groups, an efficient estimate will be $m' = \dfrac{M\left[\bar{u} - \frac{s_{uv}}{s_v^2}\left(\bar{v} - \frac{N}{M}\right)\right]}{N}$. The efficiency of m' relative to the conventional estimate $\dfrac{M\bar{u}}{N}$ is $(1 - \rho_{uv}^2)^{-1}$, which ordinarily would seem to be quite large. This is easily extended to the case Π is divided into l strata with M_i groups comprising N_i individuals in the i^{th} stratum, when a random sample of m_i out of the M_i groups in each stratum is taken. Let v_{ij} be the number of individuals in the j^{th} group of the i^{th} stratum and u_{ij} denote the sum of the values of x for these v_{ij} individuals. The estimate of m_x becomes

$$m'' = \frac{\sum_{i=1}^{l} M_i\left[\bar{u}_i - \frac{s_{u_iv_i}}{s_{v_i}^2}\left(\bar{v}_i - \frac{N_i}{M_i}\right)\right]}{N}$$

If $\sum_{i=1}^{l} m_i = m$ is fixed, the large sample variance of m'' will be a minimum if m_i is proportional to $M_i\sigma_{u_i}\sqrt{1 - \rho_i^2}$, where ρ_i is the correlation between u and v in the i^{th} stratum.

In conclusion, the writer wishes to thank Professor A. Wald for his advice and encouragement, and Mr. Henry Goldberg for several suggestions.

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SIGNIFICANCE LEVELS FOR THE RATIO OF THE MEAN SQUARE SUCCESSIVE DIFFERENCE TO THE VARIANCE

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For purposes of practical application in connection with significance tests a tabulation of the argument corresponding to certain percentage points of the probability integral is usually more convenient than that of the probability integral for equal intervals of the argument. A table of probabilities for the

Values of $\frac{\delta^2}{s^2}$ for Different Levels of Significance

	Values of k			Values of k'		
n	P=.001	P=.01	P=.05	P=.95	P=.99	P=.999
4	.7864	.8341	1.0406	4.2927	4.4992	4.5469
5	.5201	.6724	1.0255	3.9745	4.3276	4.4799
6	.4361	.6738	1.0682	3.7318	4.1262	4.3639
7	.4311	.7163	1.0919	3.5748	3.9504	4.2356
8	.4612	.7575	1.1228	3.4486	3.8139	4.1102
9	.4973	.7974	1.1524	3.3476	3.7025	4.0027
10	.5351	.8353	1.1803	3.2642	3.6091	3.9093
11	.5717	.8706	1.2062	3.1938	3.5294	3.8283
12	.6062	.9033	1.2301	3.1335	3.4603	3.7574
13	.6390	.9336	1.2521	3.0812	3.3996	3.6944
14	.6702	.9618	1.2725	3.0352	3.3458	3.6375
15	.6999	.9880	1.2914	2.9943	3.2977	3.5858
16	.7281	1.0124	1.3090	2.9577	3.2543	3.5386
17	.7548	1.0352	1.3253	2.9247	3.2148	3.4952
18	.7801	1.0566	1.3405	2.8948	3.1787	3.4552
19	.8040	1.0766	1.3547	2.8675	3.1456	3.4182
20	.8265	1.0954	1.3680	2.8425	3.1151	3.3840
21	.8477	1.1131	1.3805	2.8195	3.0869	3.3523
22	.8677	1.1298	1.3923	2.7982	3.0607	3.3228
23	.8866	1.1456	1.4035	2.7784	3.0362	3.2953
24	.9045	1.1606	1.4141	2.7599	3.0133	3.2695
25	.9215	1.1748	1.4241	2.7426	2.9919	3.2452
26	.9378	1.1883	1.4336	2.7264	2.9718	3.2222
27	.9535	1.2012	1.4426	2.7112	2.9528	3.2003
28	.9687	1.2135	1.4512	2.6969	2.9348	3.1794
29	.9835	1.2252	1.4594	2.6834	2.9177	3.1594
30	.9978	1.2363	1.4672	2.6707	2.9016	3.1402

	Values of k			Values of k'		
n	P=.001	P=.01	P=.05	P=.95	P=.99	P=.999
31	1.0115	1.2469	1.4746	2.6587	2.8864	3.1219
32	1.0245	1.2570	1.4817	2.6473	2.8720	3.1046
33	1.0369	1.2667	1.4885	2.6365	2.8583	3.0882
34	1.0488	1.2761	1.4951	2.6262	2.8451	3.0725
35	1.0603	1.2852	1.5014	2.6163	2.8324	3.0574
36	1.0714	1.2940	1.5075	2.6068	2.8202	3.0429
37	1.0822	1.3025	1.5135	2.5977	2.8085	3.0289
38	1.0927	1.3108	1.5193	2.5889	2.7973	3.0154
39	1.1029	1.3188	1.5249	2.5804	2.7865	3.0024
40	1.1128	1.3266	1.5304	2.5722	2.7760	2.9898
41	1.1224	1.3342	1.5357	2.5643	2.7658	2.9776
42	1.1317	1.3415	1.5408	2.5567	2.7560	2.9658
43	1.1407	1.3486	1.5458	2.5494	2.7466	2.9545
44	1.1494	1.3554	1.5506	2.5424	2.7376	2.9436
45	1.1577	1.3620	1.5552	2.5357	2.7289	2.9332
46	1.1657	1.3684	1.5596	2.5293	2.7205	2.9232
47	1.1734	1.3745	1.5638	2.5232	2.7125	2.9136
48	1.1807	1.3802	1.5678	2.5173	2.7049	2.9044
49	1.1877	1.3856	1.5716	2.5117	2.6977	2.8956
50	1.1944	1.3907	1.5752	2.5064	2.6908	2.8872
51	1.2010	1.3957	1.5787	2.5013	2.6842	2.8790
52	1.2075	1.4007	1.5822	2.4963	2.6777	2.8709
53	1.2139	1.4057	1.5856	2.4914	2.6712	2.8630
54	1.2202	1.4107	1.5890	2.4866	2.6648	2.8553
55	1.2264	1.4156	1.5923	2.4819	2.6585	2.8477
56	1.2324	1.4203	1.5955	2.4773	2.6524	2.8403
57	1.2383	1.4249	1.5987	2.4728	2.6465	2.8331
58	1.2442	1.4294	1.6019	2.4684	2.6407	2.8260
59	1.2500	1.4339	1.6051	2.4640	2.6350	2.8190
60	1.2558	1.4384	1.6082	2.4596	2.6294	2.8120

ratio of the mean square successive difference δ^2 to the variance s^2, $P\left(\frac{\delta^2}{s^2} < k\right) = \int_0^k \omega(\delta^2/s^2)\, d(\delta^2/s^2)$, where $\omega(\delta^2/s^2)$ is the distribution of δ^2/s^2,[1] has been published recently[2] with k as argument. The following table of values of δ^2/s^2 for $P = .001$, .01 and .05 has been computed from it by interpolation.

Since the distribution of δ^2/s^2, $\omega(\delta^2/s^2)$, is symmetric[3] about $E(\delta^2/s^2)$, $P(\delta^2/s^2 < k) = P(\delta^2/s^2 > k')$ if $E(\delta^2/s^2) - k = k' - E(\delta^2/s^2)$, where $E(\delta^2/s^2) = 2n/(n - 1)$.[3] The upper levels are rarely of practical use, since large values of the ratio, δ^2/s^2, could arise only from a somewhat artificial set of observations, such as alternately high and low values of the observed variable.

The computation of this table of significance levels was made at the suggestion of Lt. Col. L. E. Simon.

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[1] For determination of $\omega(\delta^2/s^2)$ cf. JOHN VON NEUMANN, "Distribution of the ratio of the mean square successive difference to the variance," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 367–395.

[2] B. I. HART, "Tabulation of the probabilities for the ratio of the mean square successive difference to the variance," *Annals of Math. Stat.*, Vol. 13 (1942) p. 213.

[3] Loc. cit.[1] p. 372 for proof of symmetry and evaluation of $E(\delta^2/s^2)$.

A CORRECTION

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In my article "Notes on the Distribution of Roots of a Polynomial with Random Complex Coefficients" which appeared in the June 1942 issue of the *Annals of Mathematical Statistics*, the symbol $\sum_{p=1}^{n} \sum_{q=p+1}^{n}$ in formulas (13), (14), and (15) should be replaced by $\prod_{p=1}^{n} \prod_{q=p+1}^{n}$.