Signal Constellation Design for Optical Intensity Modulated Channels

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Abstract — **We define lattice codes for the optical intensity, direct-detection channel. These codes obey a non-negativity constraint and are shaped to minimize average optical power. Expressions for the coding and shaping gain of such codes over a rectangular PAM baseline are presented. Over short distances, we show that lattice codes provide significant rate gains for free-space optical links.**

I. Introduction

Optical communication systems are traditionally considered as power-limited rather than bandwidth-limited. However, in the case of wireless optical systems, which operate in a multipath environment and use inexpensive optoelectronic components, this may not always be the case. Multilevel, multidimensional schemes are useful on these types of channels to mitigate the impact of the limited channel bandwidth.

Present-day optical communication systems are unable to modulate the amplitude or phase of the optical carrier directly, instead modulating only the intensity of the carrier. Every optical intensity signal transmitted must therefore be non-negative. Since the amplitude of the received signal is proportional to the optical power, the average optical power is the average amplitude of the transmitted waveform. The channel can be modeled as linear and flat in the band of transmission with additive, signal-independent, white Gaussian noise [1].

In this work we present a means of defining multidimensional lattice codes for the optical intensity channel and for calculating their gain over a baseline scheme. Conventional lattice code constructions cannot be applied directly to the optical intensity channel since they do not take into account the non-negativity constraint of the channel nor do they minimize the average optical power.

II. Guaranteed non-negative Lattice Codes

To span the signal space we define a structured set of N orthonormal basis functions $\phi_1, \ldots, \phi_N$, each time limited to $t \in [0, T_s]$, with ϕ_1 chosen as a pulse of constant amplitude $T_s^{-1/2}$, so that the average optical power of each symbol is represented in the coordinate value of a single dimension. Unlike conventional constellations, the average optical power, P, depends on the geometry of the constellation as well as on the symbol interval and can be factored as $P = T_s^{-1/2} P_{\mathrm{g}}$.

We define the *bounding region*, Υ, of a modulation scheme as the set of all points in the signal space which give rise to transmittable (i.e., non-negative amplitude) signals. It can be shown that Υ is a convex *generalized cone* with vertex at the origin opening about the ϕ_1 axis.

An N-dimensional *lattice code* which is guaranteed to satisfy the non-negativity constraint can be constructed as $\Omega(\Lambda, \Upsilon, \Psi) = \Lambda \cap \Upsilon \cap \Psi$, where Λ is an N-dimensional lattice and Ψ is defined as the *shaping region*. We show that the optimum (gain-maximizing) shaping region Ψ is the half-space consisting of all points in the signal space with ϕ_1 coordinate not exceeding some fixed positive value.

III. Optical Power Gain

Due to the use of average optical power and not electrical energy, it is necessary to redefine the constellation figure of merit [2] as $\mathrm{CFM}(\Omega) = d_{\min}(\Omega)/P(\Omega)$. Both the bandwidth and the bandwidth efficiency of the two schemes are fixed equal since the average optical power depends directly on T_s. Let W be the fractional power bandwidth, we define ν as the "effective" number of dimensions of the constellation with respect to the baseline given by $\nu = (2WT_s)/(2W_\oplus T_{s\oplus})$, where terms with subscript $\oplus$ denote baseline quantities.

The baseline scheme, $\Omega_\oplus$, is taken as non-negative M-ary rectangular PAM. Using the continuous approximation, the asymptotic optical power gain [2] versus this baseline can be approximated as

$$G(\Omega(\Lambda, \Upsilon, \Psi)) \approx \left(\frac{d_{\min}(\Lambda)}{V(\Lambda)^{\frac{1}{\nu}}} \right) \cdot \left(\frac{\sqrt{\nu}}{2} \frac{V(\Upsilon, \Psi)^{\frac{1}{\nu}}}{P_{\mathrm{g}}(\Upsilon, \Psi)} \right),$$

where the first factor represents the *coding gain* of the chosen lattice and the second factor represents the *shaping gain* of the chosen region. Unlike the conventional definition [2], the coding depends not only on the lattice properties but also on the effective number of dimensions. It can be shown that coding gain is maximized by the densest N-dimensional lattice. The shaping gain is maximized by selecting points of lowest ϕ_1 coordinate until the desired volume is achieved, hence the optimal shaping region is the half-space described earlier.

IV. Applications

We calculate that in a channel similar to a commercially available line-of-sight infrared link, the use of a 512 point QAM constellation, transmitting at 8 Mbps with a 99% fractional power bandwidth of 40 MHz can operate at a symbol error rate of 10^{-8} over a distance of 100 cm when transmitting at eye-safe optical power limits. If the bandwidth is increased to 80 MHz, the same scheme can transmit at 16 Mbps over a distance of 80 cm. Over the same channel, BER and eye-safety limit, a 4-PPM scheme can operate at 4 Mbps using a 99% bandwidth of 100 MHz over a distance of 200 cm. Thus, lattice codes for bandwidth limited optical channels can supply a significant rate gain for short distance free-space optical links.

References

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