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Signal Waveform Characterization in *RLC* Trees

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Abstract - Closed form solutions for the 50% delay, rise time, overshoot characteristics, and settling time of signals in an *RLC* tree are presented. These solutions have the same accuracy characteristics as the Elmore delay for *RC* trees and preserves the simplicity and recursive characteristics of the Elmore delay. The solutions introduced here cover all damping conditions of an *RLC* circuit including the underdamped response, which is not considered by the Elmore delay due to the non-monotone nature of the response. Also, the solutions have significantly improved accuracy compared to the Elmore delay for an overdamped response. The solutions introduced here for *RLC* trees can be practically used for the same application that the Elmore delay is used in *RC* trees.

I. Introduction

It has become well accepted that interconnect delay dominates gate delay in current deep submicrometer VLSI circuits [1]-[9]. With the continuous scaling of technology and increased die area, this situation is expected to become worse. In order to properly design complex circuits, more accurate interconnect models and signal propagation characterization are required. Initially, interconnect has been modeled as a single lumped capacitance in the analysis of the performance of on-chip interconnects. Currently, *RC* models are used for high resistance nets and capacitive models are used for low resistance interconnect [10], [11]. However, inductance is becoming more important with faster on-chip rise times and longer wire lengths. Wide wires are frequently encountered in clock distribution networks and in upper metal layers. These wires are low resistive wires that can exhibit significant inductive effects. Furthermore, performance requirements are pushing the introduction of new materials for low resistance interconnect [12]. Inductance is therefore becoming an integral element in VLSI design methodologies, see e.g., [6], [13], [14].

An interconnect line in a VLSI circuit is in general a tree rather than a single line. Thus, the process of characterizing signal waveforms in tree structured interconnect is of primary importance. One of the more popular delay models used within industry for *RC* trees is the Elmore delay model [15], [16]. Despite not being highly accurate, the Elmore delay is widely used by industry for fast delay estimation. With IC's composed of tens of millions of gates it is impractical to use time consuming methods to accurately evaluate the delay at each node in the circuit. The Elmore delay model is therefore used to quickly estimate the relative delays of different paths in the circuit, permitting more exhaustive simulations to be performed for only the critical paths. Also, the Elmore delay is widely used as a delay model for the synthesis of VLSI circuits such as buffer insertion in *RC* trees and wire sizing [17]-[28]. The wide use of the Elmore delay as a basis for VLSI design methodologies is primarily because the Elmore delay has a high degree of *fidelity* [17]: an optimal or near-optimal solution reached by a design methodology

based on the Elmore delay is also near-optimal based on a more accurate (e.g., SPICE-computed [24]) delay for routing constructions [25] and wire sizing optimizations [23]. Simulations [26] have shown that the clock skew derived under the Elmore delay model has a high correlation with SPICE-derived skew data.

The popularity of the Elmore delay is primarily due to the existence of a simple tractable formula for the delay [29] that has recursive properties [27], making the calculation of the circuit delays highly efficient even in large circuits. However, no equivalent formula for delay calculation has been determined for *RLC* trees which satisfies all of these properties. The absence of an equivalent delay model for *RLC* trees is primarily due to the fact that the Elmore delay does not cover non-monotone responses [15] which can occur in *RLC* circuits. The focus of this paper is therefore the introduction of a simple tractable delay formula for *RLC* trees that preserves the useful characteristics of the Elmore delay and maintain the same accuracy characteristics. The rise time of the signals in an *RLC* tree is also characterized as well as the overshoots and the settling time (for an underdamped response).

This paper is organized as follows. In section II, an equivalent second order approximation of an *RLC* tree is developed. Closed form solutions for the 50% delay, rise time, overshoot characteristics, and settling time of the signals within an *RLC* tree are introduced in section III. Finally, some conclusions are offered in section IV.

II. Second Order Approximation for *RLC* Trees

In 1948, Elmore [15] introduced a general approach for calculating the propagation delay of a linear system given its transfer function. If the transfer function of the system is $G(s)$, the normalized transfer function $g(s)$ is $G(s)/G(0)$, which can generally be described as

$$g(s) = \frac{1 + a_1 s + a_2 s^2 + \dots + a_n s^n}{1 + b_1 s + b_2 s^2 + \dots + b_m s^m}, \quad (1)$$

where a_i and b_i are real and $m > n$. For a monotone system and a unit step input Elmore showed that the 50% propagation delay can be approximated by $T_D = b_1 - a_1$. In 1987, Wyatt [16] used the relationships that b_1 and a_1 are

$$b_1 = \sum_{i=1}^m \frac{1}{p_i} \quad \text{and} \quad a_1 = \sum_{i=1}^n \frac{1}{z_i}, \quad (2)$$

respectively, where p_i and z_i are the poles and zeros of the transfer function, respectively. Thus, Wyatt treated $T_D = b_1 - a_1$ as the reciprocal of the dominant pole (the pole that has the smallest absolute value) of the system. Using this approximation, the unit step response of the system becomes

$$e(t) = 1 - \exp\left(-\frac{t}{T_D}\right), \quad (3)$$

which models a 50% propagation delay as equal to $0.693T_D$ rather than T_D as anticipated by Elmore. The Wyatt approximation is therefore more accurate than the Elmore delay. For an *RC* tree, the time constant T_{Di} at node i is calculated in [29] and given by

$$T_{Di} = \sum_k C_k R_{ik}, \quad (4)$$

where k is an index that covers each capacitor in the circuit and R_{ik} is the common resistance from the input to the nodes i and k . Thus,

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Wyatt's approximate first order transfer function for an RC tree is

$$g_i(s) = \frac{1}{s \sum_k C_k R_{ik} + 1}. \quad (5)$$

The Elmore (Wyatt) delay does not properly characterize RLC networks due to the possibility of a non-monotone response of an RLC network [15]. To illustrate this point, consider the simple single section RLC circuit depicted in Fig. 1. This circuit has a second order transfer function given by

$$g(s) = \frac{1}{s^2 LC + sRC + 1}. \quad (6)$$

Note that the coefficient of s^1 is RC , which does not include the inductance L . Thus, the Elmore time constant (and thus the Wyatt approximation) does not depend on the inductance. However, inductance can have a significant effect on the response of the circuit. To clarify the effect of inductance, the transfer function of the circuit can be reconfigured as

$$g(s) = \frac{\omega_n^2}{s^2 + s2\zeta\omega_n + \omega_n^2}, \quad (7)$$

where

$$\zeta = \frac{1}{2} \frac{RC}{\sqrt{LC}} \quad \text{and} \quad \omega_n = \frac{1}{\sqrt{LC}}. \quad (8)$$

The poles of the transfer function are

$$P_{1,2} = \omega_n [-\zeta \pm \sqrt{\zeta^2 - 1}]. \quad (9)$$

Note that if ζ is less than one, the poles are complex and oscillations occur in the response which violates the monotone response condition of the Elmore delay. In that case the response is underdamped and overshoots occur. If ζ is greater than one, the poles are real and the response is an overdamped response. If ζ is equal to one, the response is a critically damped response. ζ is called the damping factor of the system. From (8), as the inductance increases, ζ decreases which violates the assumption of a monotonic response.

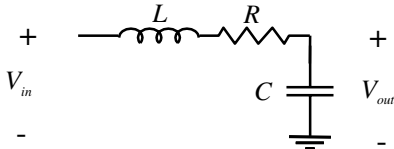


Fig. 1. Simple RLC circuit

At least a second order approximation is required to characterize a non-monotone response, because a non-monotone response involves complex poles which appear in conjugate pairs in a real system. Thus, a second order system such as (7) is used to approximate a system with a non-monotone response. It is therefore necessary to determine ζ and ω_n in order to make the second order approximation as accurate as possible as compared to the exact transfer function.

Matching the moments of a transfer function to the moments of a higher order system permits the transfer function to approximate any linear system, see *e.g.*, [30]. The greater the number of moments that are matched, the better the transfer function approximates the system. The transfer function in (7) is thus expanded in powers of s where the first two moments of the transfer function are equated to the first two moments of the system which are assumed to be m_1 and m_2 . The expansion of the transfer function in (7) is

$$g(s) = 1 - s \left(\frac{2\zeta}{\omega_n} \right) + s^2 \left(\frac{-1 + (2\zeta)^2}{\omega_n^2} \right) - \dots = 1 + m_1 s + m_2 s^2 + \dots \quad (10)$$

The parameters that characterize the second order approximation of a non-monotonic system, ζ and ω_n , can be calculated in terms of the moments of the non-monotonic system and are

$$\zeta = \frac{-m_1}{2} \frac{1}{\sqrt{m_1^2 - m_2}} \quad \text{and} \quad \omega_n = \frac{1}{\sqrt{m_1^2 - m_2}}. \quad (11)$$

Hence, for a system with a non-monotonic response a second order approximation can be found if the first and second moments of the system are known.

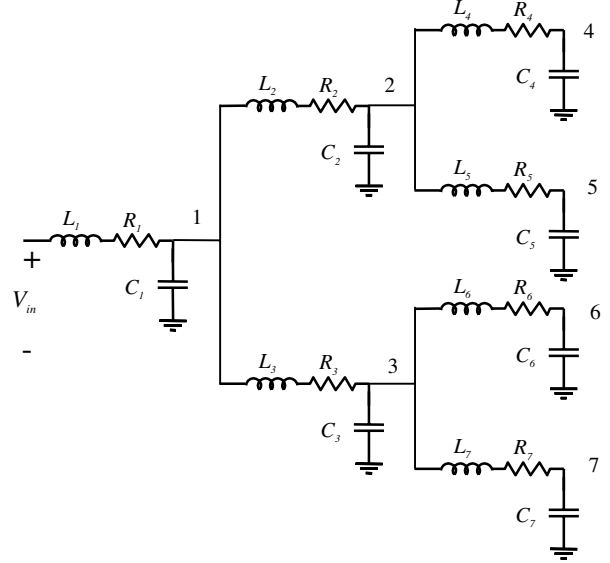


Fig. 2. General RLC tree

For the general RLC tree shown in Fig. 2, the voltage drop at any node i as compared to the input voltage is

$$V_{in}(s) - V_i(s) = \sum_k C_k V_k(s) s [R_{ki} + L_{ki} s], \quad (12)$$

where L_{ki} is the common inductance from the input to the nodes i and k . If the input is a unit impulse, $V_{in}(s)$ is equal to 1.0 and the voltages at the nodes of the tree are the unit impulse responses of these nodes. Thus, the normalized transfer function $g_i(s)$ at node i is given by $V_i(s)$ and is

$$g_i(s) = 1 - \sum_k C_k V_k(s) s [R_{ki} + L_{ki} s] = 1 + m_1^i s + m_2^i s^2 + \dots \quad (13)$$

The first and second moments at node i can be found from

$$m_1^i = \left. \frac{dg_i(s)}{ds} \right|_{s=0} \quad \text{and} \quad m_2^i = \left. \frac{1}{2!} \frac{d^2 g_i(s)}{ds^2} \right|_{s=0}. \quad (14)$$

Differentiating (13) with respect to s and substituting $s = 0$,

$$m_1^i = - \sum_k C_k R_{ik} V_k(s) \Big|_{s=0}, \quad (15)$$

$$m_2^i = - \sum_k C_k R_{ik} \left. \frac{dV_k(s)}{ds} \right|_{s=0} - \sum_k C_k L_{ik} V_k(s) \Big|_{s=0}.$$

Note that $V_k(s) \Big|_{s=0} = 1$, and that $dV_k(s)/ds \Big|_{s=0} = m_1^k$ since $V_k(s) = g_k(s) = 1 + m_1^k s + m_2^k s^2 + \dots$. Thus, the first and second moments of a general RLC tree at node i are

$$m_1^i = - \sum_k C_k R_{ik}, \quad (16)$$

$$m_2^i = \sum_k \sum_j C_k R_{ik} C_j R_{kj} - \sum_k C_k L_{ik}.$$

Since Elmore (Wyatt) approximates the first term in m_2^i by

$\left(\sum_k C_k R_{ik} \right)^2$ (this approximation can be seen in the coefficient of s^2 if (5) is expanded into powers of s), a similar approach is used here. Thus, the second moment is approximated by

$$m_2^i = \left(\sum_k C_k R_{ik} \right)^2 - \sum_k C_k L_{ik}. \quad (17)$$

Substituting the first and second moments of a general RLC tree into (11), ζ_i and ω_{ni} which characterize a second order approximation of the transfer function at node i are

$$\zeta_i = \frac{1}{2} \frac{\sum_k C_k R_{ik}}{\sqrt{\sum_k C_k L_{ik}}} \quad \text{and} \quad \omega_{ni} = \frac{1}{\sqrt{\sum_k C_k L_{ik}}}. \quad (18)$$

Note the analogy with ζ and ω_n for a single RLC section in (8). The time constants RC and \sqrt{LC} are replaced by the summations of the equivalent time constants in the tree. Note also that (18) becomes (8) for a single section. This second order approximation has the same accuracy characteristics as that of the Elmore (Wyatt) approximation for an RC tree. For a step input and a supply voltage of V_{DD} , the time domain response at node i derived from the second order approximation is

$$S_i(t) = V_{DD} + \frac{V_{DD}}{2\sqrt{\zeta_i^2 - 1}} \left[\frac{\exp[\omega_{ni}t(-\zeta_i + \sqrt{\zeta_i^2 - 1})] - \exp[\omega_{ni}t(-\zeta_i - \sqrt{\zeta_i^2 - 1})]}{-\zeta_i + \sqrt{\zeta_i^2 - 1}} - \frac{\exp[\omega_{ni}t(-\zeta_i + \sqrt{\zeta_i^2 - 1})] - \exp[\omega_{ni}t(-\zeta_i - \sqrt{\zeta_i^2 - 1})]}{-\zeta_i - \sqrt{\zeta_i^2 - 1}} \right]. \quad (19)$$

This closed form solution is compared to AS/X [31] simulations of the tree shown in Fig. 2 at output node 7. The simulations are shown in Fig. 3 for a balanced tree with several values of ζ_7 (the equivalent damping factor at node 7). The Elmore (Wyatt) solution is also shown for comparison. Note the accuracy that the solution exhibits as compared to the AS/X simulations for the case of a balanced tree. The error in the propagation delay is less than 3% for this balanced tree example. This solution becomes less accurate for unbalanced RLC trees. In general this solution for RLC trees has the same accuracy characteristics as that of the Elmore (Wyatt) delay with respect to RC trees. Note also that the solution in (19) tends to the Elmore (Wyatt) approximation for large ζ_i (low inductance effects), which shows that the general solution introduced here for an RLC tree includes the special case of an RC tree.

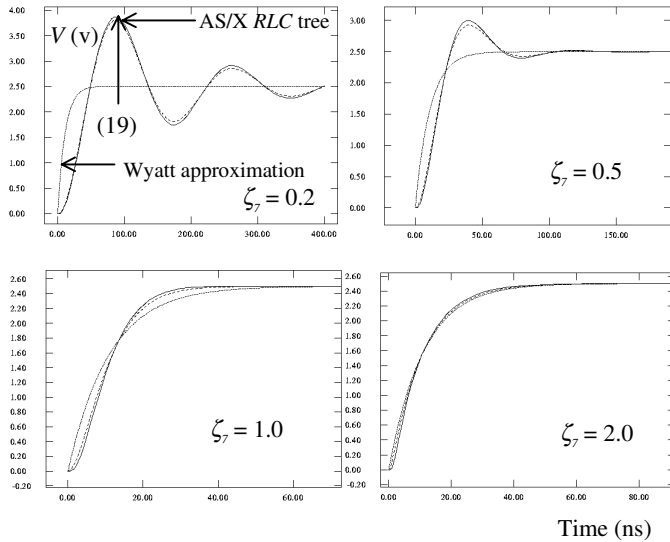


Fig. 3. AS/X simulations as compared to (19) for several values of ζ . The Elmore (Wyatt) solution is also shown.

III. Signal Characterization in RLC Trees for a Step Input

The time domain step response in (19) is used to characterize the 50% delay and the rise time of signals within an RLC tree. The rise time is defined here as the time for the signal to rise from 10% to 90% of the final value. The overshoots and the settling time for the case of an underdamped response are also characterized. In the step response described by (19), note that time is always multiplied by ω_{ni} .

Thus, if the time is scaled by ω_{ni} , the step response at node i with a supply voltage of V_{DD} becomes a function of only one variable ζ_i and is

$$S_i'(t) = V_{DD} + \frac{V_{DD}}{2\sqrt{\zeta_i^2 - 1}} \left[\frac{\exp[t'(-\zeta_i + \sqrt{\zeta_i^2 - 1})] - \exp[t'(-\zeta_i - \sqrt{\zeta_i^2 - 1})]}{-\zeta_i + \sqrt{\zeta_i^2 - 1}} - \frac{\exp[t'(-\zeta_i + \sqrt{\zeta_i^2 - 1})] - \exp[t'(-\zeta_i - \sqrt{\zeta_i^2 - 1})]}{-\zeta_i - \sqrt{\zeta_i^2 - 1}} \right], \quad (20)$$

where $S_i'(t)$ is the time scaled response at node i and t' is time scaled by ω_{ni} . The time scaled 50% delay and rise time can be calculated by equating $S_i'(t)$ to $0.5V_{DD}$, $0.1V_{DD}$, and $0.9V_{DD}$, respectively. The time scaled 50% delay at node i and the rise time are only functions of one variable, ζ_i . The 50% delay and the rise time calculated for several values of ζ_i are plotted as functions of ζ_i in Fig. 4. A curve fitting method is applied to characterize the time scaled 50% delay and rise time as functions of ζ_i and these functions are

$$t'_{pdi} = 1.047e^{-\frac{\zeta_i}{0.85}} + 1.39\zeta_i, \quad (21)$$

$$t'_{ri} = 6.017e^{-\frac{\zeta_i^{1.35}}{0.4}} - 5e^{-\frac{\zeta_i^{1.25}}{0.64}} + 4.39\zeta_i, \quad (22)$$

where t'_{pdi} and t'_{ri} are the time scaled 50% delay and rise time at node i , respectively. The 50% delay and rise time at node i can be found by dividing t'_{pdi} and t'_{ri} by ω_{ni} and are

$$t_{pdi} = (1.047e^{-\frac{\zeta_i}{0.85}} + 1.39\zeta_i) / \omega_{ni}, \quad (23)$$

$$t_{ri} = (6.017e^{-\frac{\zeta_i^{1.35}}{0.4}} - 5e^{-\frac{\zeta_i^{1.25}}{0.64}} + 4.39\zeta_i) / \omega_{ni}. \quad (24)$$

Note that the 50% delay and the rise time at node i can be described as

$$t_{pdi} = (1.047e^{-\frac{\zeta_i}{0.85}}) / \omega_{ni} + 0.695 \sum_k C_k R_{ik}, \quad (25)$$

$$t_{ri} = (6.017e^{-\frac{\zeta_i^{1.35}}{0.4}} - 5e^{-\frac{\zeta_i^{1.25}}{0.64}}) / \omega_{ni} + 2.195 \sum_k C_k R_{ik}. \quad (26)$$

For large ζ_i (low inductance effects), these solutions approach the Elmore (Wyatt) approximation of the 50% delay and the rise time for an RC tree at node i . This relationship demonstrates that the general solutions for the 50% delay and the rise time introduced here includes the Elmore (Wyatt) delay for the special case of an RC tree. Note also that the general solutions introduced here includes all types of responses (underdamped non-monotone, critically damped, and overdamped) in one continuous equation, which is useful in applications such as buffer insertion, wire sizing, and other VLSI-based design, synthesis, and analysis methodologies.

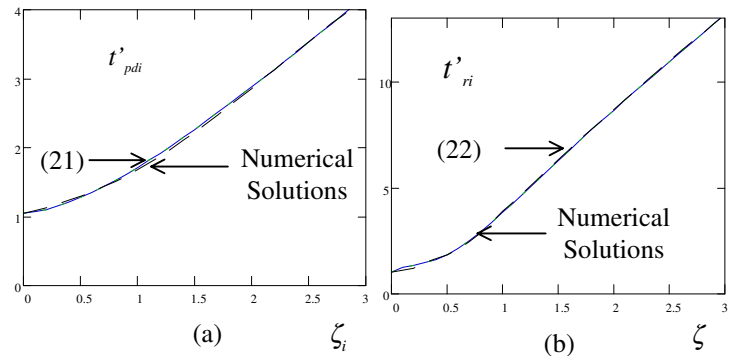


Fig. 4. Numerical solutions as compared to (21) and (22) versus ζ_i . a) The time scaled 50% delay, t'_{pdi} , compared to (21). b) The time scaled rise time, t'_{ri} , compared to (22).

For the case of an underdamped non-monotone response ($\zeta_i < 1$), overshoots and undershoots occur which must also be characterized. Also, another parameter can be used to characterize non-monotone responses which is defined as the time when the oscillations about the

steady state are smaller than x of the steady state value. This parameter is usually called the settling time and x is typically chosen to be 0.1 [32]. The value of the maximum or minimum oscillations can be found by differentiating (19) with respect to time and equating the result to zero. The values for the maximum or minimum oscillations at node i as a percentage of the final value are given by

$$\%O_i = (-1)^{n+1} \cdot 100 \exp\left(-\frac{n\pi\zeta_i}{\sqrt{1-\zeta_i^2}}\right) \quad n = 1, 2, \dots, \quad (27)$$

where $\%O_i$ represents the maximum overshoots for n odd and minimum overshoots for n even at node i . The time at which the n^{th} overshoot occurs at node i is given by

$$t_{Oi} = \frac{n\pi}{\omega_{ni}\sqrt{1-\zeta_i^2}}. \quad (28)$$

The settling time can be calculated by equating $\%O_i$ to $x*100$ to determine the value of n which represents the first overshoot that is less than x times the steady state value. The time of this overshoot is the settling time and can be calculated by substituting n from $\%O_i = x*100$ in (29). Thus, the settling time at node i is given by

$$t_{si} = \frac{-\ln(x)}{\zeta_i\omega_{ni}}. \quad (29)$$

For $x = 0.1$, t_{si} is

$$t_{si} = \frac{2.3}{\zeta_i\omega_{ni}}. \quad (30)$$

IV. Conclusions

A general method to characterize the response of a linear non-monotone system that is equivalent to the Elmore delay is presented. The generated delay expressions for RLC trees have the same accuracy characteristics that the Elmore (Wyatt) approximation has for RC trees. Simple analytical expressions of signals in an RLC tree are provided for the 50% delay, rise time, overshoot characteristics, and settling time. These expressions consider both monotone and non-monotone signal responses. The delay expressions are continuous and hence are useful for design, analysis, and synthesis in VLSI-based design methodologies.

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