Sign Change Fault Attacks On Elliptic Curve Cryptosystems

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Content

Fault Model

- Sign Change Faults

Attack

- on Elliptic Curve Scalar Multiplication

Countermeasure

- against Sign Change Attacks

Elliptic Curves: Notation

- Set of points $(x : y : z)$ satisfying

$$\Rightarrow E_p : y^2 z \equiv x^3 + Axz^2 + Bz^3 \bmod p \qquad (1)$$

(Weierstraß-Equation in projective coordinates)

- We only consider E defined over $\mathbb{F}_p$, p prime
- Group of points: All points satisfying (1)
  - $(x : y : z) = (\lambda x : \lambda y : \lambda z)$ for $\lambda \neq 0$
  - $\mathcal{O} = (0 : 1 : 0) \longrightarrow$ “point at infinity“

Previous Work

Approaches:

- Analysis of faults in curve and field parameters (Biehl, Meyer, Müller 2000 & Ciet, Joye 2003)

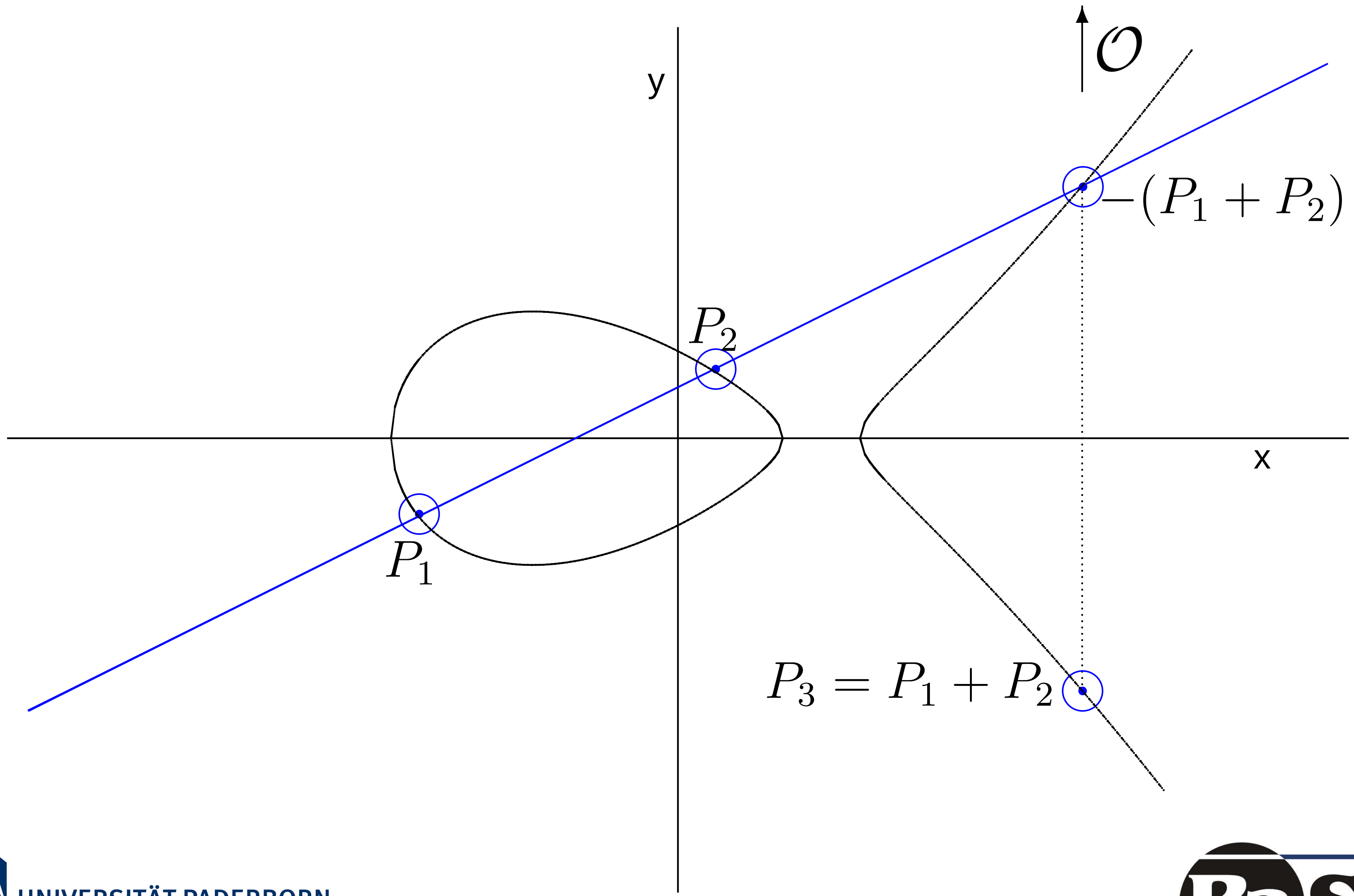
Results:

- With overwhelming probability, a (random) fault results in a final result that is not on the curve.

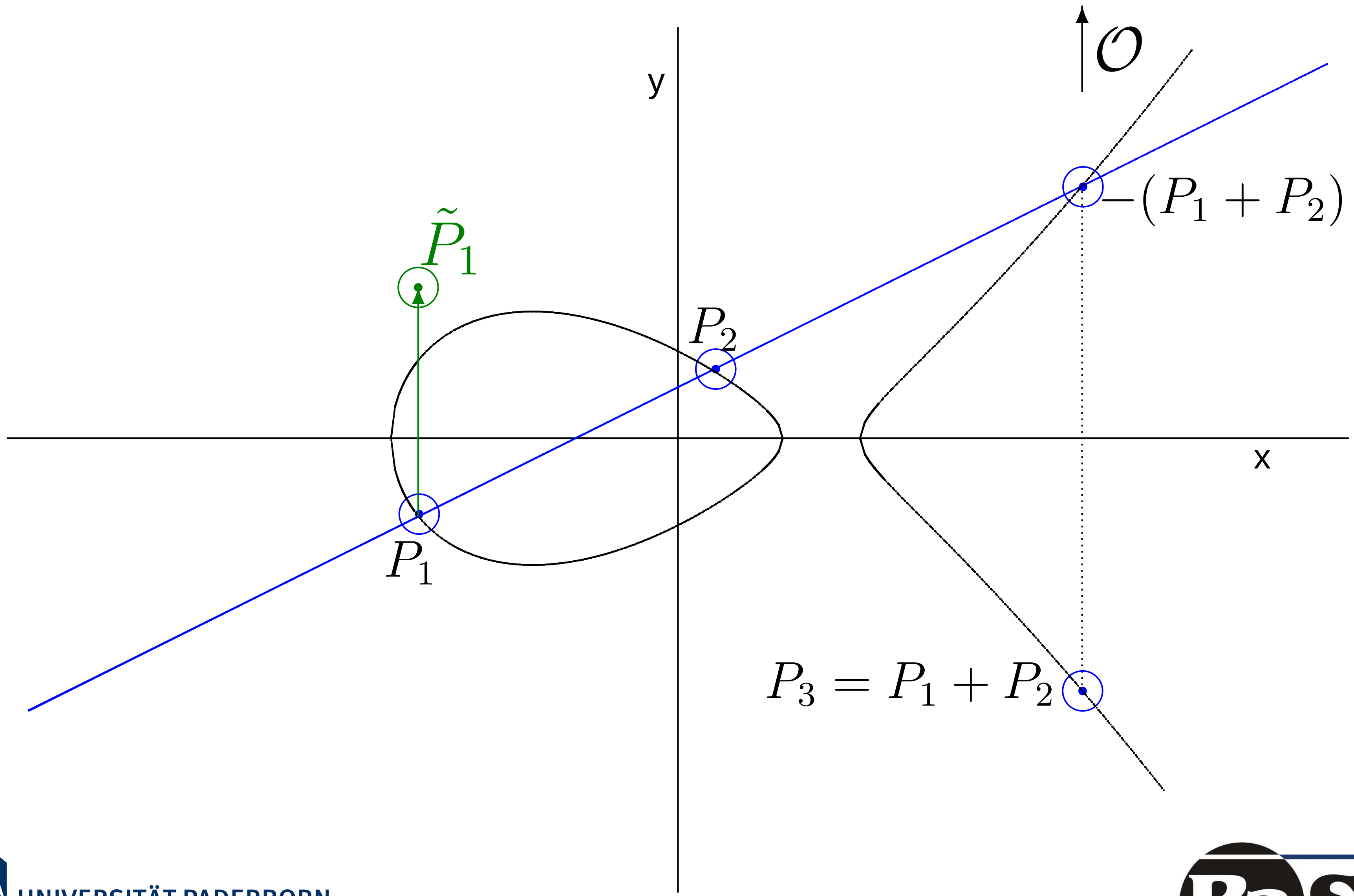
Natural Countermeasure:

- Check result before output: Is result a valid point on curve?

Fault Attacks on Elliptic Curve Addition



Fault Attacks on Elliptic Curve Addition



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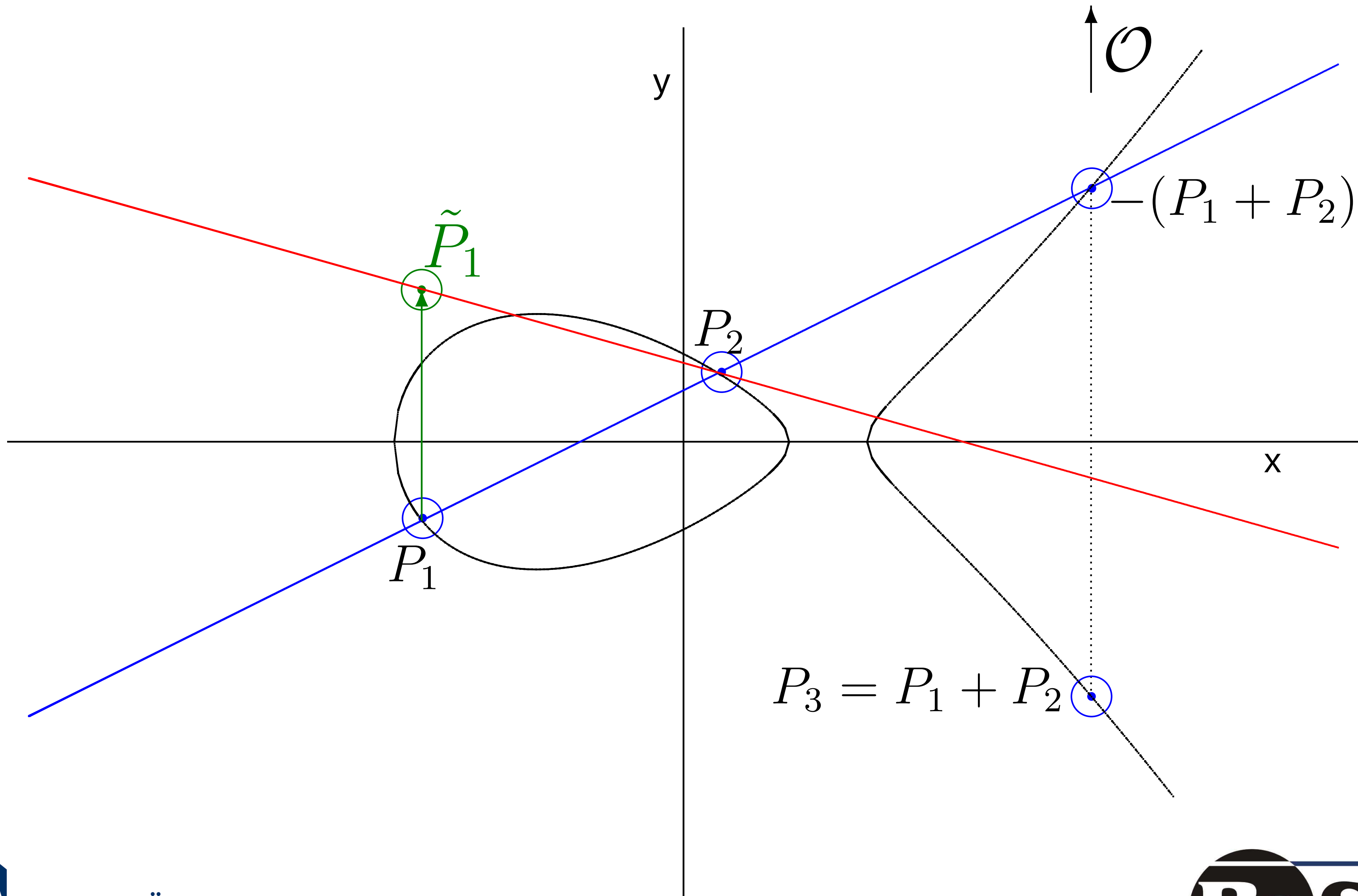

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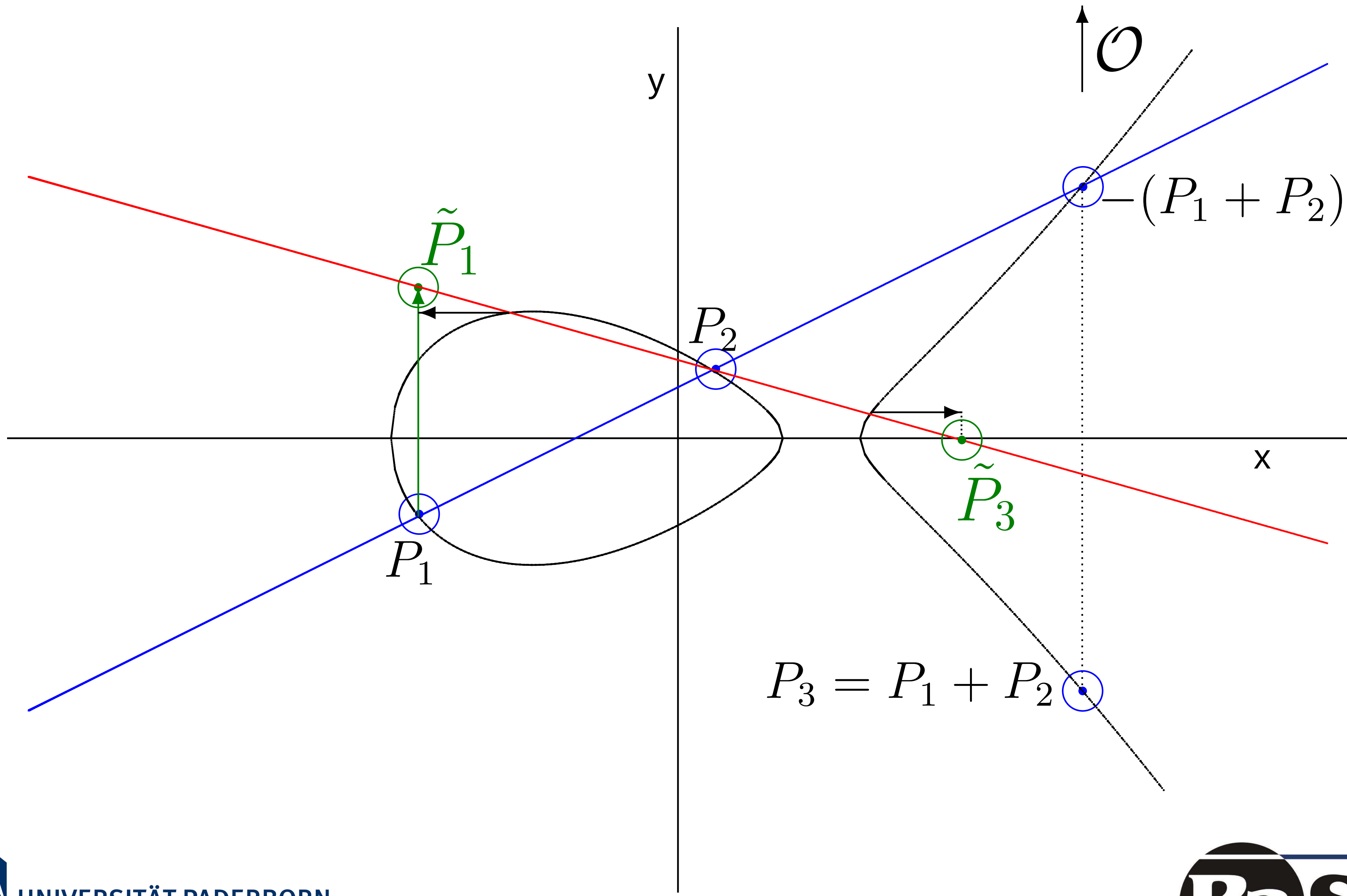
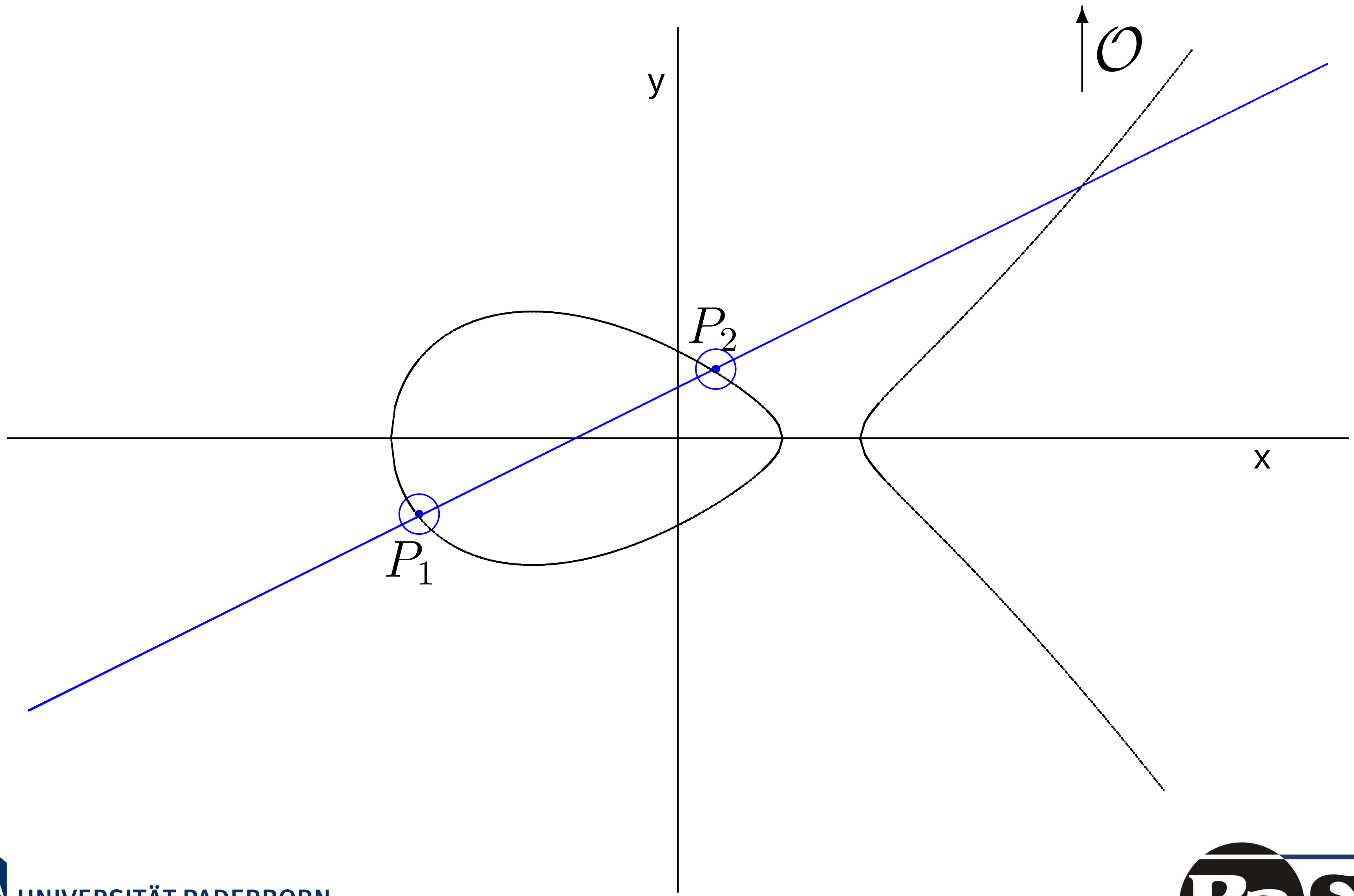
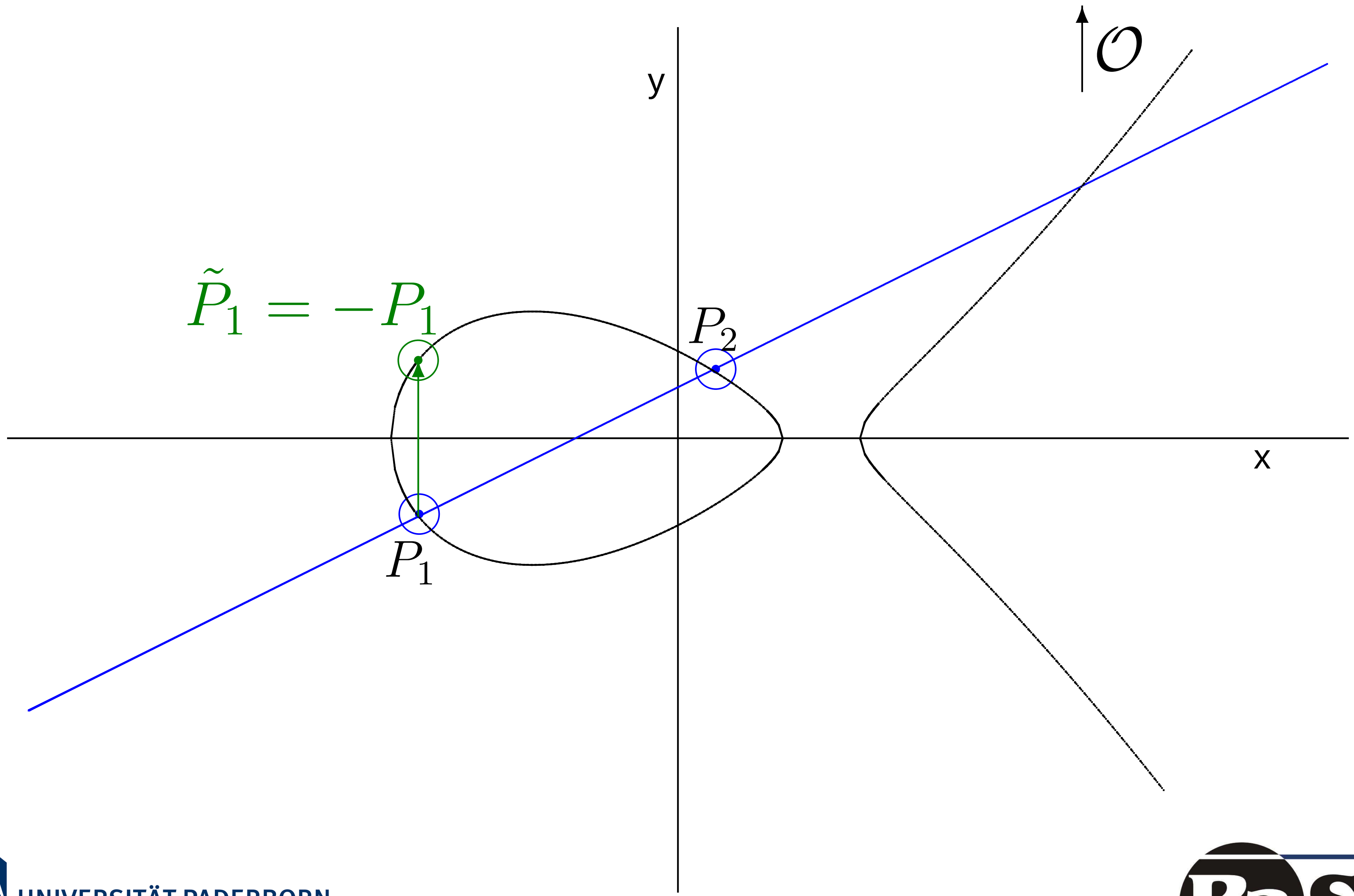


Sign Change Faults



Sign Change Faults



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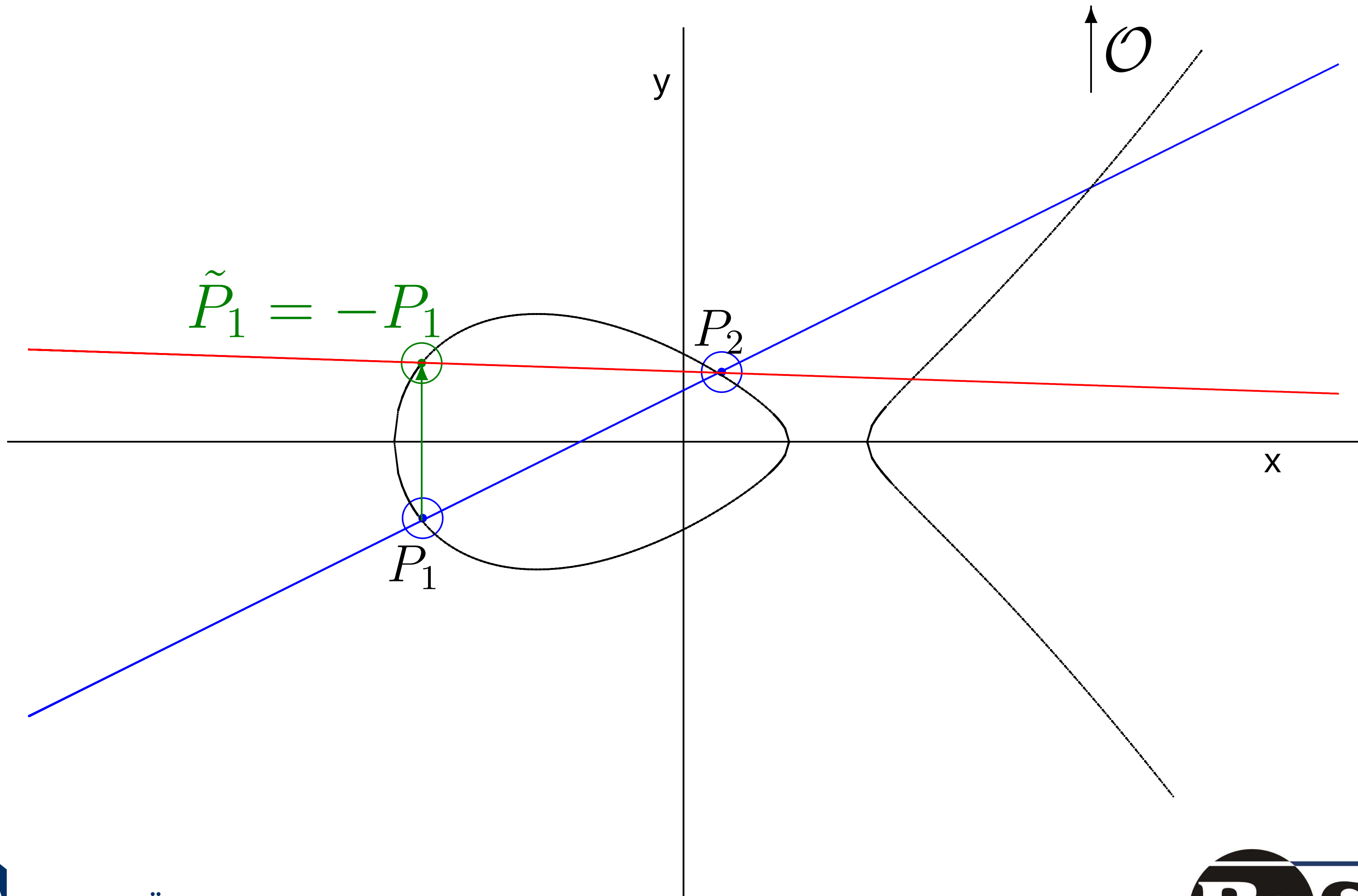
Sign Change Faults



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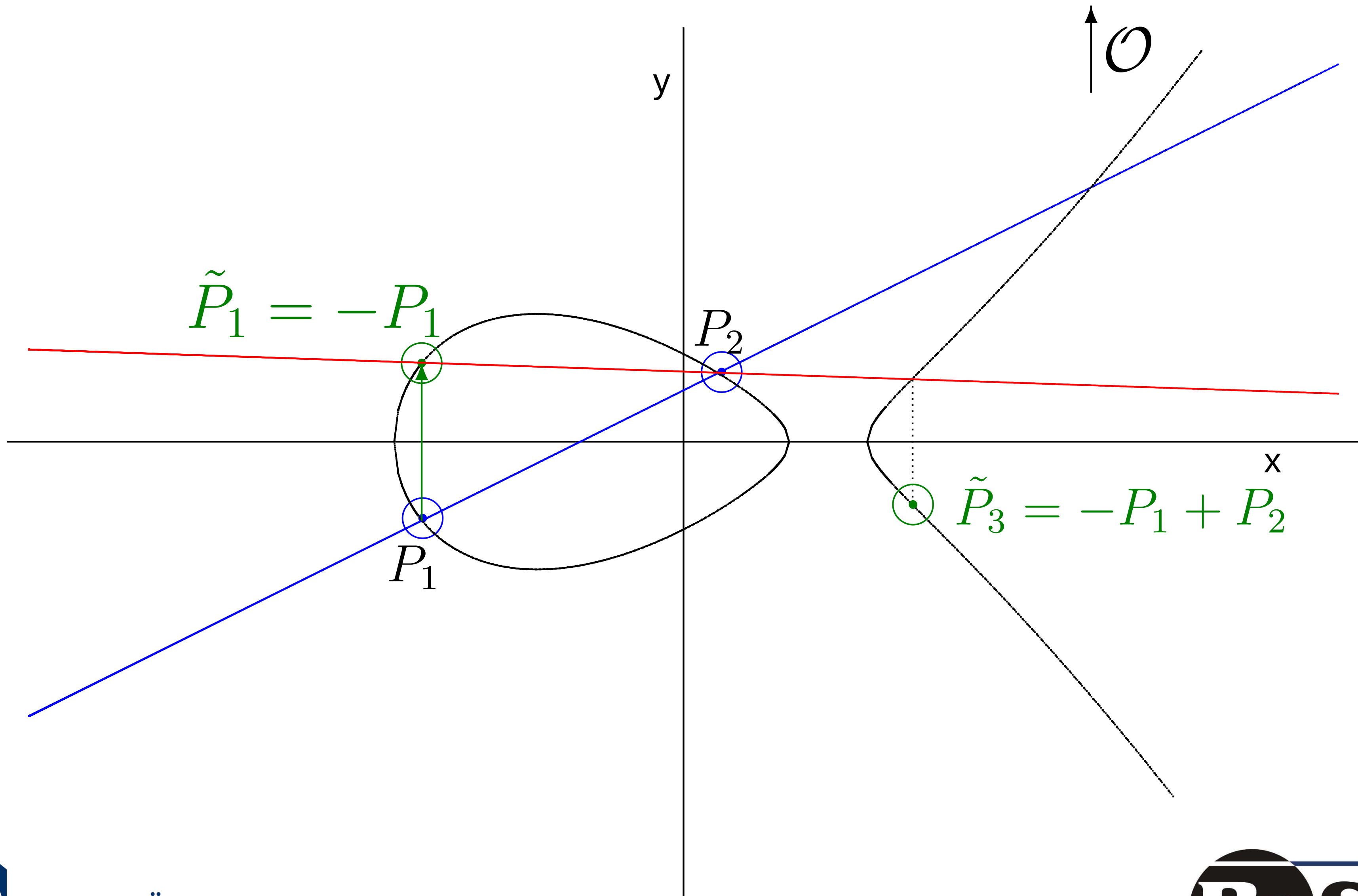
Sign Change Faults



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Can we achieve Sign Change Faults?

Yes, we can! Examples:

- NAF-based scalar multiplication: attack a key bit
- Attacks during point addition, e.g., affine addition:

$$\lambda = \frac{y_1 - y_2}{x_1 - x_2}, \qquad \begin{aligned} x_3 &= \lambda^2 - x_1 - x_2 \\ y_3 &= -y_1 + \lambda \cdot (x_1 - x_3) \end{aligned}$$

  - Change y_2 to achieve a Sign Change Fault
  - Attack ALU s.t. argument is inverted
- Success depends on the implementation (hardware and software)

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Fault Attacks With Sign Change Faults (1)

Compute $Q = kP$ on E_p:		
0 Set $n := l(k)$		
1 Set $Q_n := \mathcal{O}$		
2 For i from n-1 to 0 do		
3 Set $Q_i' := 2 \cdot Q_{i+1}$		
4 If (k_i = 1) then	set $Q_i := Q_i' + P$	
else	set $Q_i := Q_i'$	
5 Return Q_0		

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Fault Attacks With Sign Change Faults (1)

Compute $Q = kP$ on E_p:

```
Set n := l(k)
Set Q_n := O
For i from n-1 to 0 do
    Set Q'_i := 2 · Q_{i+1}
    If (k_i = 1) then  set Q_i := Q'_i + P
                 else  set Q_i := Q'_i
Return Q_0
```

Attacking

$$Q'_i \overset{⚡}{\longmapsto} -Q'_i$$

at random iteration i

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Fault Attacks With Sign Change Faults (1)

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else set $Q_i := Q'_i$
5 Return Q_0

Attacking

$Q'_i \overset{\not\;}{\longmapsto} -Q'_i$

at random iteration i

Step 1: Describe faulty final result

$\tilde{Q} = -Q + 2 \cdot L_i(k)$, where $L_i(k) := \sum_{j=0}^{i} k_j 2^j \cdot P$

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Fault Attacks With Sign Change Faults (2)

Step 2: Collect many faulty final results

- Choose block size $m \Rightarrow O(2^m)$ operations:
- Mount $(n/m)\log(2n)$ many attacks to hit every possible block with Prob. at least $1/2$

Fault Attacks With Sign Change Faults (2)

Step 2: Collect many faulty final results

- Choose block size $m \Rightarrow O(2^m)$ operations:
- Mount $(n/m)\log(2n)$ many attacks to hit every possible block with Prob. at least $1/2$

Step 3: incremental computation of k

- Assumption: all s lowest bits of k are known
- try all possibilities with up to $s+m$ bits:

$$\tilde{Q} \stackrel{?}{=} -Q + 2 \cdot L_{s+m-1}(k)$$

- Compare to gathered faulty final results

Fault Attacks With Sign Change Faults (3)

Step 4: Proof of correctness

- guessed pattern describes a faulty final result $\tilde{Q}$: we show: pattern correct
- if no pattern describes $\tilde{Q}$:
  - “Zero Block Failure“: k has block of zeros

Summary of Attack

Theorem: Secret scalar k of length n is recovered with

$$O(n \cdot 2^m \cdot t)$$

scalar multiplications with probability at least $1/2$ inducing $t = (n/m)\log(2n)$ Sign Change Faults.

- This attack also applies to other scalar multiplication algorithms (NAF-LR/RL, Montgomery Ladder)
- Attack layout derived from first fault attack on RSA (Boneh, DeMillo, Lipton 1997)

Countermeasure Against Sign Change Attacks

What we want

- check final result efficiently for correctness

Idea

- Check using a “small“ curve:
  Choose prime t and (A_t, x_t, y_t) to define

$$E_t : y^2 z \equiv x^3 + A_t x z^2 + B_t z^3 \bmod t$$
$$P_t = (x_t : y_t : 1)$$

  such that order of E_t is prime

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A “combined“ curve

- Determine $E_{pt} : y^2 z \equiv x^3 + A_{pt} x z^2 + B_{pt} z^3 \mod pt$

$$P_{pt} = (x_{pt} : y_{pt} : 1)$$

- Requirement: $A_{pt} \equiv A \mod p$

$$A_{pt} \equiv A_t \mod t \text{ etc.}$$

- Using the Chinese Remainder Theorem (CRT): $A_{pt} :=$ CRT(A, A_t) etc.

- First, compute $Q_{pt} := kP_{pt}$ on E_{pt}

We have $Q_{pt} \equiv kP \mod p$ and

$$Q_{pt} \equiv kP_t \mod t$$

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A New SCF-Secure Algorithm for $k \cdot P$

SCF-Secure Scalar Multiplication $Q = kP$	
#	Precomputation (during production time of device)
1	Choose prime t and “small“ curve E_t
2	Determine the “combined“ curve E_{pt}
#	Main (computations on the device)
3	Set $Q := kP_{pt}$ on E_{pt}
4	Set $R := kP_t$ on E_t
5	If $R \neq Q \bmod t$ then **output** „failure“ else **output** Q on E_p

- Analysis: Order of E_t is security parameter
- undetectable faults: Adversary needs $O(2^{\mathrm{ord}(E_t)})$ guesses

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Conclusion

Summary:

- New Sign Change Attacks
- New Countermeasure

Open Problems:

- Extend the idea to curves over binary fields
- Other specialized fault types?

Thank you!

Appendix

On Choosing E_t

Theorem 1: (Hasse)
Given $E_p : y^2 z \equiv x^3 + Axz^2 + Bz^3 \mod p$, it is

$$p + 1 - 2\sqrt{p} \leq \#E_p \leq p + 1 + 2\sqrt{p}.$$

Fact 2: $\exists\ p^2 - p$ different elliptic curves over $\mathbb{F}_p$.

Theorem 6: (Deurich)
There exists a constant $c > 0$ such that there are at least $c \cdot (p\sqrt{p})/\log(p)$ many elliptic curves for every given group order.

On Choosing E_t

Conjecture 7: (Cramer and Goldwasser/Kilian)
There exist constants $c_1, c_2 > 0$ such that
$\pi(t + 2\sqrt{t}) - \pi(t - 2\sqrt{t}) \geq c_2\sqrt{t}/\log^{c_1}(t)$.

Theorem 8: Choose $(A_t, x_t, y_t) \in \mathbb{Z}_t^3$ uniformly at random. (A_t, x_t, y_t) defines E_t uniquely. If Conjecture 7 is true, then $\exists c > 0$ such that the probability that E_t has prime order is at least

$$\frac{c \cdot c_2}{\log^{1+c_1}(t)},$$

where c_1, c_2 are as in Conjecture 7.

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Montgomery's skalar Multiplikation

Montgomery Algorithm: $Q = k \cdot P$

init $P1_{(n-1)} := P$ and $P2_{(n-1)} := 2P$ and $n := bits(k)$

main for i from n-2 downto 0 do

if ($k_i = 0$) then set $P2_{(i)} := P1_{(i+1)} + P2_{(i+1)}$

$P1_{(i)} := 2P1_{(i+1)}$

if ($k_i = 1$) then set $P1_{(i)} := P1_{(i+1)} + P2_{(i+1)}$

$P2_{(i)} := 2P2_{(i+1)}$

output $Q = P1_{(0)}$

$\tilde{Q} = \left(\frac{l_i(k)}{2^i} - 1\right) \cdot (Q - l_i(k)P) + l_i(k)P$, where $l_i(k) = \sum_{j=0}^{i} k_j 2^j$

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Projektive Addition

$$P_1 = (x_1 : y_1 : z_1),\ P_2 = (x_2 : y_2 : z_2),\ P_1 + P_2 = P_3 = (x_3 : y_3 : z_3)$$

$$\begin{aligned} x_3 &:= r^2 - tw^2 \\ 2y_3 &:= vr - mw^3 \\ z_3 &:= z_1 z_2 w, \end{aligned}$$

where $u_1 := x_1 z_2^2, \quad s_1 := y_1 z_2^3, \quad w := u_1 - u_2, \quad r := s_1 - s_2,$

$u_2 := x_2 z_1^2, \quad s_2 := y_2 z_1^3, \quad t := u_1 + u_2, \quad m := s_1 + s_2,$

and $v := tw^2 - 2x_3$

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