j_{il}^r}(t) \cdot z^{\phi_P(ws_{1,j_{i1}^r},...,ws_{n_i^w, j_{in_i^w}^r})/n_i^w}\right)$$
$$= \sum_{j_i^r=1}^{K_i^r} p_{j_i^r}(t) \cdot z^{WS_{i,j_i^r}} \qquad (14)$$

where $p_{j_i^r}(t)$ and WS_{i,j_i^r} are respectively the probability at time t and performance indicator for the reliability state j_i^r , $j_i^r = 1,...,K_i^r$. $WS_{i,j_i^r} = 1$ represents all the WTGs are operating normally, $WS_{i,j_i^r} = 0$ represents the total failure of $MWF_i(t)$.

The combination of (11) and (14) can be used to obtain the power output distribution of the $MWF_i(t)$ considering both the stochastic variation of wind and the random failures of the WTGs. Applying the multiplication operator $\Omega_{\phi m}$ over $u_i^W(z,t)$ and $u_i^r(z,t)$, we can obtain:

$$u_i^{\hat{W}}(z,t) = \Omega_{\phi s}\left\{u_i^W(z,t),u_i^r(z,t)\right\}$$

$$= \Omega_{\phi m}\left\{\sum_{j_i^w=1}^{K_i^w} p_{j_i^w}(t)\cdot z^{WP_{i,j_i^w}}, \sum_{j_i^r=1}^{K_i^r} p_{j_i^r}(t)\cdot z^{WS_{i,j_i^r}}\right\}$$

$$= \sum_{j_i^w=1}^{K_i^w}\sum_{j_i^r=1}^{K_i^r} p_{j_i^w}(t)\cdot p_{j_i^r}(t)\cdot z^{(WP_{i,j_i^w}\times WS_{i,j_i^r})} \qquad (15)$$

$$= \sum_{j_i^{\hat{W}}=1}^{K_i^{\hat{W}}} p_{j_i^{\hat{W}}}(t)\cdot z^{WP_{i,j_i^{\hat{W}}}}$$

where $p_{j_i^{\hat{W}}}(t)$ and $WP_{i,j_i^{\hat{W}}}$ are the probability at time t and the power output of the wind farm for state $j_i^{\hat{W}}$, which is determined by both the stochastic variation of wind and the random failures of the WTGs. The wind farm has $K_i^{\hat{W}}$ states, which equals to $K_i^w \cdot K_i^r$.

B. RELIABILITY MODEL OF CONVENTIONAL GENERATOR

The basic reliability model for a conventional online generating unit is a binary-state representation [15], where the unit resides either in the perfect functioning state or in the complete failure state. The simple binary-state models for large generating units, however, leads to an accurate decrease and pessimistic appraisals in generating capacity adequacy assessment. Multi-state representations of generating units, which have been used by many utilities [10], provide a more accurate and flexible tool for assessing power system reliabilities than the conventional binary-state models.

A typical example to consider is a coal fired unit with a nominal generating capacity of 576 MW used in the real world [10]. As shown in Fig. 4, the coal fired unit is represented as a four-state Markov model.
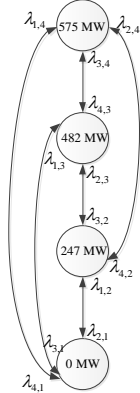

Fig. 4 State space diagram for the coal fired unit

State transitions from the "nominal capacity" state to derated states, including the complete failure state and vice versa, are caused by failures and repairs of the unit. The generating capacity of derated states and state transition rates can be evaluated by the method developed in [10] based on the observed data of the last 5 years.

In general, a conventional online generating unit l can have K_l^g states, $K_l^g \geq 2$. Each unit state j_l^g, $j_l^g = 1,...,K_l^g$, has its available generating capacity GC_{l,j_l^g}. State K_l^g has the nominal generating capacity of the unit. Unit evolution in its state space produces the stochastic capacity process $GC_l(t) \in \{GC_{l,1},...GC_{l,j_l^g}...,GC_{l,K_l^g}\}$ as shown in Fig.5. Let $p_{l,j_l^g}(t)$, $j_l^g = 1,...,K_l^g$ be the state probabilities of the capacity process $GC_l(t)$ at time t of the unit l:

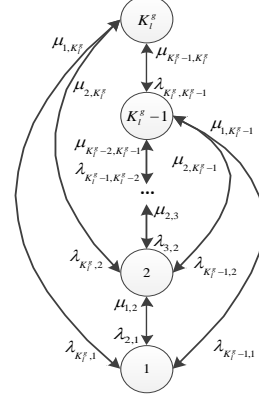$$p_{l,j_l^g}(t) = \Pr\left\{GC_l(t) = GC_{l,j_l^g}\right\}, \ j_l^g = 1,...,K_l^g, t \geq 0 \qquad (16)$$

Fig. 5 State space diagram for online multi-state generating unit

The state probabilities $p_{l,j_l^g}(t)$, $j_l^g = 1,...,K_l^g$ for the homogeneous Markov process $GC_l(t)$ can be evaluated by solving the differential equations [16]. Suppose the state K_l^g is the initial state of the unit l. The corresponding differential equations considering a generating unit model with failures and repairs can be represented as:

$$\begin{cases} \dfrac{dp_{l,K_l^g}(t)}{dt} = \sum_{s_l^g=1}^{K_l^g-1}\mu_{s_l^g,K_l^g}\cdot p_{l,s_l^g}(t) - p_{l,K_l^g}(t)\cdot\sum_{s_l^g=1}^{K_l^g-1}\lambda_{K_l^g,s_l^g} \\[2ex] \dfrac{dp_{l,j_l^g}(t)}{dt} = \left(\sum_{s_l^g=j_l^g+1}^{K_l^g}\lambda_{s_l^g,j_l^g}\cdot p_{l,s_l^g}(t) + \sum_{s_l^g=1}^{j_l^g-1}\mu_{s_l^g,j_l^g}\cdot p_{l,s_l^g}(t)\right) - ... \\[2ex] \qquad (\sum_{s_l^g=j_l^g+1}^{K_l^g}\mu_{j_l^g,s_l^g} + \sum_{s_l^g=1}^{j_l^g-1}\lambda_{j_l^g,s_l^g})\cdot p_{l,j_l^g}(t), \quad j_l^g = 2,...,K_l^g-1 \\[2ex] \dfrac{dp_{l,1}(t)}{dt} = \sum_{s_l^g=2}^{K_l^g}\lambda_{s_l^g,1}\cdot p_{l,s_l^g}(t) - p_{l,1}(t)\cdot\sum_{s_l^g=2}^{K_l^g}\mu_{1,s_l^g} \end{cases} \qquad (17)$$

By solving the differential equations (17) under the initial conditions, e.g. $p_{l,1}(0) = 1$, $p_{l,j_l^g}(0) = 0$, $j_l^g = 2,...,K_l^g$, the state probabilities for the unit l can be obtained.

The UGF representing the capacity distribution of online generating unit l at bus i for time t can be obtained as:

$$u_{il}^g(z,t) = \sum_{j_{il}^g=1}^{K_{il}^g} p_{l,j_{il}^g}(t)\cdot z^{GC_{l,j_{il}^g}} \qquad (18)$$

where $p_{l,j_{il}^g}(t)$ and GC_{l,j_{il}^g} are probability at time t and available generating capacity of unit state j_{il}^g, $j_{il}^g = 1,...,K_{il}^g$, respectively. The UGF (18) corresponds to the stochastic process $GC_l(t) \in \{GC_{l,1},...GC_{l,j_{il}^g}...,GC_{l,K_l^g}\}$, which describes the unit evolution in its state space.

The generation bus i can have multiple conventional online units. The reliability model of these units can be represented as a multi-state conventional generation provider at bus i for time t ($\mathrm{MCGP}_i(t)$) using UGF equivalent techniques. The UGF for a $\mathrm{MCGP}_i(t)$ can be obtained based on the individual UGF of each conventional online unit using the parallel composition operator $\Omega_{\phi p}$ [16]:

$$
\begin{aligned}
u_i^g(z,t) &= \Omega_{\phi p}\left\{u_{i1}^g(z,t),...,u_{i,n_i^g}^g(z,t)\right\} \\
&= \Omega_{\phi p}\left\{\sum_{j_{i1}=1}^{K_{i1}^g} p_{1,j_{i1}}(t)\cdot z^{GC_{1,j_{i1}}},...,\sum_{j_{i,n_i^g}=1}^{K_{i,n_i^g}^g} p_{n_i^g,j}(t)\cdot z^{GC_{n_i^g,j_{i,n_i^g}}}\right\} \\
&= \sum_{j_{i1}=1}^{K_{i1}^g}...\sum_{j_{i,n_i^g}=1}^{K_{i,n_i^g}^g}(\prod_{l=1}^{n_i^g} p_{l,j_{il}^g}(t)\cdot z^{\phi_P(GC_{1,j_{i1}},...,GC_{n_i^g,j_{i,n_i^g}})}) \\
&= \sum_{j_i^G=1}^{K_i^G} p_{j_i^G}(t)\cdot z^{AG_{i,j_i^G}}
\end{aligned}
\tag{19}
$$

where $p_{j_i^G}(t)$ and AG_{i,j_i^G} are respectively the probability at time t and total available capacity of conventional generators at bus i for the state j_i^G, $j_i^G=1,...,K_i^G$.

The structure function, which defines the parallel composition operator $\Omega_{\phi p}$ by using (19), takes the form:

$$
AG_{i,j_i^G}=\phi_p(GC_{1,j_{i1}},...,GC_{n_i^g,j_{i,n_i^g}})=\sum_{l=1}^{n_i^g}GC_{l,j_{il}^g}
\tag{20}
$$

The above model assumes that all conventional generating units considered are online. A more general model of AG_{i,j_i^G} considering commitment state of generating units can be represented as:

$$
AG_{i,j_i^G}=\sum_{l=1}^{n_i^g}GC_{l,j_{il}^g}\cdot\zeta_{i,l}
\tag{21}
$$

where $\zeta_{i,l}$ represents the commitment state of the unit l, where 1 means on and 0 means off. The unit commitment (UC) is used to determine the commitment state of generating units [25].

C. RELIABILITY MODEL OF RAPID START-UP GENERATOR

The rapid start-up generating units such as gas and hydro turbines require a very short lead time to start, synchronize and carry load [15], thus providing operating reserve. Usually the rapid start-up units are only deployed when they are needed. The frequent start-up results in extra starting stress of generating units. The reliability model of the online generating units does not consider, however, the impact of start–up failures of units.

Four-state reliability models for rapid start-up or peaking generating units have been proposed in [15], which considers the ready-for-service (reserve shutdown) state and start-up failures. For operational reliability analysis, the four-state model can be reduced to a three-state model [26] which is illustrated in Fig. 6.
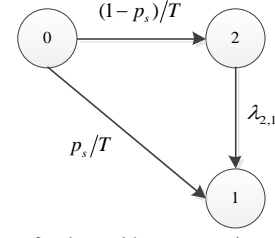

Fig. 6 State space diagram for the rapid start-up unit

As shown in Fig. 6, a rapid start-up generating unit has three states: State 0 represents the ready-for-service state; state 2 represents the in service state; state 1 represents the failure state. Once committed, a unit initially in the ready-for-service state (state 0) can either start up successfully and transit to the in-service state (state 2) at a rate $(1-p_s)/T$ or fail to start-up and goes into the failure state (state 1) at a rate p_s/T. T, p_s and $\lambda_{2,1}$ represent the average reserve shut-down time between periods of need, the probability of a start-up failure and transition and failure rates from state 2 to state 0, respectively [26]. In this reduced model, the repair can be neglected for a short operating period.

The state probabilities of rapid start-up unit l at time t can be obtained as in reference [26]:

$$
\begin{cases}
p_{l,2}(t)=1+\dfrac{\lambda_{2,1}T-p_s}{1-\lambda_{2,1}T}e^{-t/T}+\dfrac{1-p_s}{\lambda_{2,1}T-1}e^{-\lambda_{2,1}t} \\[2mm]
p_{l,1}(t)=\dfrac{1-p_s}{1-\lambda_{2,1}T}\left(e^{-\lambda_{2,1}t}-e^{-t/T}\right) \\[2mm]
p_{l,0}(t)=e^{-t/T}
\end{cases}
\tag{22}
$$

Suppose the commitment decision of rapid start-up unit l is made at t_l^0 and the start-up time (lead time) is LT_l. Before the time $t_l^0+LT_l$, the unit l will reside in the ready-for-service state with a probability of unity and it does not contribute to system generation. It is also assumed that the committed unit l in an operational period will not be sent back to the ready-for-service state. In other words, once a unit is committed it can either start up successfully or undergo a forced outage [26]. After the time $t_l^0+LT_l$, the probabilities of states 1 and 2 are represented as:

$$
p_{l,2}(t)=e^{-\lambda_{2,1}(t-t_l^0+LT_l)}-p_s\cdot e^{-\lambda_{2,1}(t-t_l^0+LT_l)}
\tag{23}
$$

$$
p_{l,1}(t)=1-e^{-\lambda_{2,1}(t-t_l^0+LT_l)}+p_s\cdot e^{-\lambda_{2,1}(t-t_l^0+LT_l)}
\tag{24}
$$

The UGF of rapid start-up generating unit l at bus i for time t can be obtained as:

$$
u_{il}^f(z,t)=A_{il}^f(t)\cdot z^{RC_{il}}+UA_{il}^f(t)\cdot z^0
\tag{25}
$$

where $A_{il}^f(t)$ and $UA_{il}^f(t)$ are the availability and unavailability respectively of the rapid start-up unit l for time t. The available capacity of the rapid start-up unit l at bus i is RC_{il}.

Before the time $t_l^0+LT_l$, the availability and unavailability of the rapid start-up unit l are 0 and 1, respectively. After the time $t_l^0+LT_l$, the probabilities of states 1 and 2 of the rapid start-up unit l represents the availability and unavailability of

the rapid start-up unit l for time t respectively, which can be evaluated by equations (23) and (24), respectively.

The generation bus i can have multiple rapid start-up units providing reserve for time t. The reliability model of these units can be represented as a multi-state rapid start-up reserve provider at bus i for time t ($\mathrm{MRRP}_i(t)$). The UGF for the $\mathrm{MRRP}_i(t)$ can be obtained by utilizing parallel operator $\Omega_{\phi p}$ over UGF representations of n_i^f units:

$$
\begin{aligned}
u_i^f(z,t) &= \Omega_{\phi p}\left\{u_{i1}^f(z,t),...,u_{i,n_i^f}^f(z,t)\right\} \\
&= \Omega_{\phi p}\left\{A_{i1}^f(t)\cdot z^{RC_{i1}} + UA_{i1}^f(t)\cdot z^0,...,A_{i,n_i^f}^f(t)\cdot z^{RC_{i,n_i^f}} + UA_{i,n_i^f}^f(t)\cdot z^0\right\} \\
&= \prod_{l=1}^{n_i^f}A_{il}^f(t)\cdot z^{\sum_{i=1}^{n_i^f}RC_{il}} + \prod_{l=1}^{n_i^f-1}A_{il}^f(t)\cdot UA_{il}^f(t)\cdot z^{\sum_{i=1}^{n_i^f-1}RC_{il}} +...+ \prod_{l=1}^{n_i^f}UA_{il}^f(t)\cdot z^0 \\
&= \sum_{j_i^f=1}^{K_i^f}p_{j_i^f}(t)\cdot z^{AC_{i,j_i^f}}
\end{aligned}
\tag{26}
$$

where $p_{j_i^f}(t)$ and AC_{i,j_i^f} are respectively the probability at time t and available capacity of the $\mathrm{MRRP}_i(t)$ for the state j_i^f, $j_i^f = 1,...,K_i^f$.

D. Reliability Model of Hybrid Generation and Reserve Provider

The combination of the $\mathrm{MWF}_i(t)$, the $\mathrm{MCGP}_i(t)$ and the $\mathrm{MRRP}_i(t)$ can be represented as a multi-state hybrid generation and reserve provider at bus i for time t ($\mathrm{MHGR}_i(t)$), as illustrated in Fig. 7.
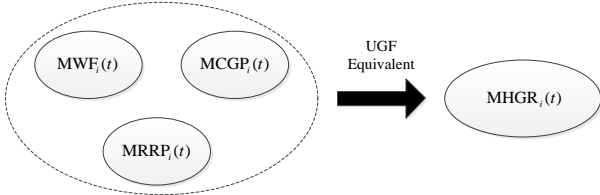

Fig. 7 Reliability model of a hybrid generation and reserve provider

The UGF for the $\mathrm{MHGR}_i(t)$ can be obtained by utilizing parallel operator $\Omega_{\phi p}$ over UGF representations of the $\mathrm{MWF}_i(t)$, the $\mathrm{MCGP}_i(t)$ and the $\mathrm{MRRP}_i(t)$:

$$
\begin{aligned}
u_i^G(z,t) &= \Omega_{\phi p}\left\{u_i^{\hat{W}}(z,t),u_i^g(z,t),u_i^f(z,t)\right\} \\
&= \Omega_{\phi p}\left\{\sum_{j_i^{\hat{W}}=1}^{K_i^{\hat{W}}}p_{j_i^{\hat{W}}}(t)\cdot z^{WP_{i,j_i^{\hat{W}}}}, \sum_{j_i^G=1}^{K_i^G}p_{j_i^G}(t)\cdot z^{AG_{i,j_i^G}}, \sum_{j_i^f=1}^{K_i^f}p_{j_i^f}(t)\cdot z^{AC_{i,j_i^f}}\right\} \\
&= \sum_{j_i^{\hat{W}}=1}^{K_i^{\hat{W}}}\sum_{j_i^G=1}^{K_i^G}\sum_{j_i^f=1}^{K_i^f}p_{j_i^{\hat{W}}}\cdot p_{j_i^G}\cdot p_{j_i^f}(t)\cdot z^{\phi_p(WP_{i,j_i^{\hat{W}}},AG_{i,j_i^G},AC_{i,j_i^f})} \\
&= \sum_{j_i=1}^{K_i}p_{j_i}(t)\cdot z^{AG_{i,j_i}^*}
\end{aligned}
\tag{27}
$$

where $p_{j_i}(t)$ and AG_{i,j_i}^* are respectively the probability at time t and available generating capacity of $\mathrm{MHGR}_i(t)$ for the state j_i, $j_i = 1,...,K_i$.

The structure function, which defines the parallel composition operator $\Omega_{\phi p}$ by using (27), takes the form:

$$
AG_{i,j_i}^* = \phi_p(WP_{i,j_i^{\hat{W}}},AG_{i,j_i^G},AC_{i,j_i^f}) = WP_{i,j_i^{\hat{W}}} + AG_{i,j_i^G} + AC_{i,j_i^f} \tag{28}
$$

After obtaining the UGF for the $\mathrm{MHGR}_i(t)$, the UGF for the generation system can be obtained using the parallel composition operator $\Omega_{\phi p}$ [16]:

$$
\begin{aligned}
u^{GS}(z,t) &= \Omega_{\phi p}\left\{u_1^G(z,t),...,u_N^G(z,t)\right\} \\
&= \Omega_{\phi p}\left\{\sum_{j_1=1}^{K_1}p_{j_1}(t)\cdot z^{AG_{1,j_1}^*},...,\sum_{j_N=1}^{K_N}p_{j_N}(t)\cdot z^{AG_{i,j_N}^*}\right\} \\
&= \sum_{j_1=1}^{K_1}...\sum_{j_N=1}^{K_N}(\prod_{i=1}^{N}p_{j_i}(t)\cdot z^{\phi_P(AG_{i,j_1}^*,...,AG_{i,j_N}^*)}) \\
&= \sum_{j_G=1}^{K_G}p_{j_G}(t)\cdot z^{AG_{j_G}}
\end{aligned}
\tag{29}
$$

After obtaining the $u^{GS}(z,t)$, reliability indexes such as the expected energy not supplied caused by the generation system during the operation period T ($EENS_G(T)$) can be evaluated as:

$$
EENS_G(T) = \int_0^T\left(\sum_{j_i=1}^{K_i}p_{j_i}(t)\cdot\left(TD(t)-AG_{j_G}\right)\cdot\omega_{j_G}(t)\right)\cdot dt \tag{30}
$$

where $TD(t)$ is the total system demand at time t, $\omega_{j_G}(t)$ is a binary variable, which indicates the presence or absence of any loss of load for generation state j_G .

The following linear inequalities can be used to determine the $\omega_{j_i}(t)$ [25]:

$$
1 - \frac{TD(t)-AG_{j_G}}{IC} \le \omega_{j_i}(t) \le 1 + \frac{TD(t)-AG_{j_G}}{IC} \tag{31}
$$

where IC is the installed generation system capacity. If $\omega_{j_G}(t)$ takes the value 1, there exists loss of load for generation state j_G, otherwise it is equal to zero.

An innovative reliability-constrained UC framework considering the spinning reserve and interruptible load has been developed in [25]. In [25], $EENS$ has been used as the reliability criterion in UC. Similarly $EENS_G(T) \le EENS^{\max}$ and linear inequalities (31) can be integrated into the framework of [25] for determining the UC with penetration of wind power.

III. System Reliability Model Considering the Impact of Transmission Network

Failures of transmission network components can result in line congestion, bulk load point (BLP) isolation and customer interruptions. The impact of wind power integration in bulk power system reliability analysis has been conducted in [27]. For absorbing a significant amount of wind capacity,

transmission reinforcement planning based on reliability analysis has been studied [27]. A comprehensive procedure for evaluating locational value of a wind farm by considering transmission system constrains and losses, load delivery point interruption cost and operating cost of generating units has been proposed in [22].

The reliability model of the transmission network can be represented as a multi-state transmission provider for time t $(MTP(t))$. As shown in Fig. 8, the $MHGR_i(t)$ delivers electricity through $MTP(t)$ to customers at a BLP.
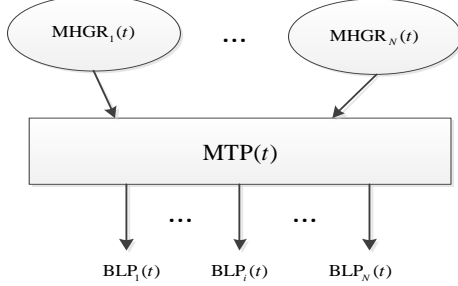

Fig. 8 System reliability modeling considering the impact of transmission network

After obtaining $u_i^G(z,t)$, the system UGF is developed for determining the distribution of load curtailment at each BLP for time t. The effect of $MTP(t)$ on the reliabilities of BLPs is also considered in the system UGF. For an N-bus system with K system states, the system UGF can be obtained by the OPF composition operator $\Omega_{\Phi OPF}$:

$$u_i^{LC}(z,t) = \Omega_{\phi OPF}\left\{u_1^G(z,t),...,u_N^G(z,t)\right\}$$
$$= \sum_{i=1}^{N}\sum_{j_i=1}^{K_i}\sum_{j_L=1}^{K_L} p_{j_i}(t)\cdot p_{j_L}(t)\cdot z^{\phi_{OPF}\left\{AG_{1,j_1}^*,...,AG_{N,j_N}^*\right\}} \quad (32)$$
$$= \sum_{j=1}^{K} p_j(t)\cdot z^{LC_{j_i}(t)}$$

where $p_j(t)$ and $LC_{j_i}(t)$ are the probability and load curtailment at BLP i for the system state j at time t, respectively. $p_{j_L}(t)$ is the probability of the transmission network state j_L at time t. It is supposed that the transmission network has K_L states.

The OPF operator used in (32) is defined as a linear optimization model for determining the load curtailment at each BLP for the system state j at time t, which is described by (33) – (37). The optimization objective is to minimize the total system load curtailment for the system state j at time t:

$$Min \quad f_j = \sum_{i=1}^{N} LC_{j_i}(t) \quad (33)$$

subject to the following constraints:

DC power flow constraints:
$$\mathbf{B_j}\boldsymbol{\theta_j}(t) = \mathbf{P_j}(t) - \mathbf{D_j}(t) \quad (34)$$

Load curtailment constraints:
$$\mathbf{LC_j}(t) = \bar{\mathbf{D}}(t) - \mathbf{D_j}(t) \quad (35)$$

Generation output limits:

$$0 \le p_{j_i}(t) \le AG_{i,j_i}^* \quad (36)$$

Line flow constraints:

$$\left|\frac{1}{x_{j_{ik}}}\left(\theta_{j_i}(t) - \theta_{j_k}(t)\right)\right| \le \left|F_{ik}^{max}\right| \quad (37)$$

where $\mathbf{B_j}$ is the admittance matrix of transmission network, $\boldsymbol{\theta_j}(t)$ is phase angle vector of bus voltages at time t, $\mathbf{P_j}(t) = [p_{j_1}(t),...,p_{j_N}(t)]^T$ is the vector of bus power generations for the state j at time t, $\mathbf{D_j}(t) = [D_{1,j}(t),...,D_{N,j}(t)]^T$ and $\bar{\mathbf{D}}(t) = [\bar{D}_1(t),...,\bar{D}_N(t)]^T$ represent the vector of the bus loads for the state j at time t and the vector of the bus loads for the normal state at time t, respectively. $\mathbf{LC_j}(t) = [LC_{1,j}(t),...,LC_{N,j}(t)]^T$ is the vector of load curtailment for the state j at time t. $p_{j_i}(t)$ is power generation of the $MHGR_i(t)$ and $\theta_{j_i}(t)$ is the phase angle of voltage at bus j at time t, $x_{j_{ik}}$ and $\left|F_{ik}^{max}\right|$ are the reactance and maximum power flow of the line between buses i and k respectively.

IV. RELIABILITY EVALUATION PROCEDURES

A. Reliability indices

The conventional reliability indexes such as loss of load probability at BLP i ($LOLP_i$), and expected energy not supplied at BLP i ($EENS_i$) are usually used to assess long-term (steady-state) reliabilities of customers at different buses. These indexes have been re-defined to evaluate the short-term and medium-term reliabilities of customers. $LOLP_i(t)$ is defined as the loss of probability at BLP i for time t, which can be evaluated as:

$$LOLP_i(t) = \sum_{j=1}^{K} p_j(t)\mathbf{I}(LC_{j_i}(t) > 0), \quad (38)$$

where $\mathbf{1}(True) \equiv 1, \quad \mathbf{1}(False) \equiv 0$.

$EENS_i(T)$ is defined as the expected energy not supplied at BLP i during the operation period T, which can be evaluated as:

$$EENS_i(T) = \int_0^T\left(\sum_{j=1}^{K} p_j(t)\cdot LC_{j_i}(t)\right)\cdot dt \quad (39)$$

After obtaining $LOLP_i(t)$ and $EENS_i(T)$, system $LOLP(t)$ and $EENS(T)$ can be evaluated by the following equations:

$$LOLP(t) = \sum_{j=1}^{K} p_j(t)\mathbf{I}\left(\sum_{i=1}^{N} LC_{j_i}(t) > 0\right) \quad (40)$$

$$EENS(T) = \int_0^T\left(\sum_{j=1}^{K} p_j(t)\cdot\left(\sum_{i=1}^{N} LC_{j_i}(t)\right)\right)\cdot dt \quad (41)$$

B. Computation Procedure for Reliability Evaluation

The basic procedures for the time varying reliability assessment of power systems are as follows:

Step1: Obtain the UGFs representing the power output distribution of WTGs at time t using equation (3) or (8).

Step2: Determine the UGF for the $\mathrm{MWF}_i(t)$ considering both the stochastic variation of wind and the random failures of the WTGs using (14).

Step3: Build the Markov process models for online generating units. Calculate the state probabilities of units at time t by solving differential equations represented by (17).

Step4: Obtain the UGFs representing the capacity distribution of online generating units at time t using equation (18).

Step5: Determine the UGF for the $\mathrm{MCGP}_i(t)$ based on the individual UGF of each online generating unit using equation (19).

Step6: Build the Markov process models for rapid start-up generating units. Calculate the state probabilities of units at time t by solving equation (22).

Step7: Obtain the UGFs of rapid start-up generating units at time t using equation (25).

Step8: Determine the UGF for the $\mathrm{MRRP}_i(t)$ based on the individual UGF of each rapid start-up generating units using equation (26).

Step9: Determine the UGF for the $\mathrm{MHGR}_i(t)$ from (27) by combining the UGFs of the $\mathrm{MWF}_i(t)$, the $\mathrm{MCGP}_i(t)$ and the $\mathrm{MRRP}_i(t)$.

Step10: Obtain the system UGF for determining the distribution of load curtailment at each BLP for time t. The OPF model developed in (32) – (37) is used to evaluate the load curtailment at each BLP for system state j at time t.

Step11: Calculate the $LOLP_i(t)$, $EENS_i(T)$, $LOLP(t)$ and $EENS(T)$ using (38) - (41), respectively.

V. SYSTEM STUDIES

The IEEE-RTS [19] has been modified to illustrate the proposed models and techniques: A large wind farm with 250 V-80 WTGs is added at bus 21. The rated power of a WTG is 2 MW. The total wind capacity is 500 MW. The cut-in, rated, and cut-out wind speeds of a V-80 WTG are 4, 15 and 25 km/h, respectively. The Markov model for the output power of a single WTG proposed in [3] is used for the studies. The MTTF and MTTR of a WTG are assumed to be 3650 hrs and 55 hrs, respectively. The online conventional generators consist of four 576-MW coal thermal generators and three 197-MW oil thermal generators. The 576-MW coal thermal generators are real generators used in industry [10], which are represented using the four-state Markov model. The four coal thermal generators are located at buses 15, 16, 18 and 23. The three oil thermal generators are represented as binary Markov models [15], and installed at bus 13. There are five 40-MW gas thermal generators working as rapid start-up units, which are located at bus 1 (three units) and bus 2 (two units), respectively. The system and customer reliabilities are evaluated for two cases.

Case 1.

In case 1, we assume that the wind speed at the wind farm is 16 km/h and WTGs are generating rated power at time $t=0$. All the generating units are in good condition at the beginning of the operating time. Three scenarios are considered in case 1. Fig. 9 and Fig. 10 illustrate the *LOLP* for a representative

load bus – bus 6 considering three scenarios from $t=0$ to $100h$ and from $t=0$ to $10h$, respectively.

Scenario A is the base case one without considering the commitment of rapid start-up units. It can be seen from Fig. 9 that the instant *LOLP* at a BLP is time variable instead of being constant: In scenario A, the instant *LOLP* increases from 0 at $t=0$ to 0.0717 at $t=100h$, which is a relatively large value.

For increasing customers' reliability~~ies~~, rapid start-up units are committed for providing reserves in scenarios B and C. In scenario B, rapid start-up units are committed for operation at $t=5h$, which immediately decreases the *LOLP*. *LOLP* at $t=100h$ is 0.01221 in scenario B, which decreases about 83.0% compared with that in scenario A. In scenario C, rapid start-up units are committed for operation at $t=10h$, which also decreases the *LOLP* from 0.01004 at $t=9h$ to 0.00295 at $t=10h$. The *EENS* for $T=100h$ are 35.6 MWh, 7.4 MWh, and 7.6 MWh for scenarios A, B and C in case 1, respectively. Clearly, it can be appreciated that early commitment of reserve units can increase customers' reliability~~ies~~ but may also lead to higher reserve cost.
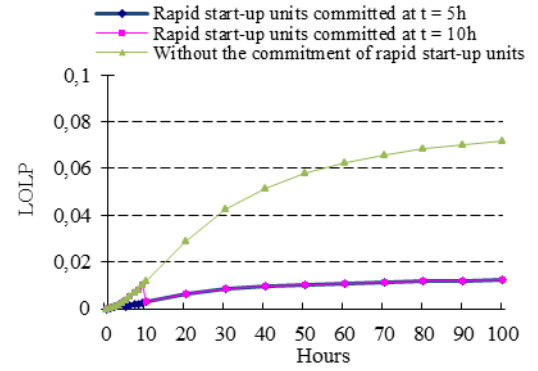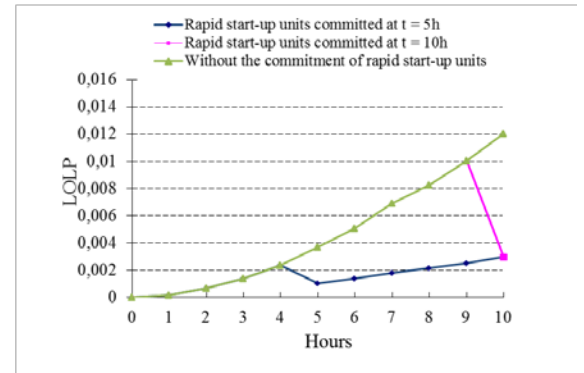

Fig. 9 Instant LOLP at bu6 for case 1 from $t=0$ to $100h$

Fig. 10 Instant LOLP at bu6 for case 1 from $t=0$ to $10h$

The system *LOLP* at time t for the three scenarios is illustrated in Table 1. For validating the accuracy of the proposed method, the Monte Carlo simulation (MCS) approach was also developed to compare the results obtained by the proposed method, which is also shown in Table 3. It can be observed from Table 1 that the proposed method has high computational accuracy: the average percentage error of the proposed method and MCS is relatively low at 1.38%.

Table 1
Instant LOLP for the system for case 1

Time (h)	Scenario A		Scenario B		Scenario C	
	Proposed method	MCS	Proposed method	MCS	Proposed method	MCS
20	0.0299	0.0304	0.0071	0.0071	0.0071	0.0071
40	0.0529	0.0511	0.0109	0.0109	0.0109	0.0108
60	0.0639	0.0615	0.0124	0.0123	0.0124	0.0123
80	0.0698	0.0685	0.0132	0.0130	0.0132	0.0130
100	0.0732	0.0733	0.0137	0.0134	0.0137	0.0134

A 2.67 GHz Fujitsu laptop was used in the simulation studies. The simulation codes were written in C. The computational time of the proposed method and MCS for case 1 is illustrated and compared in Table 2.

The average computational time of MCS took 4.33 times more than that of the proposed approach for obtaining the results.

Table 2
Computational time for case 1(seconds)

	Proposed method	MCS
Scenario A	347.1	1450.1
Scenario B	403.2	1821.9
Scenario C	351.1	1509.8

Steady state reliability analysis has been conducted in most existing research. For comparative purposes, the system *LOLP* for steady state for case 1 was also evaluated as 0.0810. Comparing the results in Table 1, this constant value cannot accurately represent the time varying reliabilities during power system operation and catch the operating behaviors of different scenarios.

Case 2.

In case 2, we assume that the wind speed at the wind farm is 3 km/h (below cut-in speed) and the power outputs of WTGs are zero at time $t=0$. Other conditions are the same as those in case 1. Three scenarios are also considered in case 2. The instant *LOLP* from $t=0$ to $100h$ and from $t=0$ to $10h,$ for bus 6 in case 2 is shown in Figs. 11 and 12, respectively.

Comparing Fig. 10 with Fig. 12, it can be observed that a change of system operating conditions may lead to a large variation of customer reliabilities. For example, in scenario A of case 2 the instant *LOLP* at $t=10h$ is 0.02998, which increases about 148.7% comparing with that in case 1.

For the system operator, it is more important to commit rapid start-up units earlier for maintaining customer reliabilities in case 2. The *EENS* for bus 6 for $T=100h$ are 39.2 MWh, 8.5 MWh, and 9.3 MWh for scenarios A, B and C in case 2, respectively.
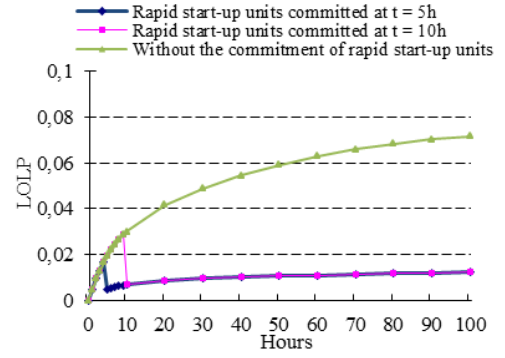

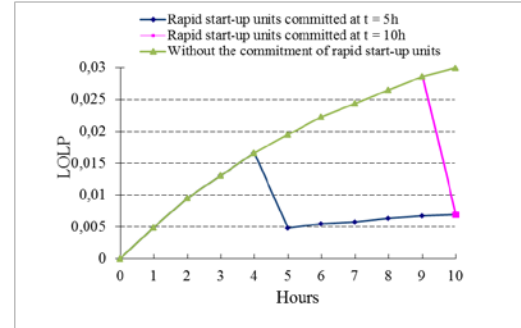Fig. 11 Instant LOLP at bu6 for case 2 from $t=0$ to $100h$

Fig. 12 Instant LOLP at bu6 for case 2 from $t=0$ to $10h$

The system *LOLP* at time t for the three scenarios of case 2 is illustrated in Table 3. The obtained results of the proposed method are also compared with those from the MCS. From Table 3, we observe that the proposed method has high computational accuracy: the average percentage error of the proposed method and MCS is about 1.78% in case 2.

Table 3
Instant LOLP for the system for case 2

Time (h)	Scenario A		Scenario B		Scenario C	
	Proposed method	MCS	Proposed method	MCS	Proposed method	MCS
20	0.0429	0.0425	0.0099	0.0094	0.0099	0.0095
40	0.0560	0.0567	0.0115	0.0112	0.0115	0.0112
60	0.0645	0.0657	0.0125	0.0123	0.0125	0.0123
80	0.0699	0.0710	0.0132	0.0132	0.0132	0.0131
100	0.0732	0.0742	0.0137	0.0136	0.0137	0.0136

The computational time of the proposed method and MCS for case 2 is illustrated and compared in Table 4. The average computational time of MCS took 5.17 times more than that of the proposed approach.

The system *LOLP* for steady state for case 2 was also evaluated as 0.0810. This value is the same as that for case 1 because the steady state reliability analysis does not consider the initial conditions of operating states, which is more important for short-term and medium-term reliability assessment.

Table 4
Computational time for case 2 (sec)

	Proposed method	MCS
Scenario A	306.2	1517.4
Scenario B	320.7	1702.1
Scenario C	307.9	1619.5

VI. CONCLUSIONS

Conventional methods for steady-state reliability evaluation have been well developed. However these methods cannot consider the time varying behavior of system operation, which becomes more critical for a power system with high fluctuation of wind power. This paper proposes a method for assessing short-term and medium-term reliabilities for power systems.

The proposed reliability evaluation can be conducted in the UC interval for estimating the risk of scheduled UC. It is possible to integrate the proposed reliability evaluation method into the framework of [25] for determining the UC with penetration of wind power. The proposed reliability evaluation can also be implemented in the interval of economical dispatch or OPF considering transmission network for the operational hour. Moreover, the system operator can conduct analysis in a few days' interval for the "medium-term" reliability assessment. However, shorter operation interval correlated with the frequency disturbance has not been considered and is out of scope of the paper.

The obtained information can be used to assist the system operator in making optimal decisions for maintaining reliability and security for a power system with high penetration of wind power. Comparing with the MCS approach, the proposed method is much faster.

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