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van Damme, E.E.C.

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SIGNALING AND FORWARD INDUCTION IN A MARKET ENTRY CONTEXT

by Eric van Damme

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SIGNALING AND FORWARD INDUCTION IN A MARKET ENTRY CONTEXT

Eric van Damme
CentER for Economic Research, Tilburg University,
The Netherlands.

December, 1989

Abstract: Recent developments in noncooperative game theory (especially those dealing with information transmission and equilibrium refinements) are illustrated by means of variations on a simple market entry game.

1 Introduction

During the last decade there has been a tremendous increase in the use of game theoretic modeling and methodology in the social sciences, especially in economics, accompanied by a considerable progress in the development of the theory itself. My aim in this paper is to illustrate some of these recent developments and to show why they were necessary for the applications to be successful. Emphasis will be on the intuitive ideas, not on the formal concepts. For a description of the latter, the reader may turn to VAN DAMME (1987).

The two areas in economics that have probably profited most from adopting game theoretic models are 'industrial organization' and 'the economics of information'. In the present paper we consider variations on a simple market entry game. This example is chosen to allow illustration of some of the basic issues in these areas, as well as of the game theoretic problems involved. In Section 2, the most simple variant of this game is considered (Fig. 1). The game of Fig. 1 is one of perfect information and illustrates the difference between Nash equilibria and subgame perfect equilibria. In Section 3 modifications of the game are introduced that have incomplete information. The examples in this section illustrate the notion of sequential equilibrium, as well as why it is necessary to refine this concept. Various such refinements are briefly discussed.

The games considered in Section 3 are so called signaling games. They have the following structure: There are two players, one informed and one uninformed; the informed party moves first and its action is observed by the uninformed; the uninformed draws inferences about which information the other has and then takes an action; the payoffs to both players depend on the actions taken and on the information. The essential question is how much information will be revealed in equilibrium. Typically, however, there exist multiple equilibria, both pooling ones (no information transfer) as well as separating ones (full information revelation) and hybrids (part of the information is revealed). More refined equilibrium notions try to capture the idea, called Forward Induction, that the uninformed party should realize that the other will reveal only that information that is profitable to him. Section 3 makes this idea more precise.

It should be clear that examples of signaling games abound. Let us just mention a few: (i) Finance (buying back shares signals that they are undervalued), (ii) Macroeconomics (Mrs T. wants to signal that she is really tough on inflation), (iii) Intelligence (how to show that you are not a double spy?), (iv) Accounting (You know you cheated but the tax inspector does not), (v) Advertising (a more extended warranty signals higher quality), (vi) Bargaining (how to show your strength?) and (vii) Politics (how can Mr Krenz show that he is "different" from Mr Honecker?, Is the opening of the Berlin Wall together with displaying the luxuries of Wandlitz enough to establish credibility? Hence, the question of how to solve these games is of some importance. (It is worthwhile to note that signaling games were first studied in SPENCE (1973).)

In Section 4 we turn to the case where the private information that a player has is not exogenously determined, but rather concerns what he will do in the future. It is shown that the idea of Forward Induction may increase the predictive power of game theory also in this case. Section 5 considers an even more elaborate model in which there is simultaneous signaling of private information about the past (i.e. the type) and the future (i.e. the actions). The model of that Section, although relatively simple, is a prototype of the so called 'reputation' models in macro-economics, i.e. how, in repeated context, one can get a reputation for being tough (or for being cooperative). Again Forward Induction is an essential element when trying to interpret signals.

The paper emphasizes the underlying ideas rather than the formalities. The discussion will make clear that many important problems in the area are still open, and some open problems are mentioned in the text. It is hoped that the material signals that this is a very challenging area to work in.

2 Market Entry: Complete Information

Consider a market in which 2 firms (firm I and firm II) contemplate entry. The market, however, is a natural monopoly. If one firm enters, it makes a profit (say of 1 unit), but if both firms enter, each makes a loss (say of a units each). If a firm stays out, it has zero profit. Let us first assume that firm I has detected the potential profitability of this market first. Hence, firm I makes its entry decision first and is committed to this choice. Firm II decides upon entry after firm I and being fully informed about firm I's choice. The situation may be modeled by the extensive form game from Figure 1.

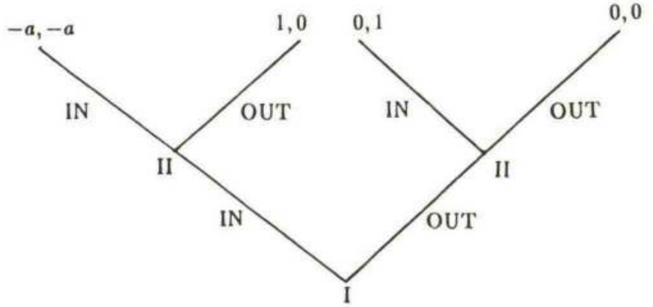

Figure 1

The solution of the game is found by straightforward backward induction (dynamic programming): Firm II will choose OUT when I has chosen IN (having 0 is better than losing a) and II will choose IN when I has chosen OUT. Knowing this it is optimal for player I to choose IN. The outcome is that firm I captures the market and that II stays out.

Using game theoretic terminology, one says that the above solution is the subgame perfect equilibrium of the game (SELTEN (1965)). This solution is also a Nash equilibrium but there exist other Nash equilibria as well. A second Nash equilibrium is the strategy pair where firm I chooses OUT and II decides to go IN irrespective of what I has done. The reason that this pair is a Nash equilibrium is that II's threat (to play IN after I has chosen IN) does not have to be executed when it is believed by I. Basically the Nash concept only requires that players behave optimally on the equilibrium path; since only ex ante expected payoffs matter for this concept, events off the equilibrium path are irrelevant as they have probability zero. However, in games, probabilities are endogenously determined, hence, an event to which one assigns zero probability ex ante does matter since during the game one may find out that it has happened after all. In the game of Figure 1, even if II expects I to choose OUT, he may observe I choosing IN and in that case II optimally chooses OUT: The threat to play IN in that event is incredible. Selten's concept of subgame perfectness strengthens Nash's notion in that it requires ex post optimality at every decision point rather than ex ante optimality. By now there is almost unanimous agreement among game theorists that those Nash equilibria that are not subgame perfect do not make sense.

Even though for games of perfect information the notion of ex post optimality is easy to define (one simply assumes that no matter what has happened in the past, players will behave rationally in the future[1], hence one obtains the standard dynamic programming procedure), things become much more intricate when information is imperfect or incomplete. The KREPS AND WILSON (1982) notion of sequential equilibrium (which is closely related to SELTEN's (1975) perfectness concept) can be seen as an attempt to extend the dynamic programming reasoning to this class of games. The basic idea is that at each decision point a player constructs beliefs about what has happened in the past and then optimizes against these beliefs. One naturally requires that beliefs are Bayes consistent with the strategies that are played and that they are consistent across time and across players. The examples from Section 3 illustrate the sequential equilibrium concept as well as the need to refine it.

3 Market Entry: Incomplete Information

Consider the market entry situation discussed in the previous section but now assume that if both firms enter the outcome is determined by a battle, the winner of which is the financially strongest firm. There are two possibilities: Firm I is either strong (in which case it wins the battle) or weak (and then it looses). Assume that the loosing firm looses a, that firm II makes an overall profit of b if it drives the weak firm I out of the market and that firm I looses x (which may be positive or negative) when it wins the battle from firm II. Assume that firm I knows which case prevails but that II only knows that I is strong with probability $1-\varepsilon$ and weak with probability ε. (ε small but positive.) Assume also that these beliefs are common knowledge. Again firm I moves first and firm I's choice is observable. Note that the essential assumption is that the market may be profitable for II even as a duopoly, but that there is only a very small probability that this is the case. The game now has one-sided incomplete information; it may be represented by a tree in which first nature determines which firm is superior, then firm I (having this information) moves and finally firm II (knowing only what I has done) chooses between IN and OUT. Note that firm I's action may signal its information. Such a game is therefore called a signaling game. A bimatrix representation of the game is given in Figure 2. (The left matrix describes the payoffs if I is superior (= strong), the right one the payoffs if I is of the weak type; I knows which matrix he is playing but II does not.)

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[1] BINMORE (1987) and others have pointed out the logical difficulties of this procedure.

$$\begin{array}{c|c|c|} & \text{IN} & \text{OUT} \\ \hline \text{IN} & -x,-a & 1,0 \\ \hline \text{OUT} & 0,1 & 0,0 \\ \hline \end{array} \qquad \begin{array}{c|c|c|} & \text{IN} & \text{OUT} \\ \hline \text{IN} & -a,b & 1,0 \\ \hline \text{OUT} & 0,1 & 0,0 \\ \hline \end{array}$$

s (prob $1-\varepsilon$) $\qquad$ w (prob ε)

Figure 2

The game is easy to solve if x is negative. In this case it is a dominant strategy for the strong firm I to enter. Firm II knows this and assesses a probability of at least $1-\varepsilon$ that it will loose a if it enters as well. Hence, if ε is small, firm II will choose to stay OUT after I has gone IN. The weak firm I, knowing this, also chooses IN. Hence, the presence of the strong firm I provides a positive externality for the weak type of this firm. In the incomplete information game, the weak type has payoff one whereas its payoff would be zero if it were common knowledge that it were weak.

Things become more interesting if $x > 0$. Intuitively one would argue that, if ε is small, the solution should not be much different from the one where it is common knowledge that firm I is strong ($\varepsilon = 0$). The latter was derived in the previous section: The strong firm I chooses IN and after this choice II decides to remain OUT (which again enables to weak firm I to also enter). Indeed if $\varepsilon \leq a/(a+b)$ there exists a sequential equilibrium in which firm I chooses IN irrespective of its type and II chooses OUT after IN. (Such an equilibrium in which the action of the informed party does not reveal any information about its type is said to be a pooling equilibrium.) However, paradoxical as it may seem, there exists a second pooling equilibrium and in this equilibrium, the outcome is completely different from the outcome derived in the previous section. In the second equilibrium, firm I chooses OUT irrespective of its type and firm II chooses IN irrespective of what I does, hence, II captures the market. Note that given this strategy of II, the behavior of I is indeed optimal (by going IN I always looses so it is better to stay OUT), and it is clearly also optimal for II to go IN when I stayed OUT. The question is whether II's threat to go IN also when I goes IN is credible. (Note that, in the equilibrium the threat does not have to be carried out, I never chooses IN.) According to the sequential equilibrium concept, this threat is credible: If II observes that I has chosen IN, II may believe that firm I is of the weak type (beliefs are arbitrary since Bayes' rule does not apply off the equilibrium path) and, if firm I is actually weak, it is ex post optimal to go IN as well. We see that, in games of imperfect information, the question of which threats (actions) are credible amounts to asking which beliefs are credible, since actions can be made credible (i.e. ex post optimal in a Bayesian sense) by adopting incredible beliefs.

The problem of how to define credible beliefs has drawn a lot of attention from game theorists since it was first formulated in KREPS AND WILSON (1982). Various formalizations have been proposed and lack of space prevents a detailed discussion here (see VAN DAMME (1987, Ch. 10)), but the main ideas may be sketched briefly (also see CHO AND KREPS (1987)). The central theme is that of Forward Induction, i.e. the question of when one observes something unexpected, then what should one deduce from the past and what should one infer for the future? The simplest formulation of this idea is due to David Kreps and is known as "the intuitive criterion". It amounts to saying that one should not believe that one is dealing with a type that cannot benefit at all by choosing the unexpected action. The criterion is quite weak, hence, frequently, it is not very helpful. This is also the case in the game of Fig. 2 (both the strong and weak type of firm I benefit from choosing IN if this leads to II staying OUT), hence, we will not discuss it further.

A much more stronger (and more controversial) concept requires that one believes one deals with those types that most easily gain from the defection. Formally, this notion requires "independence of never weak best responses" (INWBR), it is implied by the concept of stable equilibrium advanced in KOHLBERG AND MERTENS (1986). Consider, in the game of Fig. 2, the pooling equilibrium where both types of I choose OUT. To prevent the strong type to deviate to IN, firm II should after IN go IN as well with a probability p satisfying

$$-xp + 1 - p \leq 0 \tag{3.1}$$

Similarly, to force the weak type to choose OUT, we should have

$$-ap + 1 - p \leq 0 \tag{3.2}$$

If $x < a$ only the first constraint is binding, hence, the strong type is more inclined to deviate. In this case, INWBR requires that, after IN, firm II believes it is dealing with the strong firm I, hence, it should stay OUT. But if II stays OUT, I moves IN. Hence, if $x < a$ only the pooling equilibrium where firm I goes IN and II stays OUT satisfies INWBR. (It indeed satisfies this requirement; more generally, Kohlberg and Mertens have shown that there always exists a stable equilibrium outcome.)

If $x \geq a$, condition (3.2) is binding and according to INWBR, the belief that one faces the weak type if I unexpectedly chooses IN is credible. In fact, the pooling equilibrium in which both types of I choose OUT is stable (in the sense of Kohlberg and Mertens) if $x > a$. (It should be noted that also the 'intuitive outcome' is stable and that there exists a third equilibrium (in which the strong type randomizes the weak type chooses IN, and II goes IN after IN with probability $p = 1/(1+x)$), that is stable as well.) There exist refined equilibrium notions that exclude those equilibria where firm I does not pool at IN (see OKUNO-FUJIWARA AND POSTLEWAITE (1987) for example) but none of these is entirely satisfactory. All these concepts are based on the idea that, since it is in the interest of the types of firm I to pool at IN they will do so, hence, these concepts assume that different types of a player can cooperate to a certain extent (although they are not physically present at the same point in time) and they assume away coordination problems. Hence, the state of the art is that current refined noncooperative equilibrium concepts do not succeed in reducing the game of Fig. 2 to what (at first) seems the unique plausible outcome. Apparently some work remains to be done. To conclude this section, let us however remark that there exists an entirely different theory (viz. that of HARSANYI AND SELTEN (1988)) that does not incorporate the idea of forward induction, but that produces the 'plausible' outcome in the game of Fig. 2. This theory is based on uniform perturbations, i.e. on passive updating, hence, whenever something unexpected happens one does not deduce anything but rather one assumes that the ex ante probabilities are still valid. Therefore, if $\varepsilon < a/(a+b)$, II will respond to an unexpected IN with OUT and the 2 types of firm I can safely choose IN.

4 Advertising and Repetition

Let us return to the simple model of Section 2 but let us now assume that firms make their entry decision simultaneously, i.e. firm II cannot condition its behavior on what I has done. The bimatrix representation is given in Figure 3.

	IN	OUT
IN	$-a, -a$	$1, 0$
OUT	$0, 1$	$0, 0$

Figure 3

The game of Fig. 3 has three Nash equilibria, viz. (IN, OUT), (OUT, IN) and an equilibrium in which each firm randomizes, choosing IN with probability $1/(1+a)$. The latter equilibrium yields an expected payoff of zero for both firms.

Now let us introduce an asymmetry by assuming that, before making the entry decision, firm I (and firm I only) can start an advertising campaign. For simplicity (but without loss of generality) assume that the intensity of advertising is not a choice variable, firm I just chooses whether or not to advertise. Finally, assume that advertising costs c with $0 < c < 1$ and that firm II can observe whether I advertises or not. The question is whether firm I advertises and which firm will enter the market.

Using Forward Induction, the reasoning of firm II runs as follows. Firm I can guarantee itself a payoff of zero by not advertising and staying OUT. If firm I advertises, I (i.e. firm II) should conclude that it goes IN for otherwise it will simply have incurred an unnecessary loss of c. Hence, if firm I advertises, I (i.e. firm II) should stay OUT. Firm II, therefore, concludes that, by advertising, firm I guarantees itself a payoff of $1 - c > 0$. However, then taking the argument one step further, firm II should conclude that firm I will also go IN even if it does not advertise. Namely, staying OUT only yields zero so that I would have foregone a sure payoff of $1 - c$. Hence, II concludes that I chooses IN irrespective of whether it advertises or not, hence II stays OUT in both circumstances. Firm I, mimicking the above reasoning, concludes that there is no need to advertise and chooses IN.

The astute reader will have noted that the above Forward Induction argument amounts to nothing else than elimination of weakly dominated strategies in the normal form of the game. Indeed there is a link between the 2 concepts (see KOHLBERG AND MERTENS (1986) and VAN DAMME (1989)), Forward Induction generally is more restrictive, however[2].

The latter claim may be illustrated by considering the game in which, before making the entry decision, the 2 firms simultaneously decide whether to advertise or not. (Hence, also firm II now has the possibility to advertise, and w.l.o.g. we may assume that its advertising costs are also c.) Assume that before making the entry decision, it is common knowledge which firms advertised. The normal form of this game is an 8×8 bimatrix game and by eliminating dominated strategies it cannot be reduced that much. However, Forward Induction still allows to eliminate many equilibria and leads to the conclusion that, in any 'sensible' equilibrium both firms must advertise with positive probability. Namely, consider a subgame perfect equilibrium outcome in which no firm advertises. (The ones where only one firm advertises are disposed of just as easily.) There are just three of these: After the first stage players continue with one of the equilibria from the bimatrix of Fig. 3. Suppose they continue with (IN, OUT). Then II's payoff in equilibrium is zero. By advertising in the first stage, firm II may credibly signal that it will choose IN rather than OUT in the second stage (advertising followed by OUT leads to a sure loss, followed by IN it may give a profit if $1 - c$), hence firm I has to give in. The other possibilities are eliminated by a similar argument. (If players intended to randomize at stage 2, then each firm can credibly signal that only it should be IN by advertising.) Hence, advertising must occur. It can be checked that there exists exactly one symmetric equilibrium outcome that cannot be eliminated by Forward Induction (i.e. that is stable): In the first stage, each firm advertises with probability $1 - c$, if it happens that only firm advertises then this firm captures the market at stage 2, otherwise firms play the mixed equilibrium from Fig. 3 at stage 2. The expected payoffs in this equilibrium are zero, hence, advertising is purely dissipative.

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[2] In the literature one may find various definitions that try to capture the intuition of Forward Induction, but none is completely satisfactory (see VAN DAMME (1989)). In what follows, we will indentify Forward Induction with the INWBR criterion described in Section 3.

Let us return to the basic game from Fig. 3 without advertising. Assume that this game is repeated twice, with firms having full information about the outcome at stage 1 when they make their second entry decision. Also assume $0 < a < 1$. The 2-stage game has many subgame perfect equilibrium outcomes of which some may be eliminated by Forward Induction. Consider, for example, the outcome in which (IN, OUT) is played in both periods. Firm II has a payoff zero in this equilibrium, hence, if II deviates to IN in the first period (thereby incurring a cost a) it credibly signals that it will choose IN also in the second period since this is the only way by means of which II can recoup the cost. Firm I realizes this and indeed stays OUT in period 2, thereby enabling II to make and overall profit of $1 - a$. (Formally, the outcome in which (IN, OUT) is played twice does not satisfy INWBR in the normal form of the 2-period game.) Similarly the outcome in which only firm II is IN in both periods does not satisfy INWBR, nor does an outcome in which first one firm is IN and then there is randomization in the second period. Of the outcomes that consist of strings of one-shot pure equilibria, only two are consistent with the Forward Induction logic: The firms alternate in being in the market. Hence, there seems a tendency to fair sharing. In addition to these sharing equilibria, there also exist many inefficient equilibria in which both firms randomize in the first period.

Such equilibria are also consistent with Forward Induction since deviations cannot be detected, hence, there can be no signaling. For further results on Forward Induction in repeated games the reader is referred to OSBORNE (1987) and PONSSARD (1989). Let us mention that not much is known yet. For example, denote by $P(n)$ the set of average payoff vectors associated with stable equilibria of the n times repetition of the game from Fig. 3. One would like to know $\lim_n P(n)$, but one does not know it. (Is it the line segment from $(0,0)$ to $(^1/_2, {}^1/_2)$?)

5 Commitment and Entry Deterrence

In the basic game from Fig. 1 there is a first mover advantage: Firm I gets the market. The situation would be different if firm II could make credible the threat to go IN irrespective of what I does. If II could commit itself in advance, i.e. if II could make the choice of OUT after the IN of player I infeasible or highly unattractive, then the threat would be credible. Hence, when possible, it is attractive for II to commit itself in advance. Of course, it is also necessary that I knows that II is committed. In turn it is important that II attaches positive probability to I knowing that II is committed. The commitment of II being common knowledge is definitely sufficient for commitment being optimal. In this section we first make the above statements more precise. Thereafter, we show that, in a repeated context, it is sufficient that I attaches an arbitrarily small, but positive probability to II being committed. The latter part of the section is based on KREPS AND WILSON (1982a).

Let us first consider the situation where the commitment of II is common knowledge. The game of Fig. 1 is modified such that first II chooses to commit ($=C$), i.e. to delete his choices OUT in Fig. 1, or not ($=N$) and that I is informed of II's choice. If II chooses N, the game from Fig. 1 is played, if C is chosen they play the game in which OUT is not available for II. It is easily seen that the unique subgame perfect equilibrium prescribes that II should commit and that I should stay OUT, hence, II captures the market. The situation is different if I is not informed whether II has chosen C or N (and if II knows that I is not informed). Replacing subgames by their unique equilibria, this situation may be reduced to a simultaneous move game where I chooses between IN and OUT and II chooses between commitment or not. The bimatrix is given in Figure 4.

	C	N
IN	$-a, -a$	$1, 0$
OUT	$0, 1$	$0, 1$

Figure 4

(OUT, C) and (IN, N) are equilibria of this game, but only the latter survives elimination of dominated strategies. Therefore, when II knows that I does not know whether II is committed, it is optimal not to commit and I captures the market. Let us finally in this static context analyze what happens when II does not know what I knows: II thinks that with probability p I is informed about his choice between C or N and that with probability $1 - p$ I is not informed. If I indeed is informed or uninformed and if p is common knowledge, we have a well-defined game with incomplete information. If $p > 0$, there exists a (stable) equilibrium in which II commits and captures the market, and if $p > a/(1 + a)$ this is the only equilibrium. If $p < a/(1 + a)$, however, there also exists an equilibrium where II does not commit and I goes IN, as well as an equilibrium where both I and II randomize.

The above makes clear that, even in this simple context, the outcome crucially depends on the players' knowledge. We will return to this issue in Section 6.

Next, let us turn to repetitions of the game of Fig. 1. Assume that there are N markets in which firm II contemplates entering. Unfortunately, in each market there is a competitor (firm I_n in market n) who has the option to enter first. In each market the game from Fig. 1 is played. We assume the game starts in market N, then moves to $N - 1$ etc., until market 1, and that, when playing the game in market n, the players II and I_n are fully informed about what happened in any market k with $n < k \leq N$. In order to simplify the derivation below somewhat we will assume that $\mathrm{I}_n \neq \mathrm{I}_k$ if $n \neq k$ (i.e. different competitors in different markets) so that only II is a "long-run" player, but qualitatively the analysis would also go through with two long run players. In the game just described, there is a unique subgame perfect equilibrium: Firm I_n enters in market n (for any n) and II stays out everywhere. To some extent, this result is counterintuitive as one might have expected that II will invest to require a reputation for toughness. Specifically, firms I_n with n large may fear that if they enter, II will choose IN as well in order to convince firms I_k ($k < n$, k not too small) that they better stay out; and as a consequence firms I_n (n large) would prefer to stay out. Hence, one might have expected that II captures at least the initial markets. The fact that formal game theoretic reasoning does not capture the intuition in this case is known as the chain store paradox (SELTEN (1973)).

In the remainder of this section we show that the equilibrium may be completely different (and may be more in accordance with the intuition) if the firms I just assign a small, but positive probability to the event that II may be committed to IN. Specifically, we assume that each firm I_n believes that there is a probability ε that II is an automaton that is programmed to play always IN in the game of Fig. 1. The heuristic argument for why the outcome is different is that now reputation arguments can come into play. The argument runs as follows: Firm I_n should choose IN if the probability that II chooses IN as well is sufficiently small, otherwise it should stay out. Clearly, the probability that II chooses IN in market n is not zero: II may be committed. However, I_n should consider the probability that II chooses IN to be larger than the probability that II is committed. Namely, if player II would choose OUT after IN, II would reveal itself as not being the automaton, hence II would receive zero for the rest of the game. (When it becomes common knowledge that II is not committed, players continue with the subgame perfect equilibrium described above.) However, if II chooses IN after OUT, the firms I_k with $k < n$ may revise upward their belief that II is committed and they may conclude that it is better to stay out. Hence, if n large, firm I_n realizes that II has such a strong desire to pretend to be an automaton, that, therefore, the probability of fought entry is so large that it is better to stay out. Consequently, II will indeed capture the initial markets.

The formal analysis proceeds by backwards induction. (See KREPS AND WILSON (1982a) or VAN DAMME (1987, Ch. 10) for more details.) Since, in equilibrium, the payoffs to player II cannot be negative (II can guarantee zero by consistently choosing

OUT) it follows that II chooses IN when I_n chooses OUT. (If II would choose OUT as well its payoff would be zero, by choosing IN the payoff is at least 1.) Hence, we will concentrate on what happens when I_n chooses IN. Let p_n be the probability that I_n attaches to the event that II is an automaton, let e_n be the probability that the noncommitted firm II chooses IN after the IN of firm I_n, and let f_n be the probability I_n assigns to entry being fought, $f_n = p_n + (1 - p_n)e_n$. Finally, let v_n be the overall equilibrium payoff of the noncommitted firm II summed over the markets $1, \dots, n$ if beliefs in market n are p_n. (We will show that these payoffs are almost always unique.) We assume $0 < a < 1$.

Since player I_n is "short run", his decision is easy: Choose IN if the resulting expected payoff is larger than zero, hence

$$\text{IN if } f_n < 1/(1+a), \text{OUT if } f_n > 1/(1+a) \tag{5.1}$$

Now consider market $n = 1$. Obviously $e_1 = 0$, hence, $f_1 = p_1$. Therefore

$$v_1 = \begin{cases} 1 & \text{if } \; p_1 > 1/(1+a) \\ \in [0,1] & \text{if } \; p_1 = 1/(1+a) \\ 0 & \text{if } \; p_1 < 1/(1+a) \end{cases} \tag{5.2}$$

Next, consider market $n = 2$, assume that $p_2 > 1/(1+a)$ and that I_2 chooses IN. If II responds with IN as well, Bayesian updating forces I_1 to put $p_1 = p_2$, hence, to stay OUT. Consequently, IN yields II a payoff $1 - a > 0$, so that IN is optimal. Next, assume $p_2 < 1/(1+a)$ and I_2 chooses IN. Bayesian updating now leads to the conclusion that, if II responds with IN, its payoff is $-2a < 0$, hence, IN cannot be optimal. On the other hand, in equilibrium, we cannot have that II chooses OUT, since in this case, fought entry would signal that II is committed, hence, it would lead to I_1 staying OUT, but then II would rather pretend to be committed. We see that, in equilibrium, II must randomize if I_2 chooses IN and $p_2 < 1(1+a)$. Such randomization is optimal only if II is indifferent, and given that revealing to be not committed yields zero, we see that we must have $-a + v_1(p_1) = 0$. Hence, (5.2) yields $p_1 = 1/(1+a)$. Now, by Bayes' rule

$$p_1 = \frac{p_2}{p_2 + (1 - p_2)e_2} \tag{5.3}$$

so that

$$e_2 = \frac{ap_2}{1 - p_2} \qquad \text{if} \qquad p_2 < 1/(1+a) \tag{5.4}$$

and, therefore

$$f_2 = p_2(1+a) \qquad \text{if} \qquad p_2 < 1/(1+a) \tag{5.5}$$

Substituting the latter equality into (5.1) yields that I_2 should stay OUT if $p_2 < 1/(1+a)^2$, and v_2 can now be computed. The induction can be continued, and one finds that I_n should stay OUT if $p_n < 1/(1+a)^n$. If N is large enough, then $p_N = \varepsilon < 1/(1+a)^N$ and I_N stays out. Then $N - 1$ does not have new information, hence $p_{N-1} = p_N$ and also it stays out. We see that at least the initial competitors stay out. In particular, for fixed $\varepsilon > 0$, as $N \to \infty$ almost all competitors stay out: A little bit of uncertainty may make a lot of difference. (For more general results on long run players that are committed with small probability, see FUDENBERG AND LEVINE (1989).)

One may also imagine the situation in which the firms I_n know that II is not committed but in which they do not exactly know the profit function of II: Perhaps the market is even profitable as a duopoly for firm II. Call firm II strong in the latter case and weak if payoffs are as in Fig. 1. Assume firms I_n assign ex ante probability ε to II being strong. Intuitively this situation is very much like the one analyzed above: The strong type of firm II will always go IN and the weak type will pretend to be strong, at least initially. Hence, one expects the same outcome. This intuition is indeed confirmed by formal game theoretic analysis, but, what is perhaps a bit surprising at first, is that one needs a refinement of sequential equilibrium (i.e. a Forward Induction argument, or (formally) INWBR) to obtain this conclusion. If one does not use Forward Induction, one cannot eliminate counterintuitive equilibria in which I_n goes IN and II stays OUT irrespective of its type. For example, if p_2 is large enough (but $p_2 < 1$) it is possible that I_2 goes IN and that II stays OUT of market 2. The reason that II does not go in is that I_1 would (foolishly) interpret such fought entry as a signal that II is weak. INWBR forces I_1 to draw the proper conclusion that II is strong in such case, hence, it affords the strong type a profitable deviation, and eliminates such equilibria. (An interesting open question is to what extent the results of FUDENBERG AND LEVINE (1989) can be extended to games where the short run players are uncertain about the motives (payoffs) of the long run player.)

6 Conclusion

In this paper I have tried to make two related points:

(i) In many games that arise naturally there exists a multiplicity of equilibria. To come up with definite predictions, game theorists have had to refine their equilibrium concepts. In interesting classes of games, the multiplicity is caused by the existence of what, under closer examination, turn out to be incredible threats, either in actions or in beliefs. Several concepts that aim to exclude equilibria sustained by such incredible threats were illustrated and examples were given where even the most refined concepts do not give 'what we want', implying that either intuition is wrong or that the theory is incomplete.

(ii) Seemingly minor changes in the rules of the game may have drastic consequences on the outcome. We have played around with several variations of the basic market entry game from Section 2 and along the way we have encountered many different solutions. Hence, game theoretic predictions do not seem very robust. Closer examination, however, may reveal that the variations in the game were not minor ones at all, and that game theoretic analysis has given us the insight why such changes are essential. (Up to now, we do not yet have a satisfactory topology on games.) What should have become clear, however, is that modeling the knowledge of players is a delicate issue. This should be a point of concern for game theorists, especially since any game theoretic analysis assumes that the game itself is common knowledge. (For a nice illustration of the importance of common knowledge see RUBINSTEIN (1989).)

The issues raised above actually cast some doubt on the relevance of the refinements program. Namely, Forward Induction requires that one looks for consistent explanations of observed deviations within the given game. Since the model is narrowly defined it may indeed be possible to come up with a unique 'sensible' explanation of why a player deviated. If, however, one would allow for richer models[3] one probably would find many more consistent explanations, hence, Forward Induction may loose its power. One could actually have some kind of Uncertainty Principle: Within a given model, there exists a unique 'plausible' outcome, but over the class of plausible models, this outcome varies considerably. By tracing the class of 'plausible' models, one may trace out the set of all Nash equilibria of the original game; if one does not (or cannot) fix the game, refinement is futile. (A related point is made in FUDENBERG, KREPS AND LEVINE (1988), in my view, however, their topology on games is too coarse.)

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[3] The critique of BINMORE (1987) (see Fn. 1) also looses its force if one allows richer models.

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