Signaling Effects of Monetary Policy

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Abstract

We develop a dynamic general equilibrium model in which the policy rate signals the central bank's view about macroeconomic developments to price setters. The model is estimated with likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* as a measure of price setters' inflation expectations and real-time non-revised data from the Federal Reserve's Greenbook as a measure of the central bank's expectations about inflation and the output gap. In the 1970s, U.S. monetary policy is found to signal persistent inflationary shocks, accounting for why inflation and inflation expectations were so persistently heightened. Signaling effects of monetary policy also explain why inflation expectations adjusted more sluggishly than inflation after the robust monetary tightening of the 1980s.

Keywords: Bayesian estimation; real-time-data; higher-order beliefs; endogenous signals; price puzzle; persistent real effects of nominal shocks; marginal likelihood.

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1 Introduction

An important feature of economic systems is that information is dispersed across market participants and policymakers. Dispersed information implies that publicly observable policy actions transfer information to market participants. An important example is the monetary policy rate, which conveys information about the central bank's view on macroeconomic developments. Such an information transfer may strongly influence the transmission of monetary impulses and the central bank's ability to stabilize the economy. Consider the case in which a central bank expects that an exogenous disturbance will raise inflation in the next few quarters. On the one hand, as predicted by standard macroeconomic models, tightening monetary policy has the effect of mitigating the inflationary effects of the shock. On the other hand, raising the policy rate might also cause higher inflation if this action *signals* to unaware market participants that an inflationary shock is about to hit the economy. While the first type of monetary transmission has been intensively investigated in the economic literature, the signaling effects of monetary policy have received far less attention.

This paper develops a dynamic stochastic general equilibrium (DSGE) model to study the empirical relevance of the signaling effects of monetary policy and their implications for the propagation of policy and non-policy disturbances. In the model, price-setting firms face nominal rigidities and dispersed information. Firms observe their own specific technology conveying noisy private information about aggregate technology shocks that influence the future dynamics of firms' nominal marginal costs. Furthermore, price setters observe a noisy private signal about disturbances affecting households' discount factor (henceforth, demand shocks) as well as the policy rate set by the central bank according to a Taylor-type reaction function. The policy signal provides public information about the central bank's view on current inflation and the output gap to firms. The central bank is assumed to have imperfect information and thereby can make errors in forecasting the targeted macroeconomic aggregates. We call this model the *dispersed information model* (DIM).

The DIM features two channels of monetary transmission. The first channel is based on the central bank's ability to affect the real interest rate because of both nominal rigidities and dispersed information. Changes in the real interest rate induce households to intertemporally adjust their consumption. The second channel arises because the policy rate signals non-redundant information to firms and hence directly influences their beliefs about macroeconomic developments. We label this second channel *the signaling channel* of monetary transmission. The signaling effects of monetary policy on the propagation of shocks critically depend on how price setters interpret changes in the policy rate. For instance, raising the policy rate can be interpreted by price-setting firms in two ways. First, a monetary tightening might be read as the central bank responding to an exogenous deviation from its monetary policy rule; that is, a contractionary monetary shock or an overestimation of the rate of inflation or the output gap. Second, a higher interest rate may also be interpreted as the response of the central bank to inflationary non-policy shocks, which, in the model, are an adverse aggregate technology shock or a positive demand shock. If the first interpretation prevails among price setters, tightening (easing) monetary policy curbs (raises) firms' inflation expectations and hence inflation. If the second interpretation prevails, raising (cutting) the policy rate induces firms to expect higher (lower) inflation, and hence inflation rises (falls).

The model is estimated through likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. Furthermore, we use real-time non-revised data from the *Federal Reserve's Greenbook* as constructed by Orphanides (2004). Greenbook data provide information about the real-time estimates of the inflation rate and the output gap by the Federal Reserve, allowing us to exactly pin down the Federal Reserve's actual forecasts errors, which critically influence the signaling effects of monetary policy. The data range includes the 1970s, which were characterized by one of the most notorious episodes of heightened inflation and inflation expectations in recent U.S. economic history as well as by large and persistent mistakes by the Federal Reserve in estimating the output gap (Orphanides 2001, 2002). In the estimated model, firms rely mostly on private signals to learn about aggregate technology shocks and primarily on the policy signal to learn about demand shocks and exogenous deviations of the policy rate from the monetary rule. Furthermore, the policy rate is mostly informative about aggregate technology shocks, making it hard for firms to disentangle when the policy rate varies in response to demand shocks or when it varies in response to exogenous deviations from the policy rule.

This information structure has a two important implications for the propagation of aggregate disturbances in the DIM. First, contractionary monetary shocks end up signaling that the central bank is responding to positive demand shocks, mitigating substantially the disinflationary consequences of a monetary tightening. Second, the signaling effects of monetary policy bring about *deflationary* pressures in the aftermath of a *positive* demand shock. When the Federal Reserve raises the interest rate in response to a positive demand shock, firms attach some probability that both a contractionary monetary shock and a persistent overestimation of the output gap by the central bank might have occurred. These beliefs lower price setters' inflation expectations and hence inflation. From an econometric standpoint, the likelihood uses the signaling channel to transform demand shocks into *de-facto* supply shocks that move prices and quantities in opposite directions. Unlike an aggregate technology shock, this artificial supply shock implies a negative comovement between the federal funds rate and the rate of inflation as well as between the federal funds rate and inflation expectations. This property of the artificial supply shock helps the model fit the 1970s when the nominal federal funds rate was kept low and inflation expectations attained fairly high levels.

We introduce *Bayesian counterfactuals* to quantitatively assess the relevance of the signaling channel in explaining the dynamics of inflation and inflation expectations. It is important to emphasize that we use real-time data on inflation and the output gap from the Federal Reserve's Greenbook for this analysis. Hence, our econometric evaluation of the signaling effects of monetary policy controls for the highly accommodative monetary policy induced by the persistently large overestimation of potential output by the Federal Reserve in the 1970s. We find that the signaling effects of monetary policy explain why inflation and especially inflation expectations were so *persistently* heightened in the 1970s. The Federal Reserve's response to large negative demand shocks that occurred in that decade ended up signaling both expansionary monetary shocks and the central bank's underestimation of the current output gap, which persistently raised inflation and inflation expectations in that decade. Orphanides (2001, 2002, 2003) has argued that the Federal Reserve's overestimation of potential output in the 1970s led to over-expansionary policies, which ultimately resulted in high inflation. To the extent that monetary policy signaled the central bank's mistakes in estimating the current output gap, the signaling channel nicely enriches Orphanides' argument by substantially strengthening its ability to account for the persistent dynamics of inflation and inflation expectations in the 1970s.

Furthermore, the signaling channel plays an important role in explaining why inflation expectations fell more sluggishly and were almost always higher than the actual rate of inflation throughout the 1980s. The signaling effects associated with the central bank responding to technology shocks turn out to affect the dynamics of inflation and inflation expectations at different frequencies. While these signaling effects primarily affect the high frequency dynamics of the rate of inflation, they influence inflation expectations at lower frequencies. Negative technology shocks occurring in the late 1970s and early 1980s contributed to raise the policy rate. This policy ended up signaling contractionary deviations from the policy rule, which contributed to lower actual inflation in the short run. However, eight quarters after the realization of these technology shocks, monetary policy started signaling expansionary deviations from the Taylor rule, which exerted sizable upward pressures on inflation expectations. Nevertheless, at such low frequencies, signaling effects on inflation were dominated by more recent deflationary shocks. This differential speed of adjustment between inflation and inflation expectations to the signaling effects due to technology shocks is tightly linked to the *endogeneity* of the policy signal and the amount of information that private signals convey about each shock relative to what the policy signal conveys.

The DIM is found to fit the data better than a model in which price setters have perfect information (i.e., the perfect information model, or PIM). This finding validates the use of the DIM to study the signaling effects of monetary policy, since the PIM is a prototypical New Keynesian model that has been extensively used by scholars for conducting quantitative analysis about monetary policy (e.g., Rotemberg and Woodford 1997; Clarida, Galí, and Gertler 2000;

Lubik and Schorfheide 2004; Coibion and Gorodnichenko 2011). The central reason for why the DIM fits the data better than the PIM is the former's ability to capture the persistent dynamics of inflation and the inflation expectations (SPF). The fact that the DIM fits the SPF better than a perfect information model is not obvious. In fact, Del Negro and Eusepi (2011) find that the imperfect information model by Erceg and Levin (2003) is outperformed by a standard perfect information model in fitting the SPF. We also show that the DIM can explain remarkably well the dynamics of the nowcast inflation errors implied by the SPF. Neither this series nor the zero-quarter-ahead SPF inflation expectations are used in our estimation, and therefore, this result constitutes an important out-of-sample validation for the DIM. While the PIM implies zero inflation nowcast errors by construction, these errors are far from being negligible in the data: they reach up to 2 percentage points in the sample period and exhibit fairly high volatility even during the Great Moderation period.

This is the first paper that provides an econometric analysis on signaling effects of monetary policy based on a microfounded dynamic general equilibrium model. Using reduced-form model, Nakamura and Steinsson (2013), Romer and Romer (2000), and Tang (2015) find evidence of signaling effects of monetary policy in the U.S. Moreover, in line with Campbell et al. (2012), Nakamura and Steinsson (2013) find evidence of a "Fed information effect": Federal Open Market Committee (FOMC) announcements affect expectations not only about the evolution of monetary policy but also about future economic fundamentals.

The idea that the monetary authority sends public signals to an economy in which agents have dispersed information was pioneered by Morris and Shin (2003a, 2003b). While technical hurdles have prevented empiricists from conducting a structural investigation of the signaling effects of monetary policy so far, the theoretical literature has been flourishing quickly. The space in this section is regrettably too small to do justice to all these theoretical contributions. Angeletos, Hellwig, and Pavan (2006) study the signaling effects of policy decisions in a coordination game. Walsh (2010) shows that the (perceived or actual) signaling effects of monetary policy alter the central bank's decisions, resulting in a bias (i.e., an opacity bias) that distorts the central bank's optimal response to shocks. Unlike this paper, Walsh's study is based on a model that does not feature dispersed information. Baeriswyl and Cornand (2010) study optimal monetary policy in a DSGE model in which the central bank can use its policy instrument to disclose information about its assessment of the fundamentals. Price setters face two sources of information limitation: sticky information à la Mankiw and Reis (2002) and dispersed information of a type that is similar to that of this paper. That contribution is mostly theoretical, whereas this paper carries out a full-fledged likelihood estimation of a model in which monetary policy has signaling effects. Hachem and Wu (2014) develop a model in which firms update their heterogeneous inflation expectations through social dynamics to study the effects of central bank communication. Frenkel and Kartik (2015) provide a theoretical investigation of the signaling channel of monetary transmission.

The model studied in this paper is built on Nimark (2008). A particularly useful feature of Nimark's model is that the supply side of the model economy can be analytically worked out and characterized by an equation that nests the standard New Keynesian Phillips curve. The model studied in this paper shares this feature. Nonetheless, in Nimark (2008) the signaling channel does not arise because assumptions about the Taylor-rule specification imply that the policy rate conveys only redundant information to price setters. We introduce a method to solve the DIM that belongs to the more general class of solution methods introduced by Nimark (2011). Our solution method improves upon the one used by Nimark (2008) in that it does not require solving numerically any nonlinear equations.

This paper is also related to a quickly growing empirical literature that uses the SPF to study the response of public expectations to monetary policy decisions. Del Negro and Eusepi (2011) perform an econometric evaluation of the extent to which the inflation expectations generated by DSGE models are in line with the observed inflation expectations. There are three main differences between that paper and this one. First, in our settings, price setters have heterogeneous and dispersed higher-order expectations as they observe private signals. Second, this paper fits the model to a data set that includes the 1970s, whereas Del Negro and Eusepi (2011) use a data set starting from the early 1980s. Coibion and Gorodnichenko (2012*b*) find that the Federal Reserve raises the policy rate more gradually if the private sector's inflation expectations are lower than the Federal Reserve's forecasts of inflation. This empirical evidence can be rationalized in a model in which monetary policy has signaling effects and the central bank acts strategically to stabilize public inflation expectations. Coibion and Gorodnichenko (2012*a*) use the SPF to document robust evidence in favor of models with informational rigidities.

This paper also belongs to a quite thin literature that carries out likelihood-based analyses on models with dispersed information. Nimark (2014) estimates an island model built on Lorenzoni (2009) and augmented with man-bites-dog signals, which are signals that are more likely to be observed when unusual events occur. Maćkowiak, Moench, and Wiederholt (2009) use a dynamic factor model to estimate impulse responses of sectoral price indexes to aggregate shocks and to sector-specific shocks for a number of models, including a rational inattention model. Melosi (2014) conducts an econometric analysis of a stylized DSGE model with dispersed information à la Woodford (2002).

Bianchi and Melosi (2012) develop a DSGE model that features waves of agents' pessimism about how aggressively the central bank will react to future changes in inflation to study the welfare implications of monetary policy communication. Gorodnichenko (2008) introduces a model in which firms make state-dependent decisions on both pricing and acquisition of information and shows that this model delivers a delayed response of inflation to monetary shocks.

Trabandt (2007) analyzes the empirical properties of a state-of-the-art sticky-information DSGE model à la Mankiw and Reis (2002) and compares them with those of a state-of-the-art DSGE model with sticky prices à la Calvo.

The paper is organized as follows. Section 2 describes the dispersed information model, in which monetary policy has signaling effects, as well as a model in which firms have perfect information. Section 3 presents the empirical analysis of the paper, including the econometric evaluation of the signaling effects of monetary policy over the sample period. In Section 4, we assess the robustness of our findings to changes in model specification. In Section 5, we present our conclusions.

2 Models

Section 2.1 introduces the model with dispersed information and signaling effects of monetary policy. In Section 2.2, we present the time protocol of the model. Section 2.3 presents the problem of households. Section 2.4 presents firms' price-setting problem. In Section 2.5, the central bank's behavior and government's behavior are modeled. In Section 2.6, we introduce the information set available to firms and its rationale. Section 2.7 deals with the log-linearization and the solution of the dispersed information model. Finally, Section 2.8 presents the perfect information model, which will turn out to be useful for evaluating the empirical significance of the dispersed information model.

2.1 The Dispersed Information Model (DIM)

The economy is populated by a continuum $(0,1)$ of households, a continuum $(0,1)$ of monopolistically competitive firms, a central bank (or monetary authority), and a government (or fiscal authority). A Calvo lottery establishes which firms are allowed to reoptimize their prices in any given period t (Calvo 1983). Households consume the goods produced by firms, demand government bonds, pay taxes to or receive transfers from the fiscal authority, and supply labor to the firms in a perfectly competitive labor market. Firms sell differentiated goods to households. The fiscal authority has to finance maturing government bonds. The fiscal authority can issue new government bonds and can either collect lump-sum taxes from households or pay transfers to households. The central bank sets the nominal interest rate at which the government's bonds pay out their return.

2.2 The Time Protocol

Any period t is divided into three stages. All actions that are taken in any given stage are simultaneous. At stage 0, the central bank sets the interest rate for the current period t using a Taylor-type reaction function and after observing an imperfect measure of current inflation and the output gap. At stage 1, firms update their information set by observing (i) their idiosyncratic technology, (ii) a private signal about the demand shocks, and (iii) the interest rate set by the central bank. Given these observations, firms set their prices at stage 1. At stage 2, households learn about the realization of all the shocks in the economy and therefore become perfectly informed. Households then decide their consumption, C_t; their demand for (one-period) nominal government bonds, B_t; and their labor supply, N_t. At this stage, firms hire labor and produce so as to deliver the demanded quantity at the price they have set at stage 1. The fiscal authority issues bonds and collects taxes from households or pays transfers to households. The markets for goods, labor, and bonds clear.

2.3 Households

Households have perfect information, and hence, we can use the representative household to solve their problem at stage 2 of every period t:

$$\max_{C_{t+s},B_{t+s},N_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} g_{t+s} \left[\ln C_{t+s} - \chi_n N_{t+s}\right],$$

where β is the deterministic discount factor and g_t is an exogenous variable influencing households' discount factor. The logarithm of this exogenous variable follows an autoregressive process: $\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}$ with Gaussian shocks $\varepsilon_{g,t} \backsim \mathcal{N}(0,1)$. We refer to g_t as *demand conditions* and to the innovation $\varepsilon_{g,t}$ as the *demand shock*. Disutility from labor linearly enters the period utility function. Note that χ_n is a parameter that affects the marginal disutility of labor.

The flow budget constraint of the representative household in period t is given as follows:

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + \Pi_t - T_t, \tag{1}$$

where P_t is the price level of the composite good consumed by households and W_t is the (competitive) nominal wage, R_t stands for the nominal (gross) interest rate, Π_t is the (equally shared) dividends paid out by the firms, and T_t stands for the lump-sum transfers/taxes. Composite consumption in period t is given by the Dixit-Stiglitz aggregator $C_t = \left(\int_0^1 C_{j,t}^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}}$, where $C_{j,t}$ is consumption of the good produced by firm j in period t and ν is the elasticity of substitution between consumption goods.

At stage 2 of every period t, the representative household chooses its consumption of the good produced by firm j, labor supply, and bond holdings subject to the sequence of the flow budget constraints and a no-Ponzi-scheme condition. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump-sum transfers/taxes, and the prices of all consumption goods. It can be shown that the demand for the good produced by firm j is:

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\nu} C_t, \tag{2}$$

where the price level of the composite good is given by $P_t = \left(\int (P_{j,t})^{1-\nu}\, di\right)^{\frac{1}{1-\nu}}$.

2.4 Firms' Price-Setting Problem

Firms are endowed with a linear technology $Y_{j,t} = a_{j,t}N_{j,t}$, where $Y_{j,t}$ is the output produced by the firm j at time t, $N_{j,t}$ is the amount of labor employed by firm j at time t, and $a_{j,t}$ is the firm-specific level of technology that can be decomposed into a level of aggregate technology (a_t) and a white-noise firm-specific component ($\varepsilon^a_{j,t}$). More specifically,

$$\ln a_{j,t} = \ln a_t + \widetilde{\sigma}_a \varepsilon^a_{j,t}, \tag{3}$$

where $\varepsilon^a_{j,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$ and a_t stands for the *level of aggregate technology* that evolves according to the autoregressive process $\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon_{a,t}$ with Gaussian innovations $\varepsilon_{a,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. We refer to the innovation $\varepsilon_{a,t}$ as the (aggregate) *technology shock*.

Following Calvo (1983), we assume that a fraction θ of firms are not allowed to reoptimize the price of their at stage 1 of any period. Those firms that are not allowed to reoptimize are assumed to index their price to the steady-state inflation rate. Let us denote the (gross) steady-state inflation rate as π_*, the nominal marginal costs for firm j as $MC_{j,t} = W_t/a_{j,t}$, the time t value of one unit of the composite consumption good in period $t+s$ to the representative household as $\xi_{t|t+s}$, and the expectation operator conditional on firm j's information set $\mathcal{I}_{j,t}$ as $\mathbb{E}_{j,t}$. The information set contains both private and public signals and will be defined in Section 2.6. At stage 1 of every period t, an arbitrary firm j that is allowed to reoptimize its price $P_{j,t}$ solves

$$\max_{P_{j,t}} \quad \mathbb{E}_{j,t}\left[\sum_{s=0}^{\infty} (\beta\theta)^s\, \xi_{t|t+s} \left(\pi_*^s P_{j,t} - MC_{j,t+s}\right) Y_{j,t+s}\right],$$

subject to $Y_{j,t} = C_{j,t}$ (i.e., firms commit themselves to satisfying any demanded quantity that will arise at stage 2), to the firm j's specific demand in equation (2), and to the linear production function. When solving the price-setting problem at stage 1, firms have to form expectations about the evolution of their nominal marginal costs, which will be realized in the next stage of the period (i.e., stage 2), using their information set $\mathcal{I}_{j,t}$. At stage 2, firms produce and deliver the quantity the representative household demands for their specific goods at the prices they set in the previous stage 1. At stage 2 we assume that firms do not receive any further information or any additional signals to what they have already observed at stage 1.

2.5 The Monetary and Fiscal Authorities

The monetary authority sets the nominal interest rate according to a Taylor-type reaction function: $R_t = (r_* \pi_*) (\tilde{\pi}_t / \pi_*)^{\phi_\pi} (\tilde{x}_t)^{\phi_x} \xi_{m,t}$, where r_* is the steady-state real interest rate and $\tilde{\pi}_t$ is the inflation rate observed by the central bank at stage 0 of time t when it has to set the interest rate R_t. We assume that the central bank knows the current inflation rate π_t up to the realization of a random variable that follows an autoregressive process $\ln \xi_{\pi,t} = \rho_\pi \ln \xi_{\pi,t-1} + \sigma_\pi \varepsilon_{\pi,t}$ with Gaussian innovations $\varepsilon_{\pi,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. This exogenous process captures the central bank's nowcast errors for the inflation rate. In symbols, we write this as follows: $\tilde{\pi}_t = \pi_t \xi_{\pi,t}$. We will refer to the process $\xi_{\pi,t}$ as the *central bank's measurement error for inflation*. Analogously, $\tilde{x}_t$ denotes the output gap when the central bank is called to set the policy rate at stage 0. We assume that the central bank knows the current output gap x_t up to the realization of a random variable that follows an autoregressive process $\ln \xi_{x,t} = \rho_x \ln \xi_{x,t-1} + \sigma_x \varepsilon_{x,t}$ with Gaussian innovations $\varepsilon_{x,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. This exogenous process captures the central bank's nowcast errors for the output gap. We will refer to the process $\xi_{x,t}$ as the *central bank's measurement error for the output gap*. In symbols, we write this as follows: $\tilde{x}_t = x_t \xi_{x,t}$. Note that the actual output gap x_t is given by Y_t / Y_t^*, where Y_t^* stands for the potential level of output, which would be realized if prices were perfectly flexible and firms were perfectly informed. Furthermore, the process $\xi_{m,t}$ is an exogenous random variable that is driven by the following autoregressive process: $\ln \xi_{m,t} = \rho_m \ln \xi_{m,t-1} + \sigma_m \varepsilon_{m,t}$, with Gaussian innovations $\varepsilon_{m,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. We will refer to the process $\xi_{m,t}$ as the *state of monetary policy* and to the innovation $\varepsilon_{m,t}$ as the *monetary policy shock*.

It should be noted that we model policy inertia as a persistent monetary policy shock rather than adding a smoothing component. Rudebusch (2002, 2006) uses term-structure data to argue that monetary policy inertia likely reflects omitted variables in the rule and that such policy inertia can be adequately approximated by persistent shocks in the rule. Furthermore, this modeling choice serves the purpose of solving the dispersed information model fast enough to allow likelihood estimation.

The policy rule can then be rewritten as follows:

$$R_t = (r_* \pi_*) \left(\frac{\pi_t}{\pi_*} \right)^{\phi_\pi} x_t^{\phi_x} \eta_{r,t}, \tag{4}$$

where $\eta_{r,t} \equiv \xi_{m,t} \xi_{\pi,t}^{\phi_\pi} \xi_{x,t}^{\phi_x}$ captures the exogenous deviations of the interest rate from the monetary policy rule. These deviations may occur as a result of monetary policy shocks $\varepsilon_{m,t}$ or as a result of measurement errors by the central bank, $\varepsilon_{\pi t}$ and $\varepsilon_{x,t}$. We will refer to the process $\eta_{r,t}$ as the *exogenous deviation from the policy rule.*

The flow budget constraint of the fiscal authority in period t is represented as follows $R_{t-1}B_{t-1} - B_t = T_t$. The fiscal authority finances maturing government bonds by either collecting lump-sum taxes or issuing new government bonds. The aggregate resource constraint implies $Y_t = C_t$.

2.6 Firms' Information Set

Firms have imperfect knowledge about the history of shocks that have hit the economy. More specifically, it is assumed that firms' information set includes the history of firm-specific technology $\ln a_{j,t}$ and the history of a private signal $g_{j,t}$ on the demand conditions g_t, which evolves according to the following process: $\ln g_{j,t} = \ln g_t + \widetilde{\sigma}_g \varepsilon^g_{j,t}$, where $\varepsilon^g_{j,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. Moreover, firms observe the history of the nominal interest rate R_t set by the central bank, as well as the history of their own prices.[1] To sum up, the information set $\mathcal{I}_{j,t}$ of firm j at time t is given by

$$\mathcal{I}_{j,t} \equiv \{\ln a_{j,\tau}, \ln g_{j,\tau}, R_\tau, P_{j,\tau} : \tau \leq t\}. \tag{5}$$

Firms receive the signals in $\mathcal{I}_{j,t}$ at the price-setting stage 1. We assume that firms know the structural equations of the model and its parameters. For tractability, firms use the log-linear approximation to the model structural equations around its steady-state equilibrium to solve their signal extraction problem.[2] Finally, we assume that firms have received an infinitely long sequence of signals at any time t. This assumption substantially simplifies the task of solving the model by ensuring that the Kalman gain matrix is time invariant and the same across firms.

We follow the imperfect-common-knowledge literature (Woodford, 2002; Adam, 2007; Nimark, 2008) in modeling the highly complex process of acquiring the relevant information by price setters (including information about endogenous variables other than the policy rate, such as the quantities sold by firm j $C_{j,t}$, NIPA statistics with some lags, etc.) using a set of exogenous private signals ($\widehat{a}_{j,t}$ and $\hat{g}_{j,t}$).[3] These exogenous signals are assumed to be idiosyncratic

[1] Observing the history of their own price $\{P_{j,\tau} : \tau \leq t\}$ conveys only redundant information to firms because their price is either adjusted to the steady-state inflation rate, which is known by firms, or a function of the history of the signals that have been already observed in the past. Thus, this signal does not play any role in the formation of firms' expectations and will be called the redundant signal. Henceforth, when we refer to signals, we mean only the non-redundant signals (namely, $\ln A_{j,t}$, $\ln g_{j,t}$, and $\ln(R_t/r_*\pi_*)$).

[2] The log-linearized equations will be shown in the next section.

[3] In this respect, an important advantage of the rational inattention literature (e.g., Sims, 2003, 2006, 2010; Mackowiak and Wiederholt, 2009, forthcoming) is to go beyond this reduced-form approach by allowing agents to optimally choose their signal structure under an information-processing constraint that limits the overall amount of information the signals can convey. Nonetheless, estimating a rational inattention model is not feasible at this stage because solving the problem of how firms allocate their attention optimally would increase even more the already heavy computational burden that characterizes the solution of the DIM.

to capture the idea that price setters may pay attention to different indicators. We partially depart from this literature as we do not allow firms to observe private signals on all five exogenous state variables, which also include the three subcomponents ($\xi_{m,t}$, $\xi_{\pi,t}$, and $\xi_{x,t}$) of the overall state of monetary policy $\eta_{r,t}$. Allowing firms to observe specific exogenous signals on the central bank's measurement errors (i.e., $\xi_{\pi,t}$ and $\xi_{x,t}$) would imply allowing firms to have an information advantage about the central bank's measurement errors over the central bank itself. This assumption is clearly controversial. In the model price setters know the law of motion of the central bank's measurement errors but they have to learn the magnitude thereof in every period. A less controversial assumption is to endow the firms also with a private signal about the exogenous deviations from the policy rule $\eta_{r,t}$. However, the estimated value for the noise variance of this additional private signal turns out to be so large to become non-identifiable. The presence of a non-identifiable parameter also affects the convergence of the estimation procedure for the other parameters. Thus, we did not include this additional signal to firms' information set. It should also be emphasized that our information structure follows Woodford (2002) in assuming that firms observe a truth-plus-white-noise type of signals with serially uncorrelated noise shocks. This signal structure is arguably quite restrictive parametrically. However, these restrictions are crucial to avoid weak identification of the model parameters.

A novel ingredient of the model is to allow firms to perfectly observe the interest rate set by the central bank R_t. This assumption is based on the fact that the monetary policy rate is measured very accurately in real time and is subject neither to revisions nor to delays in reporting. These features do not extend to other aggregate endogenous variables, such as inflation or output (e.g., GDP). Moreover, Andrade et al. (2013) document that the *Blue Chip Financial Forecasts* show very small disagreement on the next quarter's federal funds rate compared with other leading macroeconomic aggregates, such as inflation and GDP.

In Section 4 we will show that the maintained information structure in (5) delivers quite plausible dynamics for inflation nowcast errors in the estimated DIM. Furthermore, we will also show that assuming that firms observe other endogenous variables, such as the quantity firms have sold, turns out to substantially deteriorate the fit of the dispersed information model.

2.7 Log-linearization and Model Solution

We solve the firms' and households' problems, described in Sections 2.3 and 2.4, and obtain the consumption Euler equation and the price-setting equation. We denote the log-deviation of an arbitrary (stationary) variable x_t from its steady-state value as $\widehat{x}_t$. As in Nimark (2008), we obtain the imperfect-common-knowledge Phillips curve that is given as follows:[4]

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{mc}_{t|t}^{(k)} + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{\pi}_{t+1|t}^{(k)}. \tag{6}$$

In this equation, $\widehat{\pi}_{t+1|t}^{(k)}$ denotes the average k-th order expectations about the next period's inflation rate, $\widehat{\pi}_{t+1}$, that is, $\widehat{\pi}_{t+1|t}^{(k)} \equiv \underbrace{\int \mathbb{E}_{j,t}\ldots\int \mathbb{E}_{j,t}}_{k}\widehat{\pi}_{t+1}dj...dj$, for any integer $k>1$. Moreover, $\widehat{mc}_{t|t}^{(k)}$ denotes the average k-th order expectations about the real aggregate marginal costs $\widehat{mc}_t \equiv \int \widehat{mc}_{j,t}dj$, which evolve according to the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$ for any integer $k>1$.

The log-linearized Euler equation is standard and is given as follows:

$$\widehat{g}_t - \widehat{y}_t = \mathbb{E}_t\widehat{g}_{t+1} - \mathbb{E}_t\widehat{y}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1} + \hat{R}_t, \tag{7}$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation operator conditional on the complete information set. The central bank's reaction function (4) can be written as follows:

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_y(\hat{y}_t - \widehat{y}_t^*) + \widehat{\eta}_{r,t}. \tag{8}$$

The demand conditions evolve according to $\widehat{g}_t = \rho_g\widehat{g}_{t-1} + \sigma_g\varepsilon_{g,t}$. The process for aggregate technology becomes $\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \sigma_a\varepsilon_{a,t}$. The exogenous process that leads the central bank to deviate from the monetary rule is defined as $\widehat{\eta}_{r,t} = \widehat{\xi}_{m,t} + \phi_\pi\widehat{\xi}_{\pi,t} + \phi_x\widehat{\xi}_{x,t}$. The subcomponents of $\widehat{\eta}_{r,t}$evolve as follows: $\widehat{\xi}_{i,t} = \rho_i\widehat{\xi}_{i,t} + \sigma_i\varepsilon_{i,t}$ with $i \in \{m,\pi,x\}$. We log-linearize the signal equation concerning the level of aggregate technology (3) and obtain $\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a\varepsilon_{j,t}^a$. The signal equation concerning the demand conditions is written $\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g\varepsilon_{j,t}^g$. The policy signal $\hat{R}_t$ evolves according to equation (8).

It is important to emphasize that the average higher-order expectations enter the specification of the Phillips curve in equation (6) because price setters *forecast the forecasts* of other price setters (Townsend 1983a, 1983b). Also note that the Calvo parameter θ determines the structure of weights for the higher-order expectations in the averages $\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{mc}_{t|t}^{(k)}$ and $\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{\pi}_{t+1|t}^{(k)}$. The smaller the Calvo parameter, the more the model dynamics are affected by the the average expectations of relatively higher orders.

A detailed description of how we solve the model is provided in Appendix B. The proposed solution algorithm improves upon the one used in Nimark (2008) as our approach does not require solving a system of nonlinear equations.[5] When the model is solved, the law of motion

[4] See Appendix A for a detailed derivation.

[5] Nimark (2009) introduces a method to improve the efficiency of these types of solution methods for dispersed

of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]'$ reads as follows:

$$\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}, \tag{9}$$

where $X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{g}_{t|t}^{(s)}, \widehat{\xi}_{m,t|t}^{(s)}, \widehat{\xi}_{\pi,t|t}^{(s)}, \widehat{\xi}_{x,t|t}^{(s)} : 0 \leq s \leq k\right]'$ is the vector of the average expectations of any order from zero through the truncation $k > 0$ about the exogenous state variables $X_t = \left(\widehat{a}_t, \widehat{g}_t, \hat{\xi}_{m,t}, \hat{\xi}_{\pi,t}, \hat{\xi}_{x,t}\right)$. The average s-th order expectations about the level of aggregate technology, $\widehat{a}_{t|t}^{(s)}$, are defined as the integral of firms' expectations about the average $(s-1)$-th order expectations across firms. In symbols, this is given as follows: $\widehat{a}_{t|t}^{(s)} = \int \mathbb{E}_{j,t}\left(\widehat{a}_{t|t}^{(s-1)}\right) dj$, for $1 \leq s \leq k$, where conventionally $\widehat{a}_{t|t}^{(0)} = \widehat{a}_t$. The average expectations about the demand conditions $\hat{g}_t$, the state of monetary policy $\hat{\xi}_{m,t}$, and the central bank's measurement errors for inflation $\hat{\xi}_{\pi,t}$ and for the output gap $\hat{\xi}_{x,t}$ are analogously defined. Note that in order to keep the dimensionality of the state vector finite, we truncate the infinite hierarchy of average higher-order expectations, considering only orders smaller than or equal to twenty. The vector of average expectations about the exogenous state variables $X_{t|t}^{(0:k)}$ is assumed to follow a Vector AutoRegressive (VAR) model of order one:[6]

$$X_{t|t}^{(0:k)} = \mathbf{M} X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t. \tag{10}$$

We solve the model by guessing and verify the dynamics of higher-order beliefs (i.e., the matrices $\mathbf{M}$ and $\mathbf{N}$). However, it is important to clarify that this is not the only approach for solving these types of models. More specifically, there exist other approaches that rely on the fact that average first-order expectations about the endogenous variables can be computed given the guessed laws of motion of the endogenous variables by using the assumption of rational expectations. In this case, the problem of solving the model boils down to find a fixed point over the parameters that characterizes the laws of motion for the endogenous variables of interest. See Maćkowiak and Wiederholt (2009) for an example of how this type of solution method works. When applied to our model, this approach turns out to be harder to combine with the estimation procedure (i.e., the Metropolis-Hastings posterior simulator), which requires a high degree of automatization of the solution routine. Furthermore, studying the higher-order

information models in which agents (e.g., firms) use lagged endogenous variables to form their beliefs. An alternative solution algorithm based on rewriting the equilibrium dynamics partly as a moving-average process and setting the lag with which the state is revealed to be a very large number is analyzed by Hellwig (2002) and Hellwig and Vankateswaran (2009). Rondina and Walker (2012) study a new class of rational expectations equilibria in dynamic economies with dispersed information and signal extraction from endogenous variables.

[6] As is standard in the literature (e.g., Woodford 2002), we focus on equilibria where the higher-order expectations about the exogenous state variables follow a VAR model of order one. To solve the model, we also assume common knowledge of rationality. See Nimark (2008, Assumption 1, p. 373) for a formal formulation of the assumption of common knowledge of rationality.

beliefs helps interpret some of the predictions of the model.

2.8 The Perfect Information Model (PIM)

If firms were perfectly informed, higher-order uncertainty would fade away (i.e., $X_{t|t}^{(k)} = X_t$ for any integer $k > 0$) and the linearized model would boil down to a prototypical three-equation New Keynesian DSGE model (e.g., Rotemberg and Woodford 1997; Lubik and Schorfheide 2004; Rabanal and Rubio-Ramírez 2005). Unlike in the dispersed information model, we add an exogenous process affecting the price markup so as to avoid stochastic singularity of this model, which would preclude estimation. The exogenous markup evolves according to the autoregressive process $\widehat{\xi}_{p,t} = \rho_p \widehat{\xi}_{p,t-1} + \sigma_p \varepsilon_{p,t}$ with Gaussian innovations $\varepsilon_{p,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$.[7] The new Keynesian Phillips curve is given as follows: $\hat{\pi}_t = \kappa_{pc} \widehat{mc}_t + \beta \mathbb{E}_t \widehat{\pi}_{t+1} + \widehat{\xi}_{p,t}$, where $\kappa_{pc} \equiv (1-\theta)(1-\theta\beta)/\theta$ with the real marginal costs given by $\widehat{mc}_t = \hat{y}_t - \widehat{a}_t$. The Euler equation and the Taylor rule are the same as in the dispersed information model. We call this prototypical New Keynesian DSGE model the perfect information model (PIM).

3 Empirical Analysis

This section contains the econometric analysis of the model and the signaling channel of monetary policy. Section 3.1 presents the data set and the state-space model for the econometrician. In Section 3.2, we discuss the prior and posterior distribution for the model parameters. In Section 3.3, we evaluate the ability of the DIM to fit the data relative to that of the PIM. In Section 3.4, we study the propagation of unanticipated structural disturbances. In Section 3.5, we run a Bayesian counterfactual experiment to assess the empirical relevance of the signaling effects of monetary policy.

3.1 The State-Space Model for the Econometrician

The data set is constructed using the following seven observable variables: U.S. per-capita real GDP, U.S. inflation rate (GDP deflator), the federal funds rate, one-quarter-ahead and four-quarter-ahead inflation expectations from the *Survey of Professional Forecasters* (SPF), the real-time output gap from the Federal Reserve's Greenbook, and the real-time inflation from the Federal Reserve's Greenbook. Data are quarterly and run from 1970:Q3 through 2007:Q4.

[7] In our estimation we use data on both the output gap and inflation. In the absence of price markup shocks, it is well-known that the three-equation perfect information model features almost perfect correlation between the output gap and inflation, causing the model to be stochastically singular. Adding a markup shock loosens this tight relation between the output gap and inflation, allowing us to estimate the perfect information model.

The measurement equations are:

$$\ln\left(\frac{GDP_t}{POP_t^{\geq 16}}\right) - HPF\left[\ln\left(\frac{GDP_t}{POP_t^{\geq 16}}\right)\right] = \widehat{y}_t - \widehat{a}_t, \tag{11}$$

$$\ln\left(\frac{PGDP_t}{PGDP_{t-1}}\right) = \ln \pi_* + \widehat{\pi}_t, \tag{12}$$

$$FEDRATE_t = \ln R_* + \widehat{R}_t, \tag{13}$$

$$\ln\left(\frac{PGDP3_t}{PGDP2_t}\right) = \ln \pi_* + \widehat{\pi}^{(1)}_{t+1|t} + \varepsilon_t^{\mu_1}, \tag{14}$$

$$\ln\left(\frac{PGDP6_t}{PGDP5_t}\right) = \ln \pi_* + \widehat{\pi}^{(1)}_{t+4|t} + \varepsilon_t^{\mu_2}, \tag{15}$$

$$\ln OGAP_t^{GB} = \widehat{y}_t - \widehat{a}_t + \widehat{\xi}_{x,t}, \tag{16}$$

$$\ln INFL_t^{GB} = \ln \pi_* + \widehat{\pi}_t + \widehat{\xi}_{\pi,t}. \tag{17}$$

For these equations, $HPF\left[\ln\left(\frac{GDP_t}{POP_t^{\geq 16}}\right)\right]$ denotes the Hodrick–Prescott (HP) filter of real per-capita GDP, which is used to compute the potential output;[8] GDP_t is the real gross domestic product computed by the U.S. Bureau of Economic Analysis (BEA) (Haver Analytics' mnemonic: *GDPC96*); $POP_t^{\geq 16}$ is the civilian non-institutional population aged 16 years old and over as computed by the U.S. Bureau of Labor Statistics (BLS) (Haver Analytics' mnemonic: *LNS10000000*); $PGDP_t$ is the GDP deflator computed by the BEA (Haver Analytics' mnemonic: *GDPDEF*); $FEDRATE$ is the average of daily figures of the effective federal funds rate (Haver Analytics' mnemonic: *FEDFUNDS*) reported by the Federal Reserve Economic Data (FRED) database managed by the Federal Reserve Bank of St. Louis; and $PGDP2_t$, $PGDP3_t$, $PGDP5_t$, and $PGDP6_t$ are the SPF's mnemonics for the median forecasts about the current, one-quarter-ahead, three-quarters-ahead, and four-quarters-ahead GDP price indexes, respectively. We relate these statistics with the first moment of the distribution of firms' expectations implied by the model.[9] To avoid stochastic singularity, we introduce two Gaussian measurement errors $\varepsilon_t^{\mu_1} \overset{iid}{\backsim} \mathcal{N}(0, \sigma_{\mu 1})$ and $\varepsilon_t^{\mu_2} \overset{iid}{\backsim} \mathcal{N}(0, \sigma_{\mu 2})$. The last two observables $\ln OGAP_t^{GB}$ and $\ln INFL_t^{GB}$ are the real-time data on the output gap and inflation from

[8] The results are robust if one computes the potential output using a quadratic trend instead of the HP filter.

[9] A more coherent way to construct the data on inflation expectations is to compute the cross-sectional mean or median of the inflation forecasts of each individual forecaster. This measure would be closer to our model concept of the average first-order expectations. The Federal Reserve Bank of Philadelphia does release the individual forecasts with an ID to track each forecaster over time. However, I decided not to construct the series for the inflation expectations starting with these individual data because how the ID is assigned and managed, especially before the Federal Reserve of Philadelphia took over, is unclear, casting serious doubts on whether these series are reliable. These concerns are detailed in Section 4 (Forecasts of Individual Participants) of the documentation of the SPF.

the Greenbook. These data are measured in *real time* (non revised) and capture the information set available to the Federal Open Market Committee. These series were constructed by Orphanides (2004) until 1995:Q4. I completed the data set using the tables kept by the Federal Reserve Bank of Philadelphia after harmonizing it.[10] A quick look at equations (11) and (16) and equations (12) and (17) shows that the two central bank's measurement errors $\widehat{\xi}_{\pi,t}$ and $\widehat{\xi}_{x,t}$ are exactly pinned down by the data. In this respect, an appealing feature of having the HP filtered output gap among the observables is that the central bank's measurement errors about the output gap exactly mimic the one in Orphanides (2004) when we estimate and evaluate the model. We believe that this helps obtain a neat assessment of the merits of the signaling channel in explaining the dynamics of inflation and inflation expectations relative to the merits of the mechanism proposed by Orphanides.

3.2 Bayesian Estimation

As is standard, we fix the value for β so that the steady-state nominal interest rate R_* is equal to its sample average. The prior and posterior statistics for the model parameters are reported in Table 1. The prior distribution for the Calvo parameter θ is centered at zero and its variance is sufficiently large to make our a-priori view about this parameter value fairly agnostic. As it will be clear, the degree of persistence of the signaling effects ultimately hinges on the persistence of the shocks monetary policy ends up signaling to firms by changing the policy rate. Therefore, the priors for the autoregressive parameters ρ_a, ρ_g, ρ_m, ρ_π, and ρ_x are set to be broad enough to accommodate a wide range of persistence degrees for the five exogenous processes. The value of the volatilities for the structural innovations (i.e., σ_a, σ_g, σ_m, σ_π, and σ_x) are also crucial as they affect firms' signaling extraction problem. Hence, we select quite broad priors for those volatilities. The noise variance regarding the exogenous private signals about aggregate technology and the demand conditions ($\widetilde{\sigma}_a$, and $\widetilde{\sigma}_g$) are crucial for the macroeconomic implications of the signaling channel as it affects the accuracy of private information and, hence, to what extent firms rely on the policy signal to learn about these non-policy shocks. To avoid conjecturing in the prior that the signaling channel strongly influences firms' beliefs about non-policy shocks, we set a loose prior over these parameters. Finally, the prior mean for the measurement errors (i.e., $\sigma_{\mu 1}$ and $\sigma_{\mu 2}$) is set so as to match the variance of inflation expectations reported in the *Livingston Survey* following the practice of Del Negro and Schorfheide (2008).

We combine the prior distribution for the parameters of the two models (i.e., the DIM

[10] The Federal Reserve Bank of Philadelphia computes the real time output gap as percent deviations of output Y_t from its potential Y_t^* (i.e., $100\,(Y_t - Y_t^*)\,/Y_t^*$). Therefore, these data must be adjusted so as to make them consistent with the data set constructed by Orphanides for the earlier quarters and with the model's concept of output gap (i.e., $100\,(\ln Y_t - \ln Y_t^*)$). An analogous transformation is made for the real-time inflation rate.

	DIM - Posterior			PIM - Posterior			Prior		
Name	Mean	5%	95%	Mean	5%	95%	Type	Mean	Std.
θ	0.3608	0.3137	0.4112	0.4241	0.3612	0.4812	$\mathcal{B}$	0.50	0.30
ϕ_π	1.6782	1.4454	2.1392	1.7044	1.4854	1.9361	$\mathcal{G}$	1.50	0.40
ϕ_x	0.6731	0.4898	0.7917	0.0172	0.0019	0.0328	$\mathcal{G}$	0.50	0.40
ρ_a	0.9764	0.9635	0.9897	0.9732	0.9582	0.9890	$\mathcal{B}$	0.50	0.20
ρ_g	0.9038	0.8663	0.9207	0.5570	0.4566	0.6536	$\mathcal{B}$	0.50	0.20
ρ_m	0.9468	0.8807	0.9748	0.3912	0.3282	0.4575	$\mathcal{B}$	0.50	0.20
ρ_π	0.3411	0.2472	0.4577	0.2644	0.1779	0.3601	$\mathcal{B}$	0.50	0.20
ρ_x	0.9541	0.9311	0.9812	0.9643	0.9378	0.9889	$\mathcal{B}$	0.50	0.20
ρ_p	–	–	–	0.9930	0.9845	0.9993	$\mathcal{B}$	0.50	0.20
$100\sigma_a$	1.4208	0.9764	2.0395	2.7888	1.1706	5.1692	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_a$	2.6068	1.5364	3.3252	–	–	–	$\mathcal{IG}$	0.80	1.50
$100\sigma_g$	3.6786	2.8764	4.0607	0.5895	0.4981	0.6872	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_g$	34.884	34.240	35.522	–	–	–	$\mathcal{IG}$	0.80	1.50
$100\sigma_m$	0.8474	0.6866	0.9842	0.5470	0.4704	0.6277	$\mathcal{IG}$	0.80	1.50
$100\sigma_\pi$	0.2686	0.2415	0.3043	0.2576	0.2296	0.2876	$\mathcal{IG}$	0.80	1.50
$100\sigma_x$	1.0448	0.9278	1.1762	1.0424	0.9217	1.1627	$\mathcal{IG}$	0.80	1.50
$100\sigma_p$	–	–	–	0.6846	0.4762	0.9277	$\mathcal{IG}$	0.80	1.50
$100\sigma_{\mu_1}$	0.1226	0.1088	0.1388	0.1132	0.0977	0.1296	$\mathcal{IG}$	0.10	0.08
$100\sigma_{\mu_2}$	0.1087	0.0963	0.1215	0.0659	0.0525	0.0788	$\mathcal{IG}$	0.10	0.08
$100\ln\pi_*$	0.6532	0.5661	0.7482	0.8590	0.7572	0.9580	$\mathcal{N}$	0.65	0.10

Table 1: Prior and Posterior Statistics for the parameters of the dispersed information model (DIM) and the perfect information model (PIM)

and the PIM) with their likelihood function and conduct Bayesian inference. As explained in Fernández-Villaverde and Rubio-Ramírez (2004) and An and Schorfheide (2007), a closed-form expression for the posterior distribution is not available, but we can approximate the moments of the posterior distributions via the Metropolis-Hastings algorithm. We obtain 250,000 posterior draws for the dispersed information model and 1,000,000 draws for the perfect information model. As far as the DIM is concerned, the posterior mean for the Calvo parameter θ implies very flexible price contracts, whose implied duration is roughly half a year. This finding suggests that the likelihood favors sources of persistence that are unrelated to sticky prices. In this regard, such a small value for the Calvo parameter implies that the average expectations of relatively higher order play an important role for the macroeconomic dynamics, as discussed in Section 2.7. Similar to Melosi (2014), the DIM in this paper relies on the sluggish adjustment of the higher-order expectations to fit the high serial correlation of the macro data, as we will show in Section 3.4.

The posterior mean for the inflation coefficient of the Taylor rule (ϕ_π) is higher than its prior mean and quite similar across models. The output gap coefficient in the Taylor rule ϕ_x is substantially larger in the DIM than in the PIM. Since the Taylor rule also plays the role of signaling equation in the DIM, a higher value for this parameter raises *ceteris paribus* the amount of information conveyed by the policy rate about the central bank's estimates of the output gap, which are exactly identified by the real-time data used in the estimation. On the contrary, the federal funds rate is found to respond very weakly to the output gap in the PIM. The other Taylor rule's parameters are very similar across the two models with the only exception of the persistence of monetary shocks ρ_m, which is substantially larger in the DIM. Note that highly persistent monetary shocks have the effect of increasing the persistence of the signaling effects of monetary policy on the macroeconomy insofar as changes in the policy rate signal this type of shocks. It should also be noted that the autoregressive parameter for the price markup ρ_p in the PIM is estimated to be very close to unity, highlighting a serious problem of the PIM in being unable to endogenously account for the persistent dynamics of inflation and inflation expectations in the data. A point to which we will return in the next section.

The posterior mean for the variance of the firm-specific technology shock $\widetilde{\sigma}_a$ implies that the posterior mean of the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a$ is 0.54. The posterior mean for the signal-to-noise ratio $\sigma_g/\widetilde{\sigma}_g$ is extremely small, suggesting that firms' private information is less accurate about demand shocks than about aggregate technology shocks. The posterior distribution implies that the policy signal is mainly informative about aggregate technology, since roughly 79 percent of the information flow conveyed by the public signal is about aggregate technology.[11] Nevertheless, firms mostly learn about aggregate technology from their private signal: the posterior median for the ratio of private information to public information about the aggregate technology is 88 percent. Conversely, firms largely rely on the policy signal $\widehat{R}_t$ to learn about the demand conditions $\widehat{g}_t$, since the private signal conveys only 21 percent of the overall information firms gathered about this exogenous state variable. These figures imply that firms rely mostly on the public signal to learn about the demand shocks ($\hat{\varepsilon}_{g,t}$) and the exogenous deviations from the Taylor rule ($\hat{\varepsilon}_{m,t}, \hat{\varepsilon}_{\pi,t}, \hat{\varepsilon}_{x,t}$). As a result, in the estimated DIM, firms find it fairly hard to tell whether observed changes in the policy rate are due to exogenous deviations from the policy rule or are instead due to the central bank's response to demand shocks. This feature will be crucial to understand most of the analysis that follows.

3.3 The Empirical Fit of the DIM

The objective of this section is to validate the DIM as a reliable modeling framework for macroeconomic analysis. To this end, we compare the goodness of fit of the DIM relative to that of the PIM, which is a prototypical New Keynesian DSGE model that has been extensively

[11] Appendix E shows how to use entropy-based measures to assess how much information is conveyed by signals to firms. These measures quantify information flows following a standard practice in information theory (Cover and Thomas 1991).

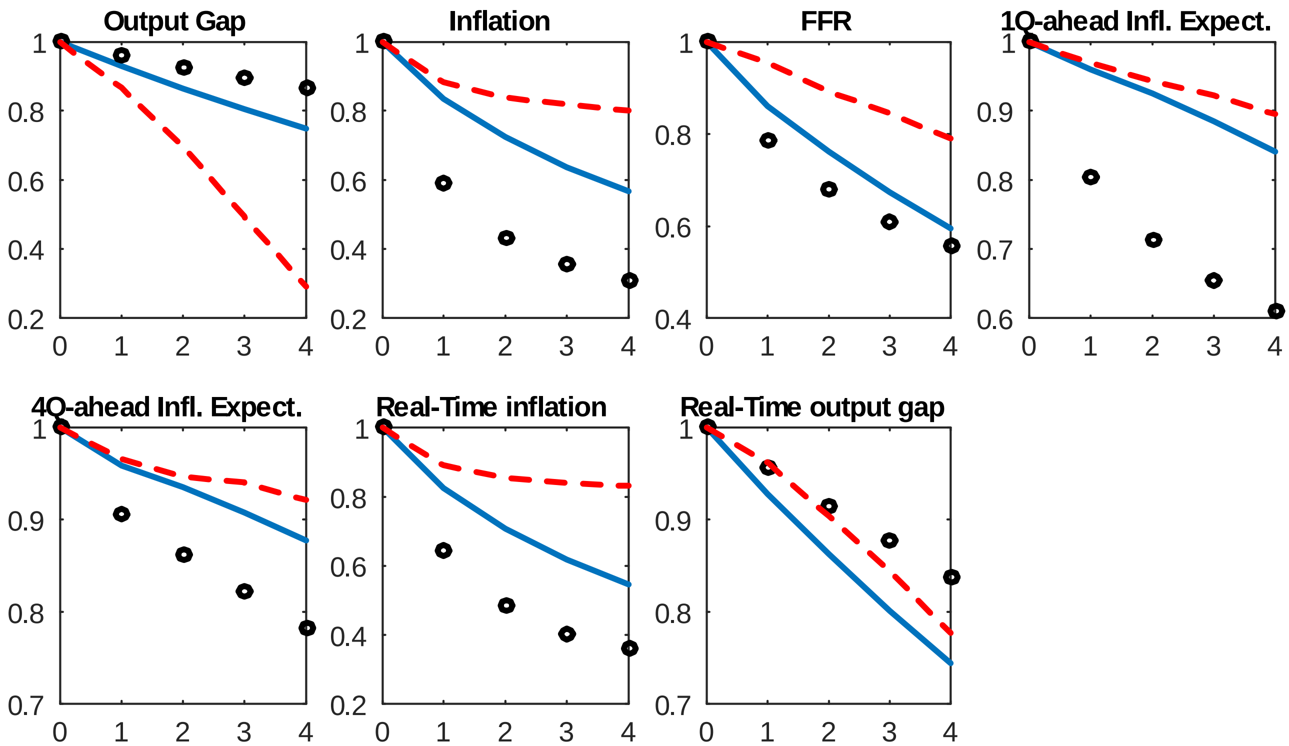

Figure 1: Autocorrelation functions for the seven observables. The red dashed line denotes the empirical autocorrelation function. The solid blue line denotes the autocorrelation function implied by the DIM. The black circles denotes the autocorrelation function implied by the PIM. The autocorrelation functions are computed by taking the mean of the autocorrelation functions evaluated at every 500 posterior draws.

used for monetary policy analysis (e.g., Rotemberg and Woodford 1997; Clarida, Galí, and Gertler 2000; Lubik and Schorfheide 2004; Coibion and Gorodnichenko 2011),

In Bayesian econometrics, non-nested model comparison is based on computing the posterior probability of the two candidate models. The marginal likelihood is the appropriate density for updating researcher's prior probabilities over a set of models. Furthermore, Fernández-Villaverde and Rubio-Ramírez (2004) show that the marginal likelihood allows the researcher to select the best model to approximate the true probability distribution of the data-generating process under the Kullback-Leibler distance. Since the marginal likelihood penalizes for the number of model parameters (An and Schorfheide 2007), it can be applied to gauge the relative fit of models that feature different numbers of parameters, such as the DIM and the PIM. The DIM has a log marginal likelihood equal to -319.89, which is higher than that of the PIM (-336.75). It follows that starting with fifty percent prior probability over each of the two competing models, the posterior probability of the DIM turns out to be extremely close to one. Restricting the autocorrelation parameter for the price markup ρ_m to zero would lower the marginal likelihood of the PIM by roughly a hundred log-points. Since the PIM has one more aggregate shock compared with the DIM, this result has to be interpreted as fairly strong evidence in favor of the ability of the DIM to fit the data relatively well.

To investigate the reasons why the DIM fits the data better than the PIM, we compare the autocorrelation functions of the observable variables implied by the two competing models. Figure 1 plots the posterior mean of the autocorrelation functions implied by the two competing models for the seven observables against the sample autocorrelation function (red dashed line).[12] These lines, which are often called *posterior predictive checks*, are obtained by simulating the model at each posterior draw for the model parameters, computing the statistic of interest, and averaging this statistic across posterior draws. The solid blue lines, which are the autocorrelation functions implied by the DIM, are always closer to the empirical autocorrelation functions (the red dashed lines) than the black circles, which stand for the autocorrelation functions implied by the PIM. In particular, one can observe that the PIM does a relatively poor job at accounting for the high persistence that characterizes the dynamics of inflation and inflation expectations in the sample. It should also be noted that the DIM captures remarkably well the high persistence of inflation expectations both at the one-quarter horizon and at four-quarter horizon. Both models cannot fully match the high empirical persistence of inflation, real-time inflation, and the federal funds rate. However, the DIM seems to do a relatively better job at fitting the persistence of inflation and real-time inflation. Both models overpredict the persistence of the output gap. The two stylized models find it hard to reconcile the sizable difference in the persistence that characterizes the output gap and the real-time output gap. Furthermore, the DIM seems to do remarkably better than the PIM at fitting some cross-correlations, such as the cross-correlation between the two inflation expectations.

3.4 Impulse Response Functions

In the previous section we have shown that the DIM outperforms the PIM in explaining the high persistence of inflation and inflation expectations in the data. To investigate the reasons why the DIM accounts so well for the persistent macroeconomic fluctuations, we turn our attention to the propagation of the structural shocks in the DIM. In Section 3.4.1, we analyze the propagation of monetary shocks to the macroeconomy. In Section 3.4.2, we deal with the transmission of non-policy shocks (i.e., aggregate technology shocks and demand shocks).

3.4.1 Propagation of Monetary Shocks

Figure 2 shows the impulse response functions (and their 95 percent posterior credible sets in gray) of the level of real output (GDP), the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a monetary shock that raises the interest rate by 25 basis points. Three features of these impulse

[12] The confidence bands are very tight for both models and hence are not reported.

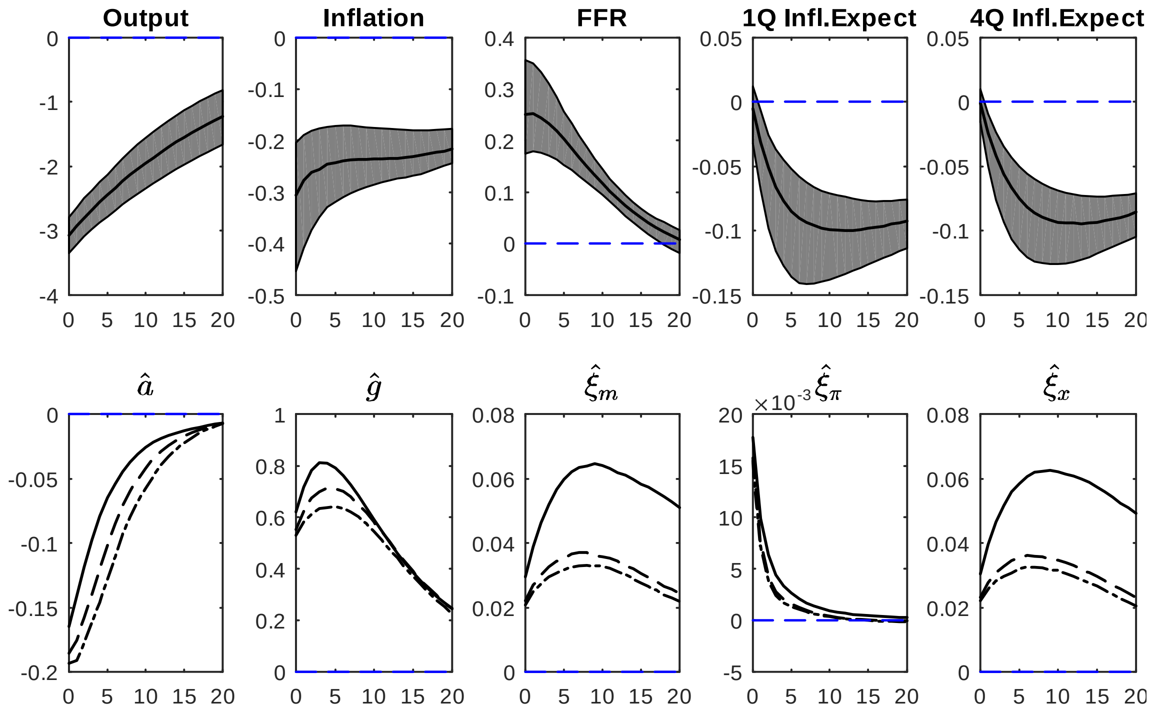

Figure 2: Impulse Response Function to a Contractionary Monetary Shock. *Upper graphs:* Impulse response function of output deviations from balanced growth (in percent), inflation, federal funds rate, one-quarter-ahead inflation expectations, and four-quarter-ahead inflation expectations in annualized percentage deviations from their steady-state value to a monetary shock that raises the federal funds rate by 25 bps. The solid line denotes posterior means computed for every 200 posterior draws. The gray areas denote 90-percent credible sets. The horizontal axis in all graphs measures the number of quarters after the shock. *Lower graphs:* Response of the average expectations about the five exogenous state variables in percentage deviations from their steady-state level. Black solid lines denote the average first-order expectations. Dashed black lines denote the average second-order expectations. Dashed-dotted lines denote the average third-order expectations.

response functions have to be emphasized. First, inflation and inflation expectations seem to react fairly sluggishly, even though the estimated degree of nominal rigidities is quite small. Second, the DIM predicts fairly strong real effects of money. Sluggish adjustments in prices imply a lower path for households' inflation expectations. Consequently, the expected path of the real interest rate shifts upward after the contractionary monetary shock, leading the Euler equation (7) to predict a large drop in real activity.[13] Third, firms' inflation expectations respond positively to contractionary monetary shocks with some posterior probability.

In the lower graphs, we report the response of the average higher-order expectations (from the first order up to the third order). It is important to notice that the signaling channel induces firms to partially believe that the rise in the interest rate is due to either a positive demand

[13] It should be noted that the log-linearized Euler equation (7) can be expanded forward to obtain $\widehat{x}_t = -\sum_{k=0}^{\infty}\left(\widehat{R}_{t+k} - E_t\widehat{\pi}_{t+k} - \widehat{r}^n_{t+k}\right)$, where $\left(\widehat{R}_t - E_t\widehat{\pi}_t\right)$ denotes the real interest rate and $\widehat{r}^n_t$ denotes the natural rate, which is a function of aggregate technology shocks and demand shocks.

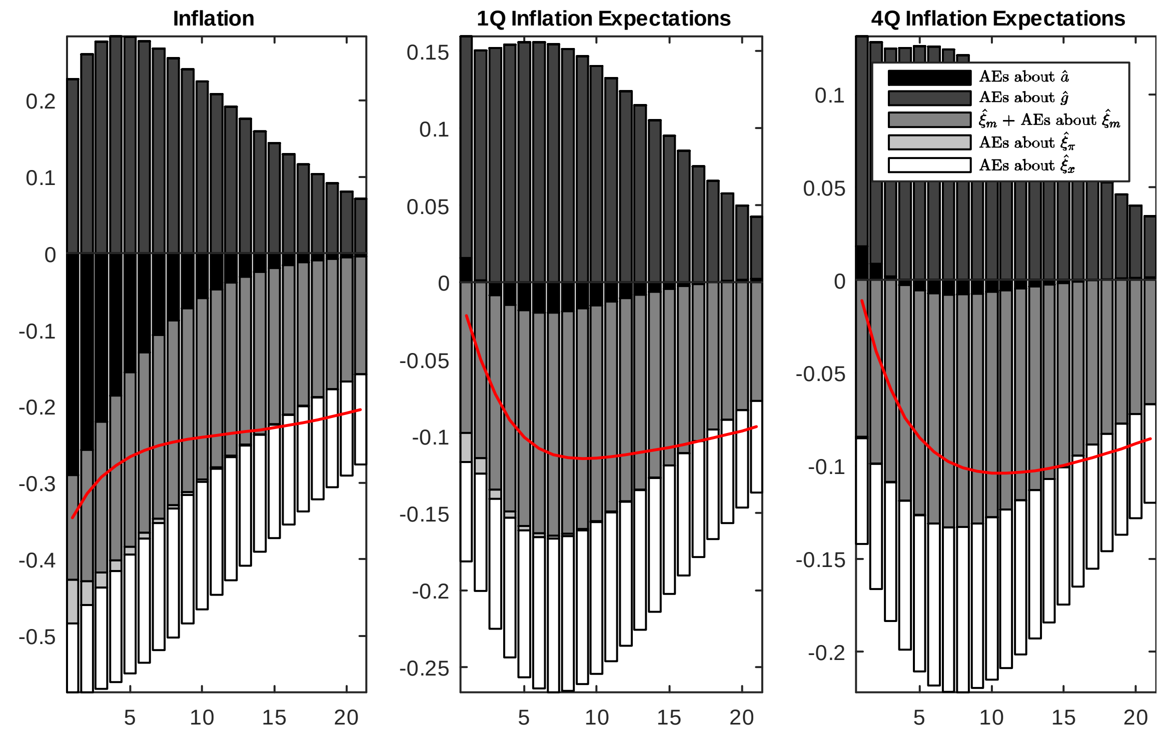

Figure 3: Contributions of average expectations to the impulse response functions of inflation and inflation expectations to a monetary shock that raises the interest rate by 25 bps. Parameter values are set equal to the posterior mean. The solid red line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph). The vertical bars capture the contribution of the actual shocks and the average expectations about the level of aggregate technology $\widehat{a}_t$, the demand conditions $\widehat{g}_t$, and the three types of deviations from the monetary rule $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, and $\widehat{\xi}_{x,t}$ to inflation and inflation expectations.

shock or a negative technology shock or an overestimation of the output gap by the central bank. These signaling effects are not surprising given the poor quality of the private signal about the demand conditions relative to the public signal and the large amount of information about aggregate technology the policy rate conveys as discussed in Section 3.2.

An important feature of the impulse response function in Figure 2 is the large degree of persistence. The DIM's ability to generate highly persistent fluctuations in inflation and inflation expectations is key for explaining its good empirical performance relative to the PIM as discussed in Section 3.3. Also recall that the sluggish adjustment of inflation to monetary disturbances is the reason for the fairly strong real effects of money in the estimated DIM. To investigate the sources of persistence in the estimated DIM, we show the contribution of the average expectations $X_{t|t}^{(0:k)}$ about the five exogenous state variables to the response of inflation and inflation expectations. The vertical bars in Figure 3 show the response of inflation (left graph) and inflation expectations (middle and right graphs) to a contractionary monetary shock obtained by simulating the DIM using only one of the five exogenous state variables and the associated average expectations. The sum of the five vertical bars equals the response of inflation and inflation expectations (i.e., the solid red line) evaluated at the posterior mean reported in Table 1. It should be noted that the persistent adjustment of inflation and inflation expectations to monetary shocks is mainly led by the monetary policy signaling persistent monetary shocks and long-lasting forecasts errors for the output gap by the central bank.

Furthermore, it is important to emphasize that raising the policy rate signals that the central bank may be responding to a positive demand shock, dampening the deflationary consequences associated with the contractionary monetary shock and, hence, raising their real effects. These signaling effects are captured by the dark gray vertical bars that persistently lie in positive territory in the three graphs of Figure 3. The average expectations about aggregate technology (the black bars) and those about the central bank's measurement errors about the output gap (the white bars) reinforce the deflationary pressures associated with the contractionary monetary shocks.[14] These deflationary pressures entirely stem from the signaling channel in that those shocks have been signaled to price setters by the central bank in raising the policy rate.

While real effects of money in the estimated DIM are stronger than what the VAR literature typically finds, this literature generally relies on schemes to identify monetary shocks (e.g., Cholesky) that are inconsistent with the presence of signaling effects associated with policy actions. Nakamura and Steinsson (2013) use unexpected changes in interest rates over a thirty-minute window surrounding scheduled Federal Reserve announcements to identify monetary policy shocks in a reduced-form model. These scholars find that the response of inflation is small and delayed. They use this evidence to estimate the key parameters of a workhorse perfect-information New Keynesian model and find that the implied real effects of money are quantitatively larger than what is usually found by the VAR literature.

As shown in the lower graphs of Figure 2, some average expectations respond very sluggishly to monetary shocks. While these persistent adjustments are crucial for such a stylized DSGE model to deliver a degree of persistence in line with the data (Figure 1), it may also raise concerns about what may appear to be an implausibly long period for firms to learn the true value of the exogenous state variables. These concerns will be addressed in Section 4. Shocks to the forecast error regarding the output gap $\varepsilon_{x,t}$ propagate across the macroeconomy very

[14] To understand why signaling an adverse technology shock has deflationary consequences, recall that the average expectations about the real marginal cost in the incomplete-information Phillips curve (6) are given by $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)},\ k \geq 1$. Note that because shocks are orthogonal, $\partial\widehat{a}_{t|t}^{(0)}/\partial\varepsilon_{m,t} \equiv \partial\widehat{a}_t/\partial\varepsilon_{m,t} = 0$. Since expecting an adverse technology shock leads firms to expect a fall in output ($\hat{y}_{t|t}^{(1)}$), the average first-order expectations about the real marginal costs $\widehat{mc}_{t|t}^{(1)} = \hat{y}_{t|t}^{(1)} - \widehat{a}_t$ would fall, driving down inflation and inflation expectations. If this first-order effect ($\widehat{mc}_{t|t}^{(1)} = \hat{y}_{t|t}^{(1)} - \widehat{a}_t$) dominates the higher-order effect ($\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)},\ k \geq 2$), then expecting a negative technology shock will bring about deflationary pressures.

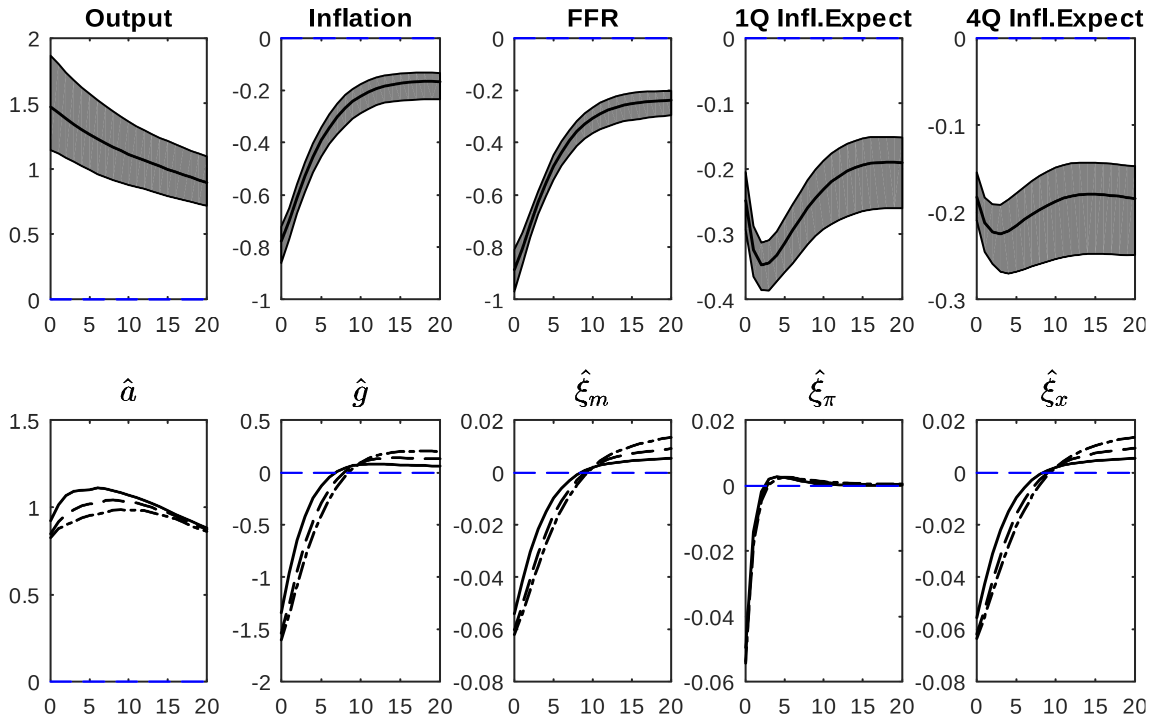

Figure 4: Impulse Response Function to a Positive Technology Shock. *Upper graphs:* Impulse response function of output deviations from balanced growth (in percent), inflation, federal funds rate, one-quarter-ahead inflation expectations, and four-quarter-ahead inflation expectations in annualized percentage deviations from their steady-state value to a one-standard deviation positive technology shock. The solid line denotes posterior means computed for every 200 posterior draws. The gray areas denote 90-percent credible sets. The horizontal axis in all graphs measures the number of quarters after the shock. *Lower graphs:* Response of the average expectations about the five exogenous state variables in percentage deviations from their steady-state level. Black solid lines denote the average first-order expectations. Dashed black lines denote the average second-order expectations. Dashed-dotted lines denote the average third-order expectations.

similarly to monetary shocks $\varepsilon_{m,t}$, which have been discussed in this section.[15]

3.4.2 Propagation of Non-Policy Shocks

Figure 4 shows the response of the level of real output (GDP), inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a one-standard-deviation positive aggregate technology shock. In the aftermath of a positive aggregate technology shock, real GDP increases, while both inflation and inflation expectations fall. The lower graphs of Figure 4 report the responses of average expectations about the five exogenous state variables. A drop in the policy rate owing to a positive technology shock induces firms to believe that the central bank is responding to either an expansionary deviations from the monetary policy rule ($\widehat{\xi}_{m,t} < 0$ and $\widehat{\xi}_{x,t} < 0$) or a negative demand shock. To the extent that

[15] The propagation of real-time measurement errors regarding inflation is less interesting for the objectives of the paper and is omitted. The results are available upon request.

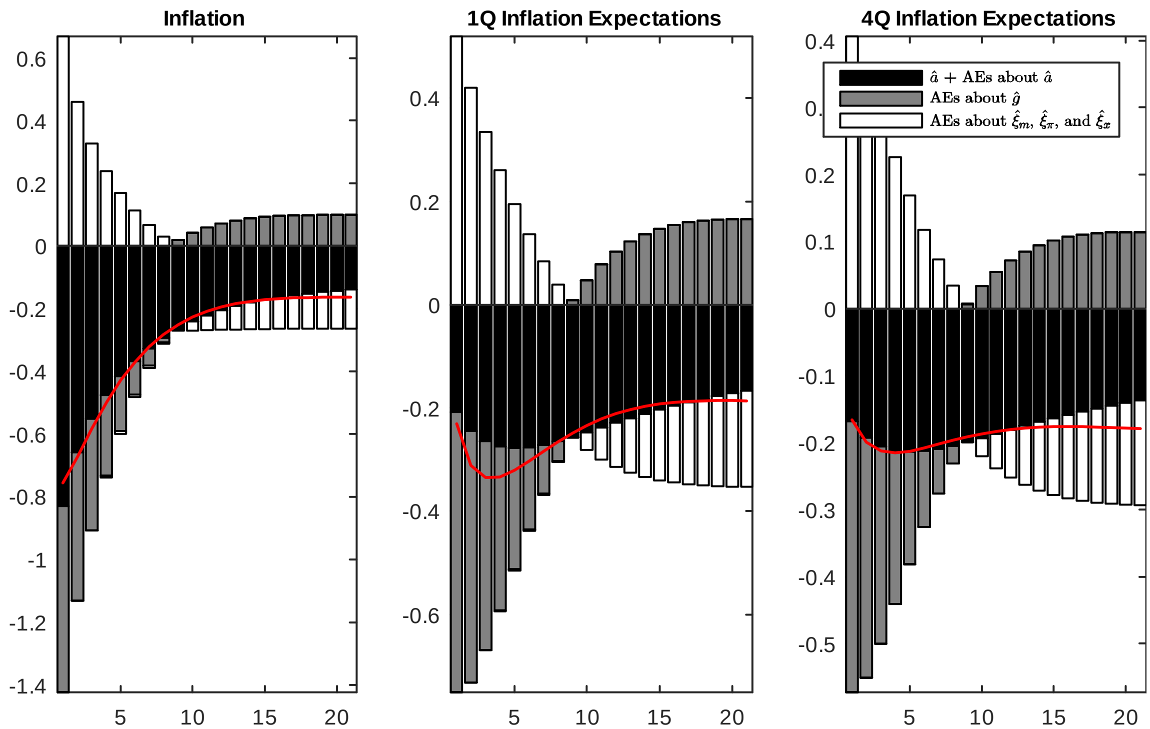

Figure 5: Contributions of average expectations to the impulse response functions of inflation and inflation expectations to positive technology shock. Parameter values are set equal to the posterior mean. The solid red line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph). The vertical bars capture the contribution of the actual shocks and the average expectations about the level of aggregate technology $\widehat{a}_t$, the demand conditions $\widehat{g}_t$, and the three types of deviations from the monetary rule $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, and $\widehat{\xi}_{x,t}$ altogether to inflation and inflation expectations.

firms are persuaded that an expansionary deviation from the rule has occurred, the negative response of inflation and inflation expectations to the technology shock will become smaller. This effect is non-negligible as the white bars lying in positive territory in Figure 5 show.[16] However, the monetary easing due to the positive technology shock leads firms to believe that a negative demand shock may have hit the economy, lowering firms' inflation expectations and inflation. This effect is captured by the gray bars lying in negative territory in Figure 5.

It is interesting to notice that in Figure 5 the response of the average expectations about exogenous deviations from the monetary rule (i.e., the white bars) and those about the demand conditions (i.e., the gray bars) contribute to the adjustment of inflation and inflation expectations by similar amounts at any period after the technology shock. Two facts are behind this result. First, firms have inaccurate private information about demand shocks and, hence, have to mainly rely on the policy signal to learn about this type of shock. Second, the policy signal

[16] To improve the readability of the graphs, we have aggregated the contribution of the three exogenous deviations from the monetary rule (i.e., $\xi_{m,t}$, $\xi_{\pi,t}$, and $\xi_{x,t}$).

is mainly informative about technology shocks. The first condition implies that firms have to jointly learn demand shocks and exogenous deviations from the monetary rule by observing the policy signal. However, the second condition implies that the policy signal provides little information about demand shocks and exogenous deviations from the monetary rule, making it quite hard for firms to figure out which of these shocks has prompted the central bank to raise the rate.

It is important to realize that Figure 5 does not imply that signaling effects associated with technology shocks are necessarily tiny. In fact, the signaling channel also affects the contribution of the average expectations (AE) about the level of aggregate technology $a_t^{(0:k)}$ (i.e., the black vertical bars). Signaling effects associated with technology shocks will be precisely quantified in the next section. What Figure 5 shows is that the sluggish adjustment of inflation and inflation expectations in the aftermath of a technology shock is mainly due to the high persistence characterizing the average expectations about aggregate technology, which, as we shall see, the signaling channel significantly contributes to generate.

The propagation of a one-standard-deviation positive demand shock is described in Figure 6. It is important to emphasize that inflation and inflation expectations respond *negatively* to demand shocks, while output responds positively. Note that the central bank raises its policy rate in the aftermath of a positive demand shock leading to two types of signaling effects. First, the monetary tightening induces firms to believe that a contractionary deviation from the monetary policy rule has happened. Second, the observed rise in the federal funds rate induces firms to believe that a negative technology shock might have occurred. Figure 7 shows that both of these effects push inflation down, countering the rise in inflation due to the positive demand shock captured by the gray bars. While the second effect (captured by the black bars in Figure 7) has quantitatively a fairly small impact on inflation expectations, the first effect (captured by the white bars) appears to substantially contribute to pushing inflation expectations down. Furthermore, the second effect is relative shorter-lived than the first one. The first effect is very persistent indeed, reflecting the fact that firms find it hard to disentangle whether changes in the policy rate are due to exogenous deviations from the monetary rule or are instead due to demand shocks for reasons that were analyzed in Section 3.2. Similar to what is seen in the impulse response to monetary shocks, signaling adverse technology shocks brings about deflationary pressures because expecting an adverse technology shock leads firms to expect a fall in output, as discussed in Section 3.4.1.

The fairly long-lasting deflation after a positive demand shock is mainly due to the fact that monetary policy ends up signaling persistent contractionary deviations from the monetary rule. The high persistence of the exogenous state variables $\widehat{\xi}_{m,t}$ and $\widehat{\xi}_{x,t}$ in the estimated DIM (Table 1) drives this finding. Quite interestingly, the signaling channel transforms demand shocks ($\hat{\varepsilon}_{g,t}$) into supply shocks that move output and inflation in opposite directions. See Figure 6.

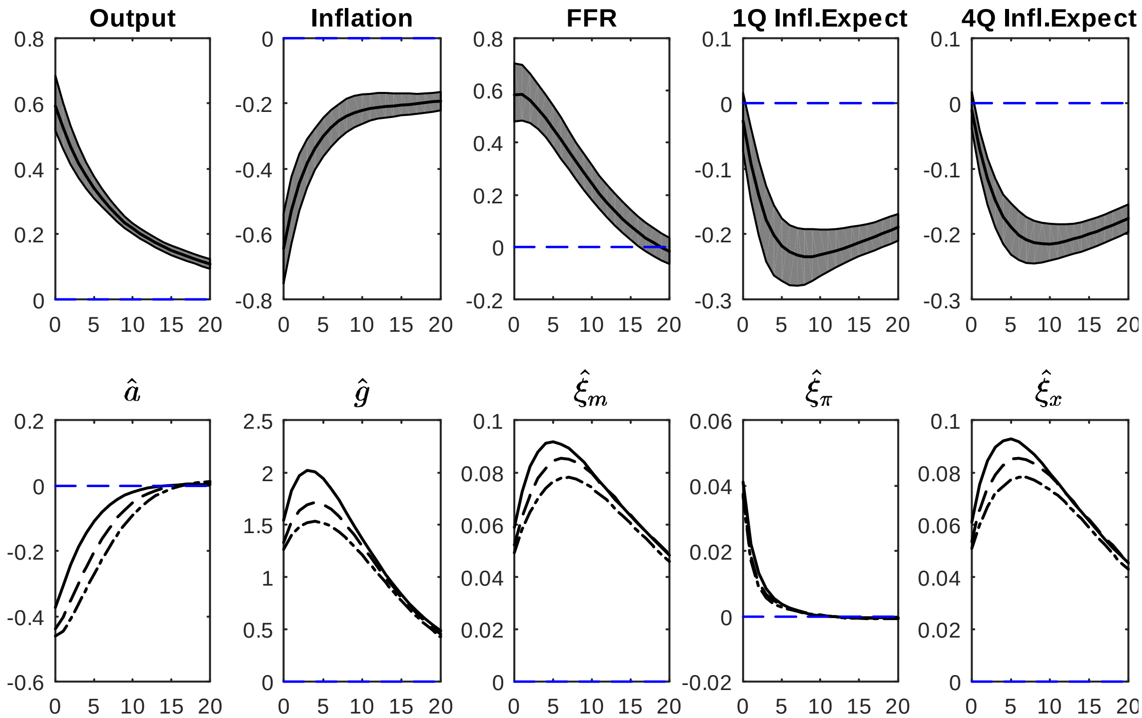

Figure 6: Impulse Response Function to a Positive Demand Shock. *Upper graphs:* Impulse response function of output deviations from balanced growth (in percent), inflation, federal funds rate, one-quarter-ahead inflation expectations, and four-quarter-ahead inflation expectations in annualized percentage deviations from their steady-state value to a one-standard deviation positive demand shock. The solid line denotes posterior means computed for every 200 posterior draws. The gray areas denote 90-percent credible sets. The horizontal axis in all graphs measures the number of quarters after the shock. *Lower graphs:* Response of the average expectations about the five exogenous state variables in percentage deviations from their steady-state level. Black solid lines denote the average first-order expectations. Dashed black lines denote the average second-order expectations. Dashed-dotted lines denote the average third-order expectations.

Unlike technology shocks, this artificial supply shock implies a negative comovement between the federal funds rate and the rate of inflation, as well as between the interest rate and inflation expectations. This property helps the model fit the 1970s, when the policy rate was low while inflation and inflation expectations attained quite high values.

3.5 The Signaling Effects of Monetary Policy

In this section, we use the DIM to empirically assess the signaling effects of monetary policy on inflation and inflation expectations. To this end, we run a *Bayesian counterfactual experiment* using an algorithm that can be described as follows. In Step 1, for every posterior draw of the DIM parameters, we obtain the model's predicted series for the five structural shocks (the aggregate technology shock $\varepsilon_{a,t}$, the demand shock, $\varepsilon_{g,t}$, the monetary shock $\varepsilon_{m,t}$, the shocks to the central bank's measurement errors $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$) using the two-sided Kalman filter and

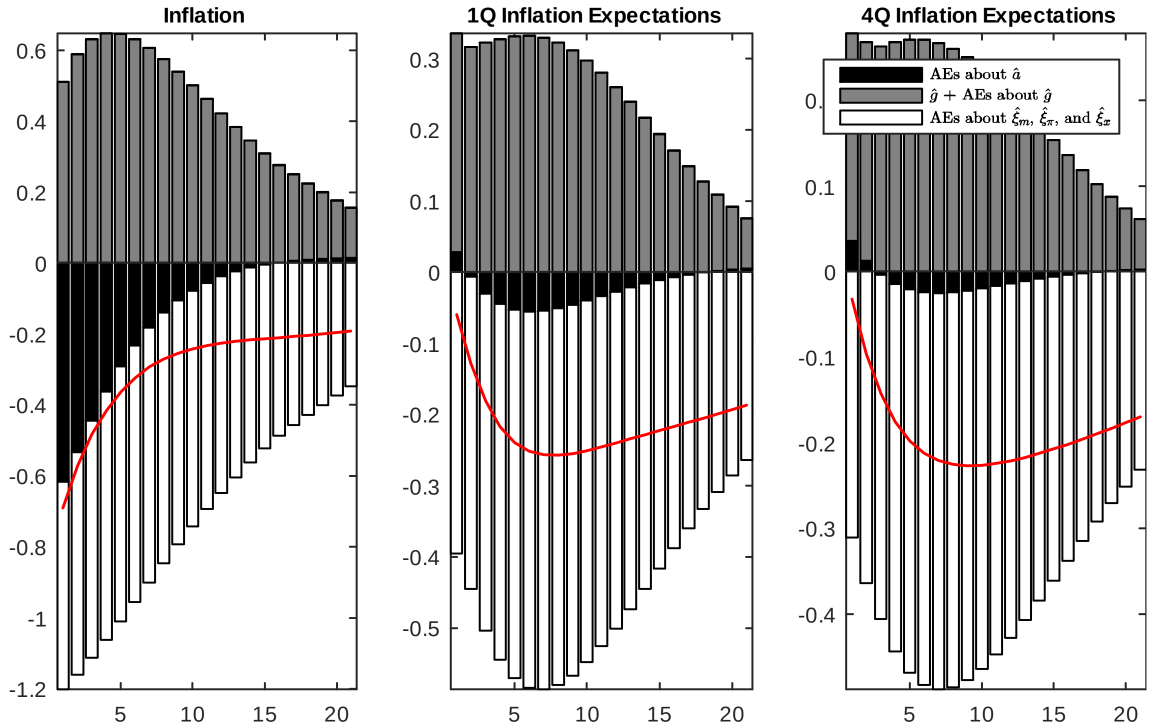

Figure 7: Contributions of average expectations to the impulse response functions of inflation and inflation expectations to a one-standard-deviation positive demand shock. Parameter values are set equal to the posterior mean. The solid red line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph). The vertical bars capture the contribution of the actual shocks and the average expectations about the level of aggregate technology $\widehat{a}_t$, the demand conditions $\widehat{g}_t$, and the three types of deviations from the monetary rule $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, and $\widehat{\xi}_{x,t}$ altogether to inflation and inflation expectations.

the seven observable variables introduced in Section 3.1. In Step 2, these filtered series of shocks are used to simulate the rate of inflation and inflation expectations from the following two models: (i) the DIM and (ii) the *counterfactual* DIM, in which monetary policy has *no signaling effects*. The latter model is obtained from the DIM by assuming that the history of the policy rate does not belong to firms' information set (i.e., $R^t \notin \mathcal{I}_{j,t}$ for all periods t and firms j). This assumption implies that the signaling channel is inactive, and hence, firms form their expectations by using only their private information (i.e., the history of the signals $\widehat{a}_{j,t}$ and $\widehat{g}_{j,t}$). In Step 3, we compute the mean of the simulated series across posterior draws for the two models.

It is important to emphasize that the shocks are filtered in Step 1 by using the final (Hodrick-Prescott-filter-based) output gap and the real-time output gap from the Greenbook. Therefore, both the simulation from the actual DIM and that from the counterfactual DIM take into account that the Federal Reserve's nowcasts of the output gap were persistently lower than the actual output gap in the 1970s and for large part of 1980s, exactly as measured by Orphanides

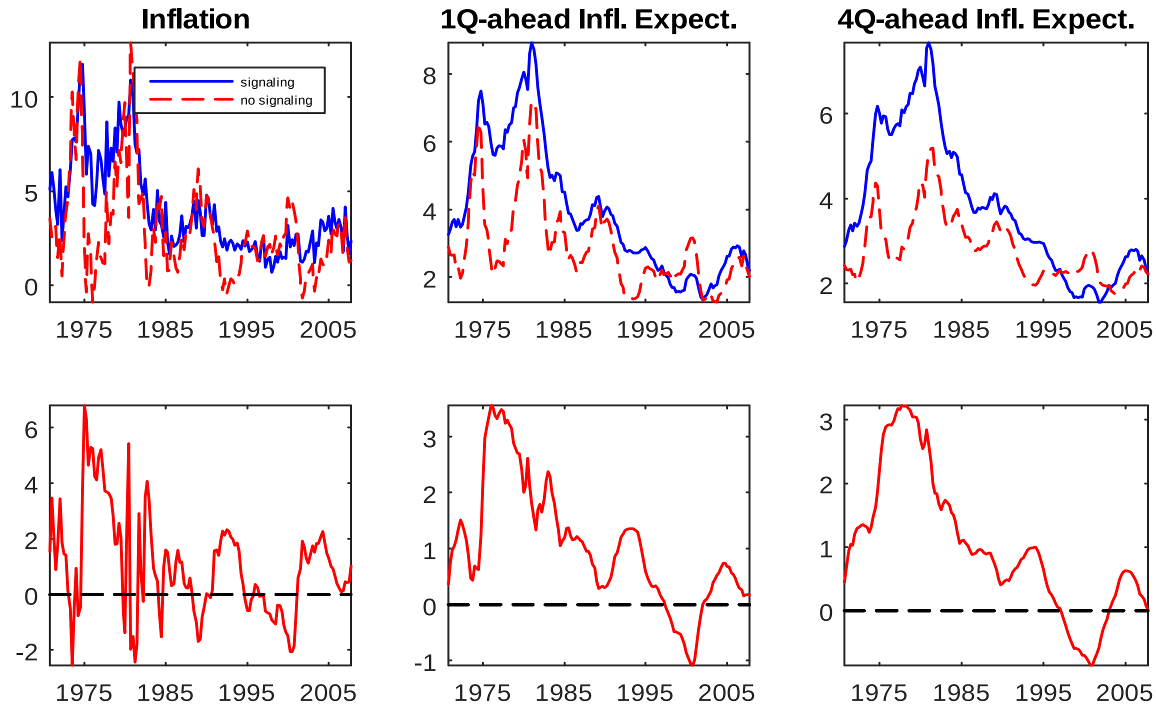

Figure 8: Signaling Effects of Monetary Policy on Inflation and Inflation Expectations. *Upper graphs:* Solid blue line: inflation rate (left graph) and inflation expectations (middle and right graphs) simulated from the estimated dispersed information model (DIM) using the two-sided filtered shocks from the estimated DIM. Red dashed line: simulated inflation rate from the counterfactual DIM, in which the signaling channel is shut down, using the two-sided filtered shocks from the estimated DIM. The vertical axis in all graphs measures units of percentage points of annualized rates. *Lower graph*: The signaling effects of monetary policy on the annualized rate of inflation (left graph) and inflation expectations (middle and right graphs) in percent.

(2004).

The solid blue line in the upper graphs of Figure 8 denotes the inflation rate (left graph) and the inflation expectations (middle and right graphs) simulated from the DIM using the two-sided filtered shocks from the estimated DIM. The simulated series of inflation is by construction the same as in the data, whereas the series of inflation expectations does not exactly replicate the actual data because of the measurement errors $\varepsilon_t^{\mu_1}$ and $\varepsilon_t^{\mu_2}$ in the observation equations (14) and (15). However, the discrepancy between these two series is rather minuscule, since *iid* measurement errors just end up smoothing out the simulated series slightly. The red dashed line denotes the series of inflation (left graph) and inflation expectations (middle and right graphs) simulated from the counterfactual DIM, in which the signaling channel is shut down. The vertical difference between the two simulated series in the upper graphs is reported in the lower graphs and captures the signaling effects of monetary policy over the sample period.

In the model signaling effects on inflation are particularly strong in the 1970s, adding up to 6.4 percentage points to the rate of inflation in that decade. Moreover, signaling effects play an important role in explaining why inflation was *persistently* heightened in the second half of the

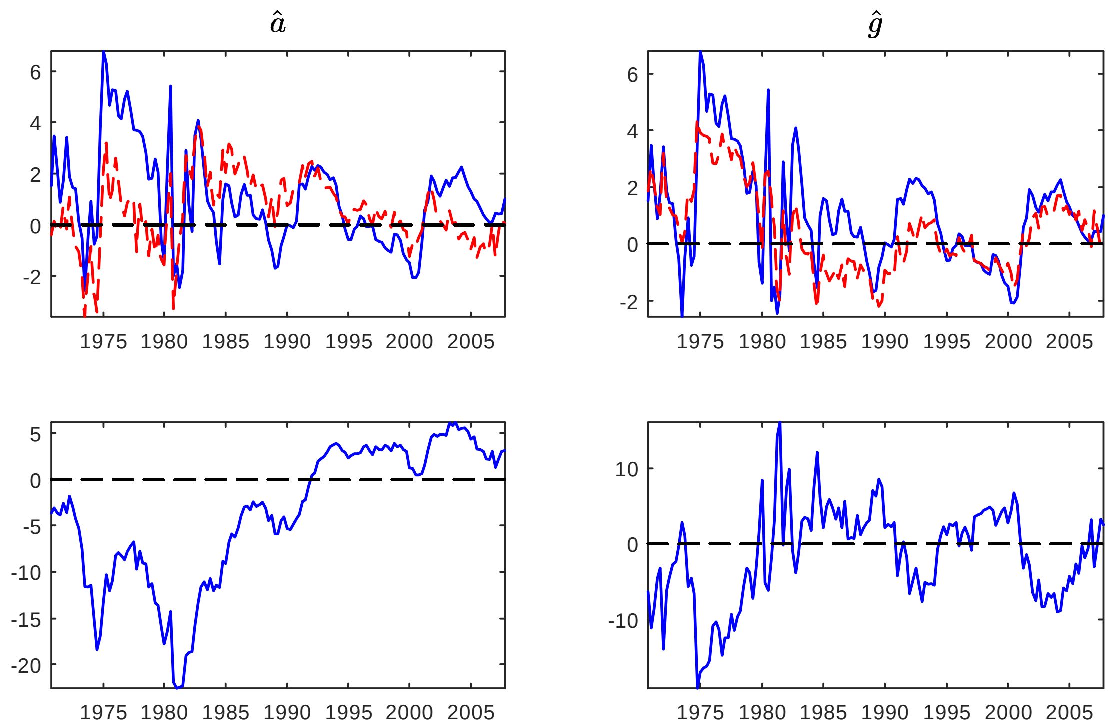

Figure 9: Contributions of Shocks to Signaling Effects on Inflation *Upper graphs:* Solid blue line: signaling effects of monetary policy on inflation. Red dashed line: signaling effects on inflation when only one type of shocks is used to simulate the DIM. Left graph: only aggregate technology shocks are used; Right graph: only demand shocks are used. *Lower graphs*: the two-sided filtered dynamics of aggregate technology $\hat{a}_t$(left) and demand conditions $\hat{g}_t$ (right).

1970s. This finding is even more pronounced when one looks at the signaling effects of monetary policy on the short- and medium-horizon inflation expectations. Furthermore, signaling effects on inflation expectations are positive until the end of the 1990s, largely explaining why in the data expectations were almost always above the rate of inflation from 1981:Q2 through the end of the 1980s.[17]

To shed light on the origin of the estimated signaling effects on inflation, we report in Figure 9 the contribution of aggregate technology shocks (left upper graph) and demand shocks (right upper graph) to the signaling effects on inflation.[18] These graphs show the dynamics of the signaling effects on inflation in the simulated DIM (the blue solid line) and in the DIM simulated by using only either smoothed estimates of technology shocks (the red dashed line in the left upper graph) or smoothed estimates of demand shocks (the red dashed line in the right upper graph). In the lower graphs of Figure 9, we show the two-sided filtered series of

[17]In that period, observed one-quarter-ahead (four-quarter-ahead) inflation expectations have been 70 bps (40 bps) higher on average than the inflation rate.

[18]The larger figure reporting the contribution to signaling effects of all the five exogenous state variables is available upon request. The omitted state variables are found to contribute only marginally to signaling effects of monetary policy on inflation and inflation expectations.

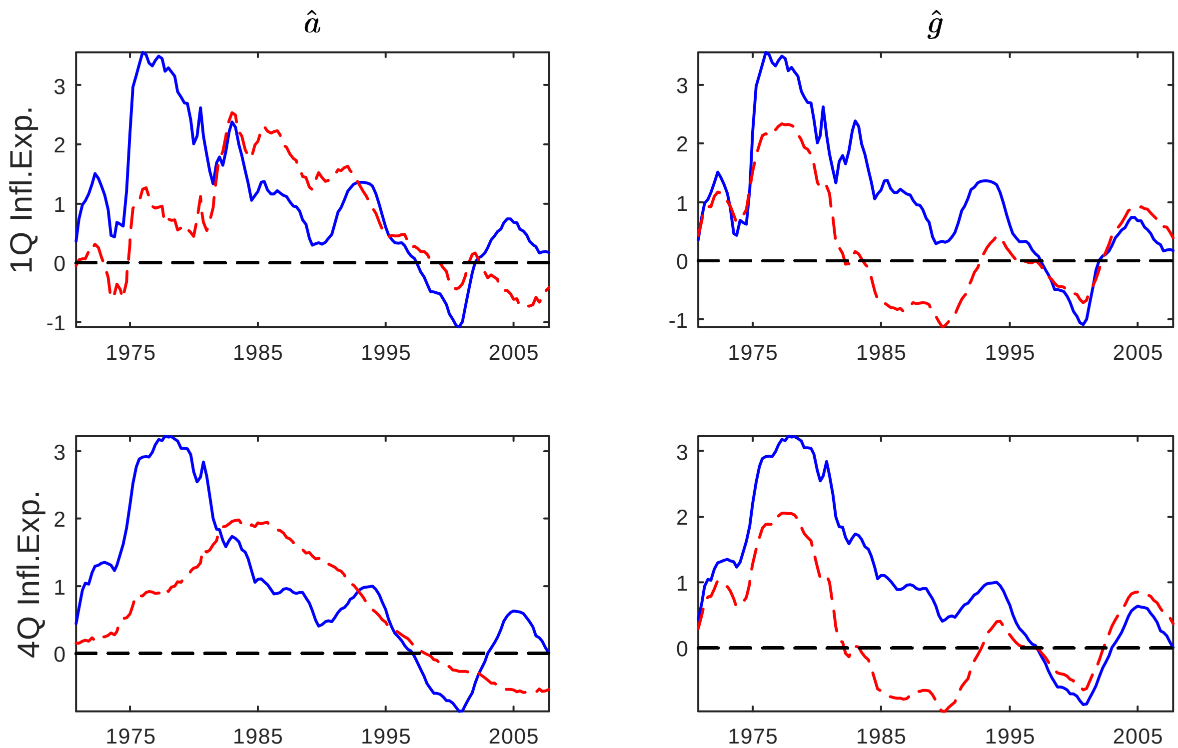

Figure 10: Contributions of Shocks to Signaling Effects on Inflation Expectations. Upper graphs: One-quarter-ahead inflation expectations. Lower graphs: Four-quarter-ahead inflation expectations. Solid blue line: signaling effects of monetary policy on inflation expectations. Red dashed line: signaling effects on inflation expectations when only one shock is used to simulate the DIM. Left graphs: only aggregate technology shocks are used. Right graph: only demand shocks are used.

the two exogenous state variables $\hat{a}_t$ (left graph) and $\hat{g}_t$ (right graph) obtained in Step 1 of the Bayesian counterfactual experiment. We observe that most of the signaling effects on inflation in the 1970s are due to negative demand conditions ($\hat{g}_t < 0$) because feeding the model with only demand shocks generates signaling effects (the red dashed line) that are similar to the ones obtained using all aggregate shocks (the blue solid line) in that decade. As shown in Section 3.4.2, negative demand shocks prompted the Federal Reserve to lower the policy rate, ending up signaling both persistent expansionary monetary shocks and long-lasting nowcast errors in measuring the output gap by the policymaker. Signal effects associated with positive technology shocks contributed to an increase in inflation of up to 3 percentage points in 1975-1976. However, this contribution was quite short-lived because of the predominance of negative technology shocks in the 1970s that brought about deflationary signaling effects as shown in the upper left graph of Figure 9. According to the model, in the 1980s and in the early 1990s, the signaling effects of monetary policy on inflation are predominantly driven by aggregate technology $\hat{a}_t$. Improvements in aggregate technology during this period induced the Federal Reserve to carry out a monetary policy that ended up signaling expansionary deviations from the monetary policy rule.

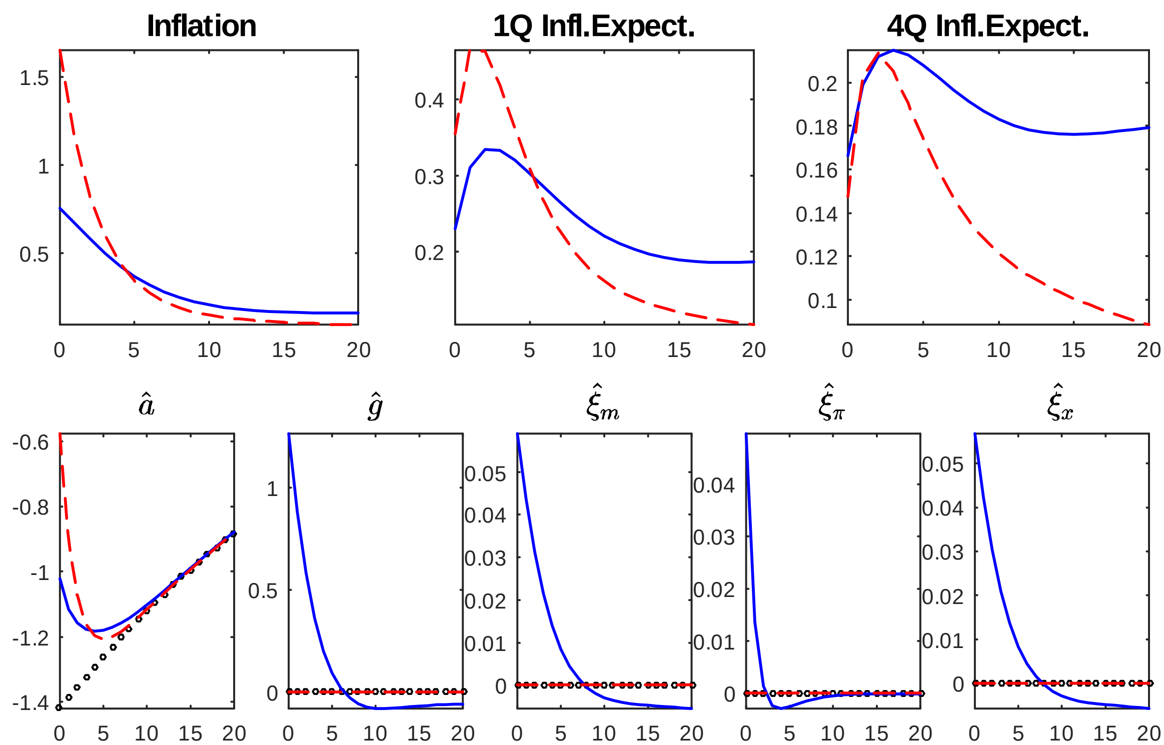

Figure 11: Signaling Effects on Inflation and Inflation Expectations Associated with a One-Standard-Deviation Negative Technology Shock. *Upper graphs*: Response of inflation (the left graph), one-quarter-ahead inflation expectations (the mddle graph), and four-quarter-ahead inflation expectations (the right graph) in the estimated DIM, with signaling effects (the solid blue line) and in the counterfactual DIM, with no signaling effects (the red dashed line). *Lower graphs:* Black circles denote the response of the five exogenous state variables to a negative technology shock. The solid blue line denotes the response of the average first-order expectations about the five exogenous states in the estimated DIM, with signaling effects. The red dashed line denotes the response of the average first-order expectations about the five exogenous states in the counterfactual DIM, with no signaling effects.

The leading sources of the signaling effects on inflation expectations are shown in Figure 10. These graphs show the dynamics of the signaling effects on the one-quarter-ahead inflation expectations (upper graphs) and on the four-quarter-ahead inflation expectations (lower graph) in the simulated DIM (the blue solid line) and in the DIM simulated by using only either smoothed estimates of technology shocks (red dashed line in the left graphs) or smoothed estimates of demand shocks (the red dashed line in the right graphs). Similar to the signaling effects on inflation, the signaling effects on inflation expectations during the 1970s, as shown on the right plots, are largely driven by demand shocks. The contribution of technology shocks to the signaling effects on the inflation expectations start building up slowly in the 1970s, which was a decade characterized by large and repeated negative technology shocks. See the red dashed line in the left graphs of Figure 10. This slow-moving pattern is suggestive of technology shocks bringing about *delayed* signaling effects on inflation expectations. It should also be observed that this pattern is fairly different from the dynamics that characterized the contribution of technology shocks to the signaling effects on inflation, which moves around the zero line during the 1970s in the upper left graph of Figure 9. The improvements in the level of aggregate technology observed from 1982 through the early 1990s slowly revert the upward trend in the technology-led signaling effects on inflation expectations in the model. However, these effects are delayed, and signaling effects on inflation expectations remain positive until the mid-1990s, largely explaining why inflation expectations were higher on average than inflation throughout the 1980s.

It is interesting to investigate why technology shocks raise inflation expectations through the signaling channel with delays. To this end, we report in Figure 11 the response of inflation (the upper left graph) and inflation expectations (the upper middle and right graphs) to a one-standard deviation negative technology shock in the estimated DIM (the solid blue line) and in the counterfactual DIM, with no signaling effects (the red dashed line). The difference between these two lines captures the signaling effects associated with the negative technology shock. Two features have to be emphasized. First, while signaling effects associated with technology shocks mainly affect inflation at short horizons, inflation expectations are primarily influenced at longer horizons. Second, signaling effects on inflation and inflation expectations become positive a few quarters past the shock. This happens because eight quarters after the technology shock, firms evaluate the policy rate to be lower than the level that they would have expected based on their beliefs about inflation and the output gap. Consequently, monetary policy starts signaling long-lasting expansionary deviations from the monetary rule, as shown in the lower graphs of Figure 11. It is important to emphasize that the *endogeneity* of the policy signal is crucial for this result to arise.

The signaling effects captured by the upper graphs of Figure 11 have two important implications. Large negative technology shocks that occurred in the late 1970s and in the early 1980s bring about signaling effects that contribute to *raise* inflation expectations with delays. Nevertheless, the signaling effects associated with the those large shocks contribute to *lower* the rate of inflation because the deflationary high-frequency signaling effects dominate the inflationary low-frequency signaling effects.

One may be concerned about the prolonged misalignment between rational beliefs (the blue line) and the actual exogenous state variables (the black circles) reported in the lower graphs of Figure 11. First of all, note that the misalignment is fairly small compared with the estimated standard deviation of the shocks reported in Table 1. As we shall show in the next section, these small but persistent misalignments of rational beliefs and the truth do not cause the inflation forecasts errors in the model to be implausibly too far from what we observe in the data (i.e., the SPF). In fact, these forecast errors are highly correlated with and similar to the actual data on average. Furthermore, it should be noted that the technology shocks are quite persistent

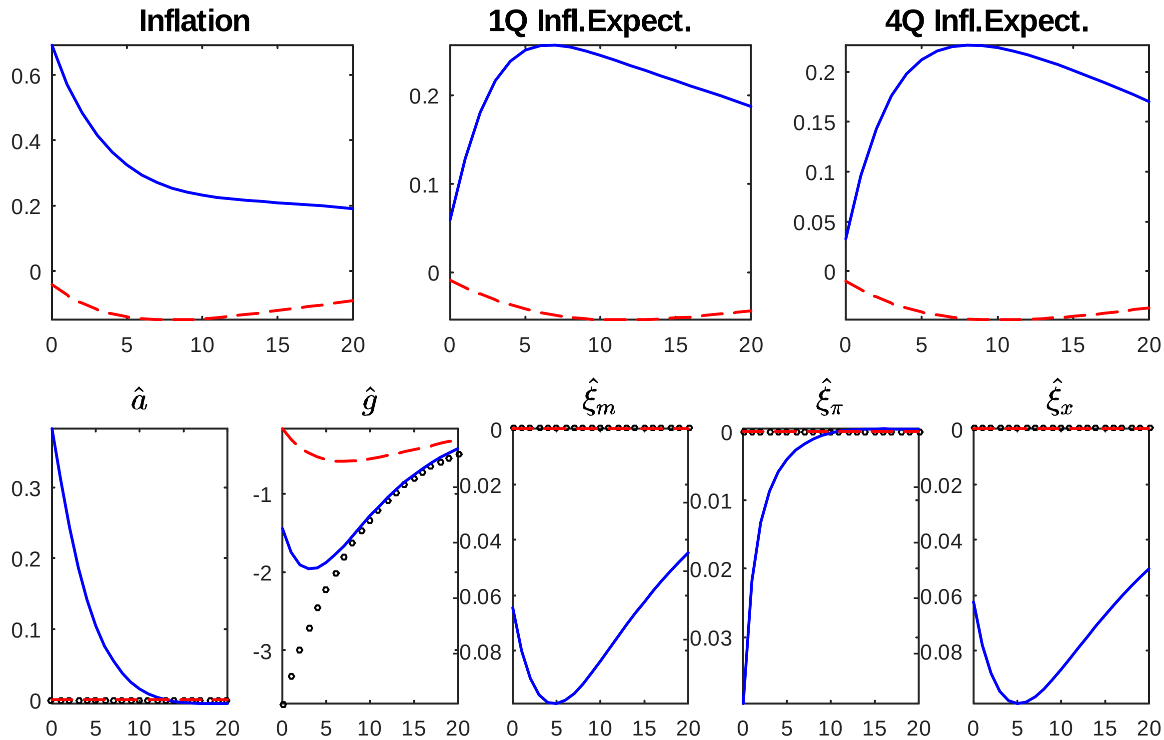

Figure 12: Signaling Effects on Inflation and Inflation Expectations Associated with a One-Standard-Deviation Negative Demand Shock. *Upper graphs*: Response of inflation (the left graph), one-quarter-ahead inflation expectations (the middle graph), and four-quarter-ahead inflation expectations (the right graph) in the estimated DIM, with signaling effects (the solid blue line) and in the counterfactual DIM, with no signaling effects (the red dashed line). *Lower graphs:* Black circles denote the response of the five exogenous state variables to a demand shock. The solid blue line denotes the response of the average first-order expectations about the five exogenous states in the estimated DIM, with signaling effects. The red dashed line denotes the response of the average first-order expectations about the five exogenous states in the counterfactual DIM, with no signaling effects.

and keep on affecting the signals observed by firms five years past the initial impact.

As shown in the upper graphs of Figure 12, demand shocks also give rise to persistent signaling effects on inflation and inflation expectations. The reason why these signaling effects on inflation and, in particular, on inflation expectations are so persistent is because negative demand shocks prompt the central bank to lower its policy rate by signaling expansionary deviations from the rule ($\hat{\xi}_{m,t}$ and $\hat{\xi}_{x,t}$) that have been estimated to be quite persistent. Firms are rational and thus expect that if the central bank deviates from the rule, this behavior will last for a fairly long time. Persistent signaling effects associated with technology and demand shocks constitute a powerful mechanism for the DIM to account for the persistent fluctuations in inflation expectations that we observe in the data (See Figure 1).

It is worthwhile emphasizing that the signaling channel of monetary policy is not the only possible channel through which the DIM could explain why inflation and inflation expectations were so persistently heightened in the 1970s. For instance, these patterns could have been

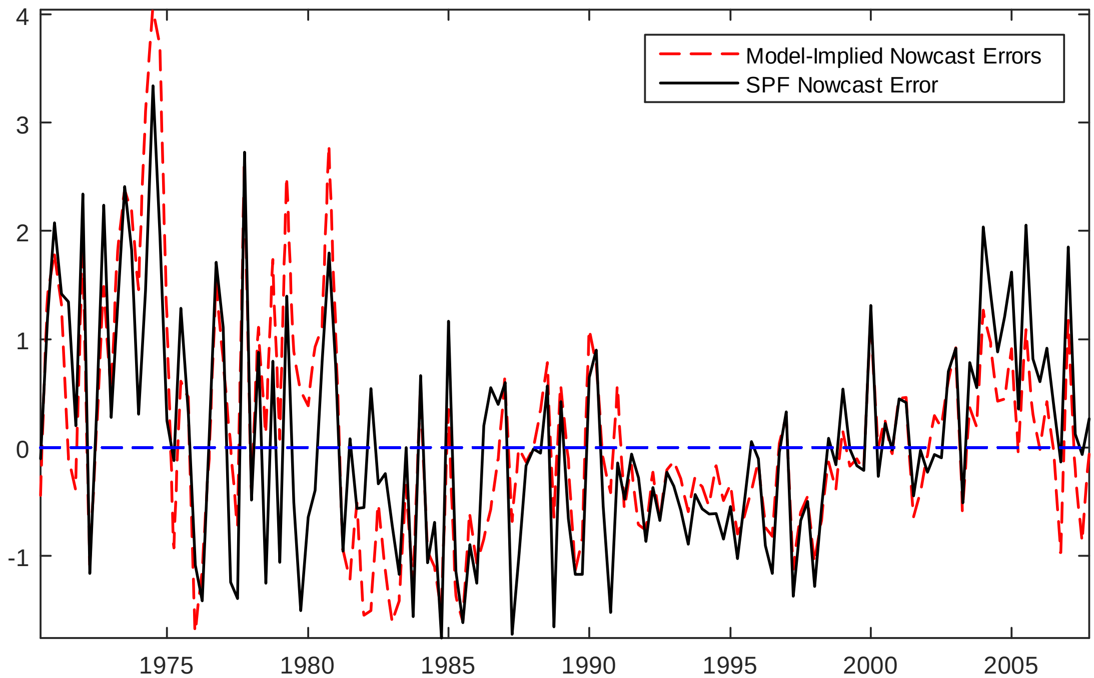

Figure 13: Nowcast Errors for Inflation. The model-implied nowcast errors are obtained by subtracting the smoothed estimates of firms' inflation nowcast (i.e., $\ln\pi_* + \hat{\pi}^{(0)}_{t|t}$) from the realized inflation rate. Smoothed estimates are obtained by setting the value of the DIM parameters at their posterior mean. Inflation errors are reported in percentage points of annualized rates.

explained in the DIM through a sequence of adverse technology shocks along with private signals that almost perfectly reveal the level of aggregate technology and the demand conditions ($\tilde{\sigma}_a \to 0$ and $\tilde{\sigma}_g \to 0$). If the likelihood had picked these estimated values for these noise variances, demand shocks would have brought about only negligible signaling effects of monetary policy because firms would not have had to rely on the policy signal to learn about non-policy shocks. In this case, the heightened inflation of the 1970s would have been explained by a combination of adverse technology shocks and the mechanism proposed by Orphanides (2002, 2003). In that case, the red dashed line in Figure 8 would have been very close to the blue solid line.

4 Discussion

The sluggish dynamics of beliefs in the DIM seem to be quite successful in explaining the persistent macroeconomic dynamics observed in the U.S. data, especially inflation expectations. However, one may argue that such persistent dynamics of beliefs imply that firms are implausibly confused about the aggregate state of the economy. To mitigate this concern, we have included one-quarter-ahead and four-quarter-ahead inflation expectations in our data set for estimation. In addition, an important check to assess the plausibility of the information set is to compare the nowcast errors for inflation predicted by the DIM ($\widehat{\pi}_t - \widehat{\pi}_{t|t}^{(1)}$) to those measured by the *Survey of Professional Forecasters.* Figure 13 shows this comparison. The two nowcast errors exhibit a great deal of comovement with a correlation coefficient of 0.82. Furthermore, the mean of the absolute nowcast errors for inflation is 0.79 in the model vis-a-vis 0.81 in the data. This result suggests that the degree of incomplete information in the estimated DIM is not implausible. It is important to emphasize that we have not used the SPF inflation nowcasts for estimation. It should also be noted that the nowcast errors in the PIM are counterfactually equal to zero.

Another concern has to do with the assumption that firms observe only one endogenous variable, the interest rate, and all the remaining private information comes from exogenous signals. As discussed in Section 2.6, our information structure is built on the imperfect-common-knowledge literature (Woodford 2002; Adam 2007; and Nimark 2008) and is critical for keeping the DIM tractable. However, one may be reasonably concerned that firms are not allowed, for instance, to use the information about the quantities they sell for price-setting decisions. The log-linear approximation to Equation (2) implies that observing the quantities sold would be one additional endogenous signal that would perfectly reveal nominal output to firms. We find that estimating a DIM in which firms pay attention to nominal output would deliver a lower marginal likelihood (-586.76<-319.89), suggesting that this alternative specification of the DIM fits the data rather poorly.

Allowing firms to perfectly observe nominal output ends up endowing them with too much information, critically weakening the ability of the dispersed information model to generate macroeconomic fluctuations with the right degree of persistence. This is particularly true for the case of the federal funds rate and for the observed inflation expectations. This empirical shortcoming of the DIM in which firms observe nominal output cannot be fixed by simply dropping the exogenous signals $a_{j,t}$ and $g_{j,t}$ from firms' information set. This finding suggests that firms may not pay attention to the nominal output when making their price-setting decisions, even though information about this variable is arguably quite cheap to obtain. This result is in line with the empirical study by Andrade et al. (2013), who document that disagreement about inflation and GDP is quite high at short horizons in the *Blue Chip Financial Forecasts.*

To keep the model tractable enough to allow likelihood estimation, we assumed that only firms have limited information, whereas households are perfectly informed. Is assuming that households have also limited information likely to overturn our finding regarding the relevance of signaling effects of monetary policy on inflation and inflation expectations in the 1970s? We argue that the answer is no. It should be noted that the dispersed information model shares with the stylized perfect information New Keynesian DSGE models a recursive structure: the Phillips curve determines inflation given firms' expected path of the output gap, whereas the

Euler equation determines the output gap given a path of the natural rate[19] $\widehat{r}_t^n = \left(1-\rho_g\right)\hat{g}_t - \left(1-\rho_a\right)\hat{a}_t$ and the actual real rate $\widehat{R}_t - E_t\widehat{\pi}_{t+1}$. Note that the natural rate solely depends on exogenous non-policy disturbances. Expanding the Euler equation forward in the DIM leads to $\widehat{x}_t = -\sum_{k=0}^{\infty}\left(\widehat{R}_{t+k} - E_t\widehat{\pi}_{t+k} - \widehat{r}_{t+k}^n\right)$. Assuming that households have incomplete information changes the Euler equation to the following one: $x_t = -\sum_{k=0}^{\infty}\left(\widehat{R}_{t+k|t}^{(1)} - \widehat{\pi}_{t+k|t}^{(1)} - \widehat{r}_{t+k|t}^{n(1)}\right)$.[20] In our discussion, we assume that households have the same information set as firms; that is, $I_{j,t}$ for $j \in (0,1)$ in (5) and, hence, the average expectations about the nominal rate are equal to the true rate; that is, $R_{t|t}^{(1)} = R_t$.

As shown in Section 3.5, signaling effects associated with demand shocks are key in explaining why inflation and inflation expectations were persistently heightened in the 1970s. Relaxing the assumption of perfectly informed households gives rise to three effects on the response of inflation in the aftermath of a negative demand shock. First, abstracting from the information received through the policy signal $\hat{R}_t$, households have imperfect *private* information about the demand conditions $\hat{g}_t$, which affect the dynamics of the natural rate of interest. Imperfect private information implies that the expected path of the future natural rates would fall less after a negative demand shock as households' beliefs will respond less than the actual variables to the shock. Compare the red dashed line that captures the response of the average first-order expectations about the demand conditions when firms observe only private signals (i.e., the signaling channel is shut down) to the black circles that capture the response of the actual demand conditions in the second-from-left lower graph of Figure 12. Second, signaling effects cause households' expectations about future demand conditions $\hat{g}_{t+h|t}^{(1)}$ to fall more and expectations about the dynamics of aggregate technology $\hat{a}_{t+h|t}^{(1)}$ to rise. This can be seen by comparing the solid blue line and the red dashed line in the first two lower graphs from the left in Figure 12. Hence, signaling effects lower the expected path of the future natural rates. Therefore, these two effects on the expected future path of the natural rate go in opposite directions, suggesting that the assumption of perfectly-informed households does not necessarily overstate the magnitude of signaling effects on inflation and inflation expectations in the 1970s. Third, the signaling channel leads households to expect future expansionary deviations of the policy rate from the monetary rule. See the lower graphs in Figure 12. These beliefs would imply an even lower expected path for the policy rate. The Euler equation would then imply that the output gap would fall less and, hence, inflation and inflation expectations are expected to rise even more than they would in the case of perfect information. Thus, assuming

[19] The natural interest rate is the real interest rate that would arise in the model under perfect information and no nominal rigidities.

[20] We abstract from technical complications that would make this extension of the model impossible to evaluate using only pencil and paper, such as the fact that having heterogenous households would lead the distribution of bond holdings to enter the state vector of the economy. Furthermore, we follow standard assumptions in the literature that studies models with learning to ensure that the aggregate resource constraint is satisfied.

imperfect information on the side of households does not necessarily lower the signaling effects on inflation and inflation expectations associated with demand shocks and could even magnify these effects.

5 Concluding Remarks

This paper studies a DSGE model in which information is dispersed across price setters and in which the interest rate set by the central bank has signaling effects. In this model, monetary impulses propagate through two channels: (*i*) the channel based on the central bank's ability to affect the real interest rate due to price stickiness and dispersed information and (*ii*) the signaling channel. The latter arises because changing the policy rate conveys information about the central bank's assessment of inflation and the output gap to price setters.

We fit the model to a data set that includes the *Survey of Professional Forecasters* as a measure of price setters' inflation expectations and Greenbook real-time data as a measure of the central bank's knowledge about inflation and the output gap. We perform a formal econometric evaluation of the model with signaling effects of monetary policy. While the likelihood selects a very short average duration for price contracts, the signaling channel causes the real effects of monetary shocks to be very sizable and persistent. Furthermore, the signaling channel brings about deflationary pressures in the aftermath of positive demand shocks. We also show that signaling effects of monetary policy can account for (i) why inflation expectations have been so persistently heightened in the 1970s, (ii) why inflation expectations fell more sluggishly than inflation after the famous disinflation policy carried out by the Federal Reserve in early 1980s, and (iii) why inflation was so persistently high in the second half of the 1970s.[21]

While there exist several channels through which central banks can communicate with markets nowadays, our paper focuses on interest-rate-based communication. Interest-based communication was virtually the only form of central bank's communication until February 1994 in the U.S. (Campbell et al., 2012). The paper mainly focuses on how this form of communication influenced inflation and inflation expectations in the 1970s and 1980s. Nonetheless, the importance of this type of communication has been growing in recent years. See, for instance, the widespread endorsement of the practice to provide information about the likely future path of the policy rate, which goes under the name of *forward guidance*. While we do not study the effects of forward guidance in this paper, we have shown how to formalize interest-rate-based

[21] Other popular theories for why inflation rose in the 1970s are (*i*) the bad luck view (e.g., Cogley and Sargent 2005; Sims and Zha 2006; Primiceri 2005; and Liu, Waggoner, and Zha 2011), (*ii*) the lack of commitment view (e.g., Chari, Christiano, and Eichenbaum 1998; Christiano and Gust 2000), (*iii*) the policy mistakes view (e.g., Sargent 2001; Clarida, Galí, and Gertler 2000; Lubik and Schorfheide 2004; Primiceri 2006; Coibion and Gorodnichenko 2011), and (*iv*) fiscal and monetary interactions view (e.g., Sargent, Williams, and Zha 2006; Bianchi and Ilut 2012; Bianchi and Melosi, 2014).

communication in dynamic general equilibrium models and how to use these models to formally evaluate the macroeconomic effects of this type of communication.

Changes in the Federal Reserve's attitude toward inflation stabilization have been documented by Davig and Leeper (2007), Justiniano and Primiceri (2008), Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010) and Bianchi (2013). Time-varying model parameters allow us to study how signaling effects of monetary policy on the macroeconomy have changed over time. This fascinating topic is left for future research.

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Technical Appendix

The Appendices are organized as follows. In Appendix A, we derive of the imperfect-common-knowledge Phillips curve (6). Appendix B details an algorithm to solve the dispersed information model. In Appendix C, we characterize the transition equations for the average higher-order expectations about the exogenous state variables − that is, equation (10). In Appendix D, we characterize the laws of motion for the three endogenous state variables (i.e., inflation $\hat{\pi}_t$, real output $\hat{y}_t$ and the interest rate $\hat{R}_t$). In Appendix E, we define a set of measures to quantify the amount of information conveyed by the signals observed by firms. Measuring information flows will simplify the task of interpreting the macroeconomic implications of the signaling channel later on.

A The Imperfect-Common-Knowledge Phillips Curve

The log-linear approximation to the labor supply can be given by $\widehat{c}_t = \widehat{w}_t$. Recalling that the resource constraint implies that $\hat{y}_t = \hat{c}_t$, we can then rewrite the labor supply as follows:

$$\hat{y}_t = \hat{w}_t. \tag{18}$$

Log-linearizing the equation for the real marginal costs yields

$$\widehat{mc}_{j,t} = \hat{w}_t - \widehat{a}_t - \varepsilon^a_{j,t}.$$

We can then write

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{w}_{j,t} - \widehat{a}_t - \varepsilon^a_{j,t},$$

where $\mathbb{E}_{j,t}$ is the expectations conditioned on firm j's information set at time t ($\mathcal{I}_{j,t}$) defined in (5). Using equation (18) for replacing $\hat{w}_t$ yields

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{y}_t - \widehat{a}_t - \varepsilon^a_{j,t}.$$

By integrating across firms, we obtain the average expectations on marginal costs:

$$\widehat{mc}^{(1)}_{t|t} = \hat{y}^{(1)}_{t|t} - \widehat{a}_t.$$

The linearized price index can be written as

$$\int \hat{p}^*_{j,t} dj = \frac{\theta}{1-\theta}\hat{\pi}_t.$$

Recall that we defined $\hat{p}^*_{j,t} = \ln P^*_{j,t} - \ln P_t$ and $\hat{\pi}_t = \ln P_t - \ln P_{t-1} - \ln \pi_*$. After some algebraic manipulation, we write

$$\ln P_t = \theta \left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta) \int \left(\ln P^*_{j,t}\right) dj. \tag{19}$$

The price-setting problem leads to the following first-order conditions:

$$\mathbb{E}\left[\sum_{s=0}^{\infty} (\beta\theta)^s \frac{\xi_{j,t+s}}{P_{t+s}} \left[(1-\nu)\,\pi_*^s + \nu \frac{MC_{j,t+s}}{P^*_{j,t}}\right] y_{j,t+s} | \mathcal{I}_{j,t}\right] = 0.$$

We define the stationary variables:

$$p^*_{j,t} = \frac{P^*_{j,t}}{P_t};\ w_t = \frac{W_t}{P_t},\ mc_{j,t} = \frac{MC_{j,t}}{P_t}.$$

And then we write

$$\mathbb{E}\left\{\xi_{j,t}\left[1-\nu+\nu\frac{mc_{j,t}}{p^*_{j,t}}\right] y_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \xi_{j,t+s}\left[(1-\nu)\,\pi_*^s + \nu\frac{mc_{j,t+s}}{p^*_{j,t}}\left(\Pi_{\tau=1}^{s}\pi_{t+\tau}\right)\right] y_{j,t+s} | \mathcal{I}_{j,t}\right\} = 0. \tag{20}$$

First realize that the terms inside the square brackets are equal to zero at the steady state, and hence, we do not care about the terms outside them. We can write

$$\mathbb{E}\left[\left[1-\nu+\nu mc_{j,*} e^{\widehat{mc}_{j,t}-\widehat{p}^*_{j,t}}\right] + \sum_{s=1}^{\infty} (\beta\theta)^s \left[(1-\nu)\,\pi_*^s + \nu mc_{j,*} e^{\widehat{mc}_{j,t+s}-\widehat{p}^*_{j,t}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}}\right] |\mathcal{I}_{j,t}\right] = 0.$$

Taking the derivatives yields

$$\mathbb{E}\left[\widehat{mc}_{j,t} - \widehat{p}^*_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \left[\left(\widehat{mc}_{j,t+s} - \widehat{p}^*_{j,t} + \sum_{\tau=1}^{s} \hat{\pi}_{t+\tau}\right)\right] |\mathcal{I}_{j,t}\right] = 0.$$

We can take the term $\widehat{p}_{j,t}$ out of the sum operator in the second term and gather the common term to obtain

$$\mathbb{E}\left[\widehat{mc}_{j,t} - \frac{1}{1-\beta\theta}\widehat{p}^*_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \left(\widehat{mc}_{j,t+s} + \sum_{\tau=1}^{s} \hat{\pi}_{t+\tau}\right) |\mathcal{I}_{j,t}\right] = 0.$$

Recall that $\widehat{p}_{j,t}^{*} \equiv \ln P_{j,t}^{*} - \ln P_t$ and cannot be taken out of the expectation operator. We write

$$\ln P_{j,t}^{*} = (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t} + \frac{1}{1-\beta\theta}\ln P_t + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right]. \tag{21}$$

Rolling this equation one step ahead yields

$$\ln P_{j,t+1}^{*} = (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1} + \frac{1}{1-\beta\theta}\ln P_{t+1} + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s+1} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t+1}\right].$$

Taking firm j's conditional expectation at time t on both sides and applying the law of iterated expectations, we obtain the following:

$$\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right) = (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1} + \frac{1}{1-\beta\theta}\ln P_{t+1} + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s+1} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t}\right].$$

We can take $\widehat{mc}_{j,t+1}$ inside the sum operator and write

$$\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right) = (1-\beta\theta)\,\mathbb{E}\left[\frac{1}{1-\beta\theta}\ln P_{t+1} + \frac{1}{\beta\theta}\sum_{s=1}^{\infty}(\beta\theta)^s\,\widehat{mc}_{j,t+s} + \sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right].$$

Therefore,

$$\sum_{s=1}^{\infty}(\beta\theta)^s\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right] = \frac{\beta\theta}{1-\beta\theta}\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right) - \mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right] - \beta\theta\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]. \tag{22}$$

Hence, the equation (21) can be rewritten as:

$$\begin{aligned}\ln P_{j,t}^{*} &= (1-\beta\theta)\left\{\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right] + \frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_t|\mathcal{I}_{j,t}\right] + \sum_{s=1}^{\infty}(\beta\theta)^s\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right]\right\}\\ &\quad + (1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right].\end{aligned}$$

By substituting the result in equation (22), we obtain

$$\begin{aligned}\ln P_{j,t}^{*} &= (1-\beta\theta)\left[\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right]\right] \\ &+\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\ &+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right].\end{aligned}$$

We consider the last term and write

$$\begin{aligned}(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+(1-\beta\theta)\sum_{s=2}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] \\ &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+ \\ &\quad+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\left(\sum_{\tau=1}^{s}\left[\left(\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\right)\right]+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]\right).\end{aligned}$$

It then follows that

$$\begin{aligned}(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\ &\quad+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\ &\quad+(1-\beta\theta)\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right].\end{aligned}$$

Because $\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)=\frac{(\beta\theta)^{2}}{1-\beta\theta}$, then after simplifying, we can write that

$$\begin{aligned}(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= \beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\ &\quad+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right].\end{aligned}$$

We substitute this result into the original equation to get the following expression:

$$\begin{aligned}\ln P_{j,t}^{*} &= (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right] \\ &+\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right].\end{aligned} \tag{23}$$

Note that by definition $\hat{\pi}_{t+1} \equiv \ln P_{t+1} - \ln P_t - \ln \pi_*$. Hence, we can write

$$\begin{aligned} \ln P_{j,t}^* &= (1-\beta\theta) \cdot \mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right] + (1-\beta\theta)\,\mathbb{E}\left[\ln P_t|\mathcal{I}_{j,t}\right] \\ &\quad +\beta\theta \cdot \mathbb{E}\left(\ln P_{j,t+1}^*|\mathcal{I}_{j,t}\right) - \beta\theta \ln \pi_*. \end{aligned} \tag{24}$$

We denote firm j's average k-th order expectation about an arbitrary variable $\hat{x}_t$ as

$$\mathbb{E}^{(k)}\left(\hat{x}_t|\mathcal{I}_{j,t}\right) \equiv \int \mathbb{E}\left(\int \mathbb{E}\left(\ldots\left(\int \mathbb{E}\left(\hat{x}_t|\mathcal{I}_{j,t}\right) dj\right)\ldots|\mathcal{I}_{j,t}\right) dj|\mathcal{I}_{j,t}\right) dj,$$

where expectations and integration across firms are taken k times.

Let us denote the average reset price as $\ln P_t^* = \int \ln P_{j,t}^* dj$. Note that we can rewrite equation (19) as follows

$$\ln P_t = \theta\left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta)\ln P_t^*. \tag{25}$$

Furthermore, we can integrate equation (24) across firms to obtain an equation for the average reset price:

$$\begin{aligned} \ln P_t^* &= (1-\beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta)\ln P_{t|t}^{(1)} \\ &\quad +\beta\theta \ln P_{t+1|t}^{*(1)} - \beta\theta \ln \pi_*, \end{aligned} \tag{26}$$

where $x_{t|t}^{(1)}$ denotes the average first-order expectations about an arbitrary variable x_t of the model (e.g., the real marginal costs).

Let us plug equation (26) into equation (25) as follows:

$$\begin{aligned} \ln P_t &= \theta \ln P_{t-1} + \left(\theta - (1-\theta)\,\beta\theta\right)\ln \pi_* \\ &\quad + (1-\theta)\left[(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta)\ln P_{t|t}^{(1)} + \beta\theta \ln P_{t+1|t}^{*(1)}\right]. \end{aligned} \tag{27}$$

From the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_*$ and from the price index (19), we get the following:[22]

$$\ln P_{t+1}^{*} = \frac{\hat{\pi}_{t+1}}{1-\theta} + \ln P_t + \ln \pi_*.$$

Furthermore, the following fact is easy to establish:

$$\ln P_{t+1} = \widehat{\pi}_{t+1} + \ln P_t + \ln \pi_*.$$

Applying these three results to equation (27) yields

$$\begin{aligned} \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_* \;=\;& \theta \ln P_{t-1} + (\theta - (1-\theta)\,\beta\theta) \ln \pi_* \\ & + (1-\theta)\left[(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta)\ln P_{t|t}^{(1)} + \beta\theta\left(\frac{\hat{\pi}_{t+1|t}^{(1)}}{1-\theta} + \ln P_{t|t}^{(1)} + \ln \pi_*\right)\right]. \end{aligned} \tag{28}$$

Algebraic manipulations yield the following equation:

$$\widehat{\pi}_t = (1-\theta)(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(1)} + (1-\theta)\,\widehat{\pi}_{t|t}^{(1)} + \beta\theta\left(\widehat{\pi}_{t+1|t}^{(1)}\right). \tag{29}$$

By repeatedly taking firm j's expectation and then averaging the resulting equation across firms, we get

$$\widehat{\pi}_{t|t}^{(k)} = (1-\theta)(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(k+1)} + (1-\theta)\,\widehat{\pi}_{t|t}^{(k+1)} + \beta\theta\left(\widehat{\pi}_{t+1|t}^{(k+1)}\right).$$

Repeatedly substituting these equations for $k \geq 1$ back in equation (29) yields the imperfect-common-knowledge Phillips curve:

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^{k-1}\,\widehat{mc}_{t|t}^{(k)} + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\,\widehat{\pi}_{t+1|t}^{(k)}.$$

[22] This last result comes from observing that

$$\ln P_t = \theta(\ln P_{t-1} + \ln \pi_*) + (1-\theta)\ln P_t^*.$$

By using the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_*$:

$$\widehat{\pi}_t + \ln P_{t-1} + \ln \pi_* = \theta(\ln P_{t-1} + \ln \pi_*) + (1-\theta)\ln P_t^*.$$

Rolling one period forward, we get

$$\widehat{\pi}_{t+1} = (\theta - 1)(\ln P_t + \ln \pi_*) + (1-\theta)\ln P_{t+1}^*.$$

And finally, by rearranging the terms, we get the result in the text.

B Solving the Dispersed Information Model

We solve the model assuming common knowledge of rationality (Nimark 2008) and focusing on equilibria where the higher-order expectations about the exogenous state variables (that is, $X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{g}_{t|t}^{(s)}, \widehat{\xi}_{m,t|t}^{(s)}, \widehat{\xi}_{\pi,t|t}^{(s)}, \widehat{\xi}_{x,t|t}^{(s)} : 0 \le s \le k\right]'$) follow the VAR(1) process in equation (10). Note that we truncate the order of the average expectations at $k < \infty$. Furthermore, we guess the matrix $\mathbf{v}_0$ that determines the dynamics of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]$ in equation (9). As shown in Appendix D, the structural equations of the model can be written in the following linear form:

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \tag{30}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on a complete information set (i.e., an information set that includes the history of all structural shocks).

For a given parameter set Θ_{DIM}, take the following steps:

Step 0 Set $i = 1$ and guess the matrices $\mathbf{M}^{(i)}$, $\mathbf{N}^{(i)}$, and $\mathbf{v}_0^{(i)}$.

Step 1 Set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$ and solve the model given by equation (10) and equation (30) through a standard linear rational expectations model solver (e.g., Blanchard and Kahn 1980; Sims 2002). The solver delivers the matrix $\mathbf{v}_0^{(i+1)}$ such that $\mathbf{s}_t = \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)}$. As we will show in Appendix D, the matrices Γ_0, Γ_1, and Γ_2 in equation (30) are functions of the model parameter Θ_{DIM} as well as the guessed matrices $\mathbf{M}^{(i)}$ and $\mathbf{v}_0^{(i)}$.

Step 2 Given the law of motion (10) for $X_{t|t}^{(0:k)}$, in which we set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$, equation $\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a$ for the signal concerning the aggregate technology, equation $\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g \varepsilon_{j,t}^g$ for the signal concerning the demand conditions, and the equation

$$\widehat{R}_t = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)}$$

for the endogenous policy signal $\widehat{R}_t \in \mathbf{s}_t$ solve the firms' signal extraction problem through the Kalman filter and determine the matrices $\mathbf{M}^{(i+1)}$ and $\mathbf{N}^{(i+1)}$. Appendix C provides a detailed explanation of how we characterize these matrices.

Step 3 If $\left\|\mathbf{M}^{(i)} - \mathbf{M}^{(i+1)}\right\| < \varepsilon_m$, $\left\|\mathbf{N}^{(i)} - \mathbf{N}^{(i+1)}\right\| < \varepsilon_n$, and $\left\|\mathbf{v}_0^{(i)} - \mathbf{v}_0^{(i+1)}\right\| < \varepsilon_v$ for any $\varepsilon_m > 0$, $\varepsilon_n > 0$, and $\varepsilon_v > 0$ and small, STOP or else set i=i+1 and go to STEP 1.

Given equation (10) and equation $\mathbf{s}_t = \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$ obtained in step 1, the law of motion of the model variables is as follows:

$$\begin{bmatrix} X_{t|t}^{(0:k)} \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{(i+1)} & \mathbf{0} \\ \mathbf{v}_0^{(i+1)} \mathbf{M}^{(i+1)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{t-1|t-1}^{(0:k)} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(i+1)} \\ \mathbf{v}_0^{(i+1)} \mathbf{N}^{(i+1)} \end{bmatrix} \boldsymbol{\varepsilon}_t. \tag{31}$$

C Transition Equation of High-Order Expectations

In this section, we show how to derive the law of motion of the average higher-order expectations of the exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{g}_t$,$\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) for given parameter values and the matrix of coefficients $\mathbf{v}_0$. We focus on equilibria where the average expectations evolve according to

$$X_{t|t}^{(0:k)} = \mathbf{M}X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t, \tag{32}$$

where $\boldsymbol{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_{a,t} & \varepsilon_{g,t} & \varepsilon_{m,t} & \varepsilon_{\pi,t} & \varepsilon_{x,t} \end{bmatrix}'$. We set $\mathbf{X}_t \equiv X_{t|t}^{(0:k)}$, for notational convenience. Firms' reduced-form state-space model can be concisely cast as follows:

$$\mathbf{X}_t = \mathbf{M}\mathbf{X}_{t-1} + \mathbf{N}\boldsymbol{\varepsilon}_t, \tag{33}$$

$$\mathbf{Z}_t\left(j\right) = \mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t}, \tag{34}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_1 & \mathbf{d}_2 & \left(\mathbf{1}_3^T\mathbf{v}_0\right) \end{bmatrix}',$$

with $\mathbf{d}_1' = \left[1, \mathbf{0}_{1\times 5(k+1)-1}\right]$, $\mathbf{d}_2' = [0, 1, \mathbf{0}_{1\times 5k+3}]$, $\mathbf{1}_3^T = [0, 0, 1]$, and $e_{j,t} = \left[\varepsilon_{j,t}^a, \varepsilon_{j,t}^g\right]'$ and

$$\mathbf{Q} = \begin{bmatrix} \widetilde{\sigma}_a & 0 \\ 0 & \widetilde{\sigma}_g \\ 0 & 0 \end{bmatrix}.$$

Solving the firms' signal extraction problem requires applying the Kalman filter. The Kalman equation pins down firm j's first-order expectations about the model's state variables $\mathbf{X}_{t|t}\left(j\right)$ and the associated conditional covariance matrix $\mathbf{P}_{t|t}$:

$$\begin{aligned} \mathbf{X}_{t|t}\left(j\right) &= \mathbf{X}_{t|t-1}\left(j\right) + \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\left[\mathbf{Z}_t\left(j\right) - \mathbf{Z}_{t|t-1}\left(j\right)\right], & (35) \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}_{t|t-1}', & (36) \end{aligned}$$

where

$$\mathbf{P}_{t|t-1} = \mathbf{M}\mathbf{P}_{t-1|t-1}\mathbf{M}' + \mathbf{N}\mathbf{N}', \tag{37}$$

and the matrix $\mathbf{F}_{t|t-1} \equiv E\left[\mathbf{Z}_t\mathbf{Z}_t'|\mathbf{Z}^{t-1}\right]$, which can be shown to be

$$\mathbf{F}_{t|t-1} = \mathbf{D}\mathbf{P}_{t|t-1}\mathbf{D}' + \mathbf{Q}\mathbf{Q}'. \tag{38}$$

Therefore, combining equation (36) with equation (37) yields

$$\mathbf{P}_{t+1|t} = \mathbf{M}\left[\mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}'_{t|t-1}\right]\mathbf{M}' + \mathbf{N}\mathbf{N}'. \tag{39}$$

Define the Kalman-gain matrix as $\mathbf{K}_t \equiv \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}$. Write the law of motion of firm j's first-order beliefs about $\mathbf{X}_t$ as

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{K}_t\left[\mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t} - \mathbf{D}\mathbf{X}_{t|t-1}(j)\right],$$

where we have combined equations (35) and (34). By recalling that $\mathbf{X}_{t|t-1}(j) = \mathbf{M}\mathbf{X}_{t-1|t-1}(j)$, we obtain

$$\mathbf{X}_{t|t}(j) = (\mathbf{M} - \mathbf{K}\mathbf{D}\mathbf{M})\,\mathbf{X}_{t-1|t-1}(j) + \mathbf{K}\left[\mathbf{D}\mathbf{M}\cdot\mathbf{X}_{t-1} + \mathbf{D}\mathbf{N}\cdot\boldsymbol{\varepsilon}_t + \mathbf{Q}e_{j,t}\right]. \tag{40}$$

The vector $\mathbf{X}_{t|t}(j)$ contains firm j's first-order expectations about the model's state variables. Integrating across firms yields the law of motion of the average expectation about $\mathbf{X}_{t|t}^{(1)}$:

$$\mathbf{X}_{t|t}^{(1)} = (\mathbf{M} - \mathbf{K}\mathbf{D}\mathbf{M})\,\mathbf{X}_{t-1|t-1}^{(1)} + \mathbf{K}\left[\mathbf{D}\mathbf{M}\cdot\mathbf{X}_{t-1} + \mathbf{D}\mathbf{N}\cdot\boldsymbol{\varepsilon}_t\right].$$

Note that $X_{t|t}^{(0:\infty)} = \left[X_t, X_{t|t}^{(1:\infty)}\right]'$ and that

$$X_t = \underbrace{\begin{bmatrix} \rho_a & 0 & 0 & 0 & 0 & \mathbf{0} \\ 0 & \rho_g & 0 & 0 & 0 & \mathbf{0} \\ 0 & 0 & \rho_m & 0 & 0 & \mathbf{0} \\ 0 & 0 & 0 & \rho_\pi & 0 & \mathbf{0} \\ 0 & 0 & 0 & 0 & \rho_x & \mathbf{0} \end{bmatrix}}_{\mathbf{R}_1} X_{t-1|t-1}^{(0:k)} + \underbrace{\begin{bmatrix} \sigma_a & 0 & 0 & 0 & 0 \\ 0 & \sigma_g & 0 & 0 & 0 \\ 0 & 0 & \sigma_m & 0 & 0 \\ 0 & 0 & 0 & \sigma_\pi & 0 \\ 0 & 0 & 0 & 0 & \sigma_x \end{bmatrix}}_{\mathbf{R}_2} \cdot \boldsymbol{\varepsilon}_t.$$

So by using the assumption of common knowledge in rationality, we can fully characterize the matrices $\mathbf{M}$ and $\mathbf{N}$:

$$\mathbf{M} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{5\times 5} & \mathbf{0}_{5\times 5k} \\ \mathbf{0}_{5k\times 5} & (\mathbf{M} - \mathbf{K}\mathbf{D}\mathbf{M})|_{(1:5k,1:5k)} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}(\mathbf{D}\mathbf{M})|_{(1:5k,1:5(k+1))} \end{bmatrix}, \tag{41}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{R}_2 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}\mathbf{D}\mathbf{N}|_{(1:5k,1:5)} \end{bmatrix}, \tag{42}$$

where $\cdot|_{(n_1:n_2,m_1:m_2)}$ denotes the submatrix obtained by taking the elements lying between the n_1-th row and the n_2-th row and between the m_1-th column and the m_2-th column. Note that $\mathbf{K}$ in equation (41) and equation (42) denotes the steady-state Kalman gain matrix, which is obtained by iterating the equations (37) and (39) until convergence.

D The Laws of Motion for the Endogenous State Variables

In this section we introduce some useful results and characterize the law of motion (30) for the endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$.

D.1 Preliminaries

The *assumption of common knowledge in rationality* ensures that agents use the actual law of motion of higher-order expectations to forecast the dynamics of the higher-order expectations. The following propositions turn out to be useful for what follows:

Proposition 1 $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$ *for any* $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$. Then common knowledge in rationality implies that $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$. ■

Since we truncate beliefs after the k-th order, we define the matrix $\mathbf{T}^{(s)}$ as follows:

$$\mathbf{T}^{(s)} \equiv \begin{bmatrix} \mathbf{0}_{5(k-s+1)\times 5s} & \mathbf{I}_{5(k-s+1)} \\ \mathbf{0}_{5s\times 5s} & \mathbf{0}_{5s\times(k+1-s)5} \end{bmatrix},$$

and we approximate the law of motion for $\mathbf{s}_{t|t}^{(s)}$ as $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

Proposition 2 *The following holds true:* $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ *for any* $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_{t+h} = \mathbf{v}_0 X_{t+h|h+1}^{(0:k)}$. Given equation (10), it follows that $\mathbf{s}_{t+h} = \mathbf{v}_0 \left(\mathbf{M}^h X_{t|t}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_{t+1} \right)$. Common knowledge in rationality implies that repeatedly taking firms' expectations and then averaging across firms leads to an expression for the law of motion of the average higher-order expectations: $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ for any s. ■

Since we truncate beliefs after the k-th order, we can approximate the law of motion for the average higher-order expectations as $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

D.2 The Laws of Motion of the Endogenous State Variables

The laws of motion of the three endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$, are given by the Euler equation (7), the Phillips curve

(6), and the Taylor rule (8). We want to write this system of linear equations as

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \tag{43}$$

where $\mathbf{s}_t \equiv \left[\hat{\pi}_t, \hat{y}_t, \hat{R}_t\right]'$. It is obvious how to write equations (7) and (8) in the form (43). However, figuring how how to write the Phillips curve (6) in the form (43) requires a bit of work. First, note that given Propositions 1−2 and the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$, the imperfect-common-knowledge Phillips curve (6) can be rewritten as follows:

$$\begin{aligned}
\mathbf{a}_0 X_{t|t}^{(0:k)} &= (1-\theta)(1-\beta\theta) \sum_{s=0}^{k-1} (1-\theta)^s \mathbf{1}_2^T \left[\mathbf{v}_0 \mathbf{T}^{(s+1)} X_{t|t}^{(0:k)}\right] + \\
&- (1-\theta)(1-\beta\theta) \sum_{s=0}^{k-1} (1-\theta)^s \left[\boldsymbol{\gamma}_a^{(s)\prime} X_{t|t}^{(0:k)}\right] \\
&+ \beta\theta \sum_{s=0}^{k-1} (1-\theta)^s \mathbf{1}_1^T \left[\mathbf{v}_0 \mathbf{M} \mathbf{T}^{(s+1)} X_{t|t}^{(0:k)}\right],
\end{aligned}$$

where $\mathbf{1}_1^T = [1,0,0]$, $\mathbf{1}_2^T = [0,1,0]$, and $\boldsymbol{\gamma}_a^{(s)} = \left[\mathbf{0}_{1\times 5s}, (1,0,0), \mathbf{0}_{1\times 5(k-s)}\right]'$. The following restrictions upon vectors of coefficients $\mathbf{a}_0$ and $\mathbf{a}_1$ can be derived from the rewritten Phillips curve:

$$\widehat{\pi}_t = \left[(1-\theta)(1-\beta\theta)\left[\boldsymbol{\nu}\mathbf{m}_1 - \left(\sum_{s=0}^{k-1} (1-\theta)^s \boldsymbol{\gamma}_a^{(s)\prime}\right)\right] + \beta\theta\boldsymbol{\nu}\mathbf{m}_2\right] X_{t|t}^{(0:k)}, \tag{44}$$

where we define:

$$\begin{aligned}
\mathbf{m}_1 &\equiv \begin{bmatrix} \mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(1)} \\ (1-\theta)\left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(2)}\right] \\ \vdots \\ (1-\theta)^k \left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(k)}\right] \end{bmatrix}, \quad \mathbf{m}_2 \equiv \begin{bmatrix} \mathbf{1}_1^T \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(1)} \\ (1-\theta)\left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(2)}\right] \\ \vdots \\ (1-\theta)^k \left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(k)}\right] \end{bmatrix}, \\
\boldsymbol{\nu} &\equiv \mathbf{1}_{1\times k}.
\end{aligned}$$

E Measuring Information Flows from the Signaling Channel

A salient feature of the dispersed information model is that the policy rate R_t transfers information about the output gap and inflation to price setters. We call the avenue by which this information is transferred the *signaling channel of monetary transmission.* Price setters use the policy rate as a signal that helps them to track non-policy shocks (namely, technology shocks $\varepsilon_{a,t}$ and demand shocks $\varepsilon_{g,t}$) and, at the same time, to infer shocks to central bank's exogenous deviations from the monetary rule (i.e., monetary policy shocks $\varepsilon_{r,t}$). Following a standard practice in information theory (Cover and Thomas 1991), we use an entropy-based measure to assess how much information is provided by the signals firms observe in every period. The entropy measures the uncertainty about a random variable. For instance, the entropy associated with the level of aggregate technology $\widehat{a}_t$, which is normally distributed with (unconditional) covariance matrix $var(\widehat{a}_t)$, is defined as $H(\widehat{a}_t) \equiv 0.5 \log_2 [2\pi e \cdot var(\widehat{a}_t)]$.

We quantify the information flow conveyed by the signals as *the reduction of uncertainty* (i.e., entropy) at time t due to observing the signals in the information set $\mathcal{I}_{j,t}$.[23] For instance, the information flow about aggregate technology conveyed by the signals in the information set $\mathcal{I}_{j,t}$ can be computed as $\mathcal{H}(\widehat{a}_t; \mathcal{I}_{j,t}) = H(\widehat{a}_t) - H(\widehat{a}_t|\mathcal{I}_{j,t})$, where the conditional entropy $H(\widehat{a}_t|\mathcal{I}_{j,t}) \equiv 0.5 \log_2 [2\pi e \cdot var(\widehat{a}_t|\mathcal{I}_{j,t})]$ and $var(\widehat{a}_t|\mathcal{I}_{j,t})$ denotes the variance of aggregate technology conditional on firms having observed the signals in their information set $\mathcal{I}_{j,t}$.[24]

We measure the information flow that firms receive about aggregate technology from observing *solely the private signals* as $\mathcal{H}(\widehat{a}_t; \mathcal{I}_{j,t} \diagup R^t) \equiv H(\widehat{a}_t) - H(\widehat{a}_t|\mathcal{I}_{j,t} \diagup R^t)$ where $H(\widehat{a}_t|\mathcal{I}_{j,t} \diagup R^t)$ is the entropy conditional on firms having observed *only* their private signals. Endowed with this measure, we compute the fraction of private information about the aggregate technology $\widehat{a}_t$ as the ratio of the private information flow to the information flow from all the signals in the information set $\mathcal{I}_{j,t}$; that is, $\vartheta_a \equiv \mathcal{H}(\widehat{a}_t; \mathcal{I}_{j,t} \diagup R^t) / \mathcal{H}(\widehat{a}_t; \mathcal{I}_{j,t})$. It should be noted that $\vartheta_a \in [0,1]$. If ϑ_a is close to zero, then most of the information about aggregate technology stems from the policy signal. On the contrary, if ϑ_a is close to unity, then most of the information about aggregate technology stems from the private signal $\widehat{a}_{j,t}$.[25] Analogously, we can define the fraction of private information about the demand conditions $\widehat{g}_t$ as $\vartheta_g \equiv \mathcal{H}(\widehat{g}_t; \mathcal{I}_{j,t} \diagup R^t) / \mathcal{H}(\widehat{g}_t; \mathcal{I}_{j,t})$.

Let us define the entropy of aggregate technology conditional on firms having observed only the history of the policy signal as $H\left(\widehat{a}_t|\widehat{R}^t\right) \equiv 0.5 \log_2 \left[2\pi e \cdot var\left(\widehat{a}_t|\widehat{R}^t\right)\right]$, where $var\left(\widehat{a}_t|\widehat{R}^t\right)$ denotes the variance of aggregate technology conditional on firms having observed only the history of the policy signal R^t. We measure the information flow about aggregate technology conveyed only by the policy signal $\widehat{R}_t$ as $\mathcal{H}\left(\widehat{a}_t; \widehat{R}^t\right) \equiv H(\widehat{a}_t) - H\left(\widehat{a}_t|\widehat{R}^t\right)$.

Another useful statistic for assessing the macroeconomic effects of the signaling channel

[23] This approach is extensively followed by the literature of rational inattention pioneered by Sims (2003) and followed by Maćkowiak and Wiederholt (2009, forthcoming), Paciello and Wiederholt (2014), and Matejka (2011).

[24] The units of the measure $\mathcal{H}(\widehat{a}_t)$ are *bits* of information. The conditional variance can be pinned down by applying the Kalman-filter recursion, as shown in Appendix C. Note that having assumed that firms have received an infinitely long sequence of signals at any time t implies that the conditional covariance matrix $var(\widehat{a}_t|\mathcal{I}_{j,t})$ is time invariant and is the same across firms at any time. Hence, information flows do not vary over time or across firms and we can omit indexing the information flow $\mathcal{H}$ with j and t.

[25] Note that the other private signal (i.e., $\widehat{g}_{j,t}$) does not convey any information about the level of aggregate technology because of the assumed orthogonality of structural shocks at all leads and lags.

is the fraction of information about the exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{g}_t$, $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) conveyed by the policy signal. For instance, the fraction of information about the level of aggregate technology $\widehat{a}_t$ is computed as follows:

$$\Phi_a \equiv \frac{\mathcal{H}\left(\widehat{a}_t; \widehat{R}^t\right)}{\mathcal{H}\left(\widehat{a}_t; \widehat{R}^t\right) + \mathcal{H}\left(\widehat{g}_t; \widehat{R}^t\right) + \mathcal{H}\left(\widehat{\xi}_{m,t}; \widehat{R}^t\right) + \mathcal{H}\left(\widehat{\xi}_{\pi,t}; \widehat{R}^t\right) + \mathcal{H}\left(\widehat{\xi}_{x,t}; \widehat{R}^t\right)}, \tag{45}$$

where $\mathcal{H}\left(\widehat{a}_t; \widehat{R}^t\right) \equiv H\left(\widehat{a}_t\right) - H\left(\widehat{a}_t | \widehat{R}^t\right)$ measures the information flow about aggregate technology conveyed only by the policy signal $\widehat{R}_t$. $\mathcal{H}\left(\widehat{g}_t; \widehat{R}^t\right)$, $\mathcal{H}\left(\widehat{\xi}_{m,t}; \widehat{R}^t\right)$, $\mathcal{H}\left(\widehat{\xi}_{\pi,t}; \widehat{R}^t\right)$, and $\mathcal{H}\left(\widehat{\xi}_{x,t}; \widehat{R}^t\right)$ are the analogous objects for the demand conditions $(\widehat{g}_t)$ and the components of the of overall state of monetary policy ($\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$), respectively. The numerator quantifies the information flow about the level of aggregate technology $\widehat{a}_t$ conveyed by the public signal. The denominator quantifies the information flow about the three exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{g}_t$, $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) conveyed by the policy signal. This ratio Φ_a assumes values between zero and one. Analogously, we can define the fraction of information about the demand conditions conveyed by the policy signal as Φ_g and the fraction of information about the deviations from the monetary rule (i.e., $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) conveyed by the policy signal as Φ_m, Φ_π, and Φ_x. Note that $\Phi_a + \Phi_g + \Phi_m + \Phi_\pi + \Phi_x = 1$.

In summary, the ratio ϑ_a measures the accuracy of the private signal $\widehat{a}_{j,t}$ about the level of aggregate technology $\widehat{a}_t$ *relative to that of the policy signal*. The ratios $\Phi_a, \Phi_g, \Phi_m, \Phi_\pi$, and Φ_x evaluate the accuracy of the public signal *about each of the five exogenous state variables* $\widehat{a}_t$, $\widehat{g}_t$, $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, and $\widehat{\xi}_{x,t}$, respectively.