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Significance of the characteristic length for micromechanical modelling of ductile fracture

D.-Z. Sun, A. Hönig
Fraunhofer-Institut für Werkstoffmechanik, Wöhlerstr. 11, D-79108 Freiburg, Germany

ABSTRACT

The ductile fracture behavior of materials is considerably influenced by the characteristic microstructural length which may be related to the average inclusion spacing. However, it is not easy to introduce this critical parameter into micromechanical modelling. An additional problem met at the simulation with micromechanical models is the sensibility of numerical results to the mesh size. In this work the nonlocal damage concept developed by Bažant was introduced into the modified Gurson model. The ability of the nonlocal micromechanical concept to reduce the mesh-size dependency and the relationship between the localization limiter and the characteristic length have been studied by simulating damage development in shear bands, necking regions and at crack tips.

INTRODUCTION

The process of ductile fracture can be described by three stages: void nucleation, void growth and void coalescence. Formation of cracks or the extension of existing cracks occur by void coalescence which takes place, if the void volume fraction exceeds a critical value f_c [1]. This coalescence criterion is in agreement with that of the critical strain which depends on stress triaxiality. For description of void coalescence at a crack tip a second parameter, the characteristic length l_c, is required, as void coalescence is a discontinuous process and the length of each microscopical crack growth step determines the resistance of materials to crack extension as well [2,3]. It implies that

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it is not sufficient for crack initiation to reach a criterion at one point, but that a minimum volume of material should be involved. The characteristic length may be related to the average inclusion spacing and influences strongly the fracture behavior of materials. It is not easy to introduce the concept of the characteristic length into micromechanical modelling, because the constitutive equations are formulated only for individual material points. Another difficulty is that numerical results of micromechanical modelling are influenced by element size, especially in the case of strain localization caused by material softening. Needleman and Tvergaard [4] applied the l_c-concept for micro-mechanical modelling by specifying the size and spacing of inclusions in the analyses directly. This method requires extreme fine element sizes (not feasible in 3D) and can be only used for materials where the crack growth behavior is dominated by coalescence of voids nucleated at large inclusions. Rousselier used a simple but practical method based on the selection of a particular mesh size at the crack tip [5]. To reduce the localization effect within an element the method was modified by Sun et al. [6] by keeping the onset of coalescence in different integration points of an element at the same time. It was found that the l_c-value for this method determined by fitting the measured load vs. displacement curve of a CT specimen can be transferred to other fracture mechanics specimens [7,8]. A disadvantage of the method is that the selected element size is not directly identified with a characteristic microstructural scale having a clear physical meaning.

Progressive damage resulting in crack extension is treated as strain softening which causes mesh sensitivity and incorrect convergence when the element is refined to vanishing size. To solve these problems Bažant favored a so-called nonlocal damage theory [9]. The basic idea of this theory is that the evolution of the internal variables for damage does not only depend on the current values of state variables in this very point, but also on the values in the vicinity of this point. The parameter l_c , a localization limiter, used in a weighting function determines the size of a region, where the increases of internal variables are averaged with the weighting function. Several publications demonstrated the ability of this concept to reduce the mesh sensitivity of numerical results for simple damage models and loading situations[10,11]. However, it is not clear if the localization limiter can be identified with the characteristic microstructural scale which controls the velocity of crack extension in ductile materials. In the present work a localization limiter was introduced into the modified Gurson constitutive relations and its physical meaning for micromechanical modelling has been studied by simulating damage development in shear bands, necking regions and at crack tips.

NONLOCAL GURSON MODEL

Based on the strain-rate dependent Gurson model [4,8] a nonlocal micromechanical model was developed by averaging the increases of damage variable f^* with a weighting function. The modified plastic potential applicable to porous solids is given by

$$\Phi = \frac{3\sigma'_{ij}\,\sigma'_{ij}}{2\sigma_m^2} + 2q_1 f_v^* \cosh\left(\frac{\sigma_{kk}}{2\sigma_m}\right) - [1 + (q_1 f_v^*)^2] = 0 \,. \quad (1)$$

σ_m denotes the flow stress of the matrix material which depends on strain, strain rate and temperature. f_v^* is an averaged damage variable. From this equation the consistency condition $\Phi=0$ follows

$$\Phi_{,\sigma_{ij}}\,\dot{\sigma}_{ij} + \Phi_{,\sigma_m}\dot{\sigma}_m + \Phi_{,f_v^*}\dot{f}_v^* = 0 \,. \quad (2)$$

The rate of increase of the nonlocal internal variable can be determined by averaging the local values of f^* with the weighting function φ

$$\dot{f}_v^*(X) = \frac{1}{V_r(X)} \int_V \dot{f}^*(Y)\ \varphi(X-Y)\ dV(Y) \quad (3)$$

in which V denotes the regarded volume, **X** and **Y** are the coordinate vectors. In this work the undeformed coordinates were used to average f^*. The normalization factor $V_r(\mathbf{X})$ is

$$V_r(X) = \int_V \varphi(X-Y)\ dV(Y) \,. \quad (4)$$

According to Bažant [9] the Gaussian distribution function given by equation 5 is a suitable form for the weighting function:

$$\varphi(X) = \exp\left[-|X|^2 / l_c^2\right] \,. \quad (5)$$

Since the function φ is nearly zero for those points **Y** whose distance from point **X** exceeds $2l_c$, the length l_c determines the size of the region where damage development is averaged. This parameter should be connected with a physical parameter representing a material property. The local rate of increase of damage variable $\dot{f}^*$ used in equation 3 is given by

$$\dot{f}^* = K\,[(1-f)D^p_{kk} + a\dot{\varepsilon}] \qquad (6)$$

with K=1 for $f \leq f_c$ and $K=(f^*_u - f_c)/(f_f - f_c)$ for $f \geq f_c$. The constant $f^*_u=1/q_1$ is the value of f^* at zero stress in equation 1. In the local Gurson model f_c and f_f denote void volume fraction at the onset of coalescence and at final rupture, respectively. D^p_{kk} is the plastic part of the volumetric strain rate and $\dot{\varepsilon}$ is the rate of the equivalent plastic strain in the matrix. A numerical difficulty met at the evaluation of the averaged value of f^* (equation 3) is that the local rate of f^* of the neighboring points at the same integration time is yet unknown. To satisfy the consistency condition an iterative procedure was carried out in this work [12].

ANALYSIS OF SHEAR BANDS

To study the delocalization effect of the nonlocal Gurson model an uniaxial plane strain tensile specimen was simulated. It is well known that final fracture of this kind of specimens occurs in a shear mode induced by strain localization, while the void volume fraction outside the shear band is still small [13]. It is the question to be answered in this work if the introduced localization limiter l_c rather than element size controls the formation of shear bands. A quadrate with an edge length of 1 mm was simulated using the local and nonlocal Gurson models, respectively. The left and bottom surfaces of the rectangle were symmetry planes and the displacements of the nodes on the right surface in the horizontal direction were kept same by defining constraint equations. To trigger the localization a small geometry imperfection was generated by shifting the node in the right bottom corner about 0.0005 mm to left. The top surface of the rectangle was loaded by using prescribed displacements in the vertical direction and all cases considered were subject to plane strain conditions. In order to test the influence of the mesh size a mesh with 400 elements and a finer mesh with 1600 elements were analyzed. The parameters for the Gurson models were mainly taken from a previous study [8]. The nucleation was considered as strain-controlled with f_N=0.004, ε_N=0.3 and s_N=0.1. The parameters f_c=0.045 and f_f=0.2 determined for the steel 22NiMoCr 3 7 were also applied. To reduce the computation time f_0=0.004 instead of f_0=0 was assumed.

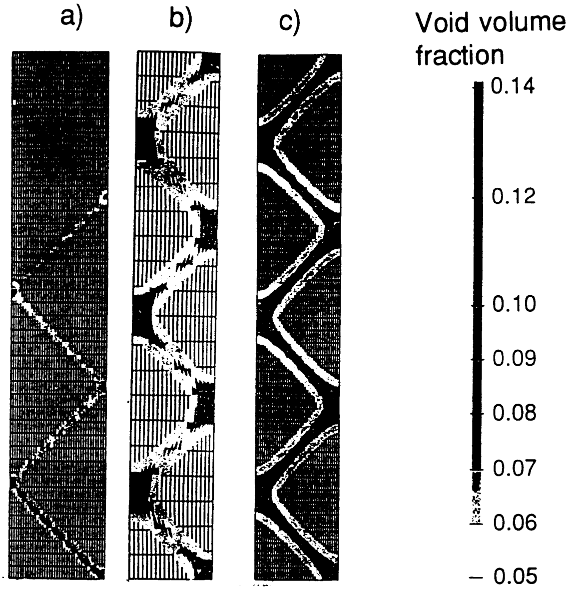

Figure 1: Void concentration in shear bands calculated with a) the local model l_o=0.025, b) the nonlocal model l_o=0.05, c) the nonlocal model l_o=0.025

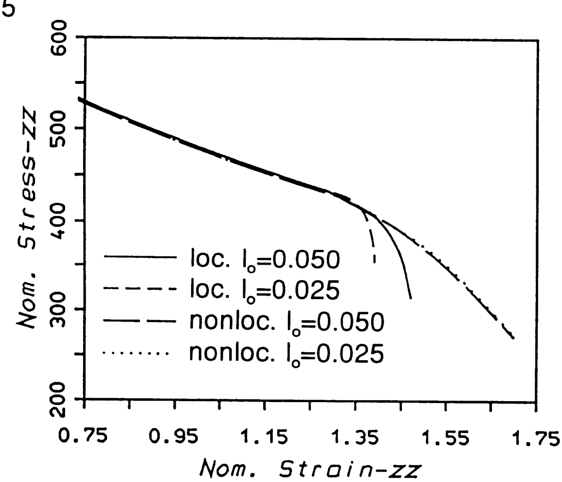

Figure 2: Nominal stress vs. strain curves of a quadrate in uniaxial plane strain tension calculated with the local and nonlocal models.

Fig. 1 compares the deformed meshes for the local and nonlocal Gurson models on which the distribution of f-value is plotted. Large strain localization can be recognized in the shear bands crossing one another at the symmetry planes. An important difference between the results of both models is that the width of the shear bands calculated

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with the nonlocal model is determined by the localization limiter (l_c=0.05 mm), not by the element size (l_o=0.05 and 0.025 mm). An additional simulation indicated that the width of shear bands was doubled when using the value of l_c =0.1 mm. Fig. 2 shows the average nominal stress σ_Z vs. strain ε_Z curves calculated with both models using different mesh sizes. The sudden drop of the average stress in the final deformation stages is caused by the rapid increase of void volume in the shear band resulting in the loss of load carrying capability of the material. Obviously, the numerical results obtained from the nonlocal Gurson model are not dependent on the mesh sizes, but those from the local model do.

SIMULATION OF FRACTURE IN NECKING REGION

A previous study with the local Gurson model [6,7] showed that the stress and strain distributions in the smooth bar are, even in the necking region, so homogeneous that the critical void volume fraction, f_c, can be evaluated without accounting for a characteristic length l_c. However, it was not analyzed how the subsequent development of damage after the onset of coalescence was influenced by the element size or the l_c-value. To check the applicability of the method for determination of the f_c-value to the nonlocal model and to study the localization effects in the final deformation stage a smooth tensile bar was simulated using both the local and nonlocal models with three different mesh sizes (l_o=0.05, 0.1 and 0.2 mm). The micromechanical parameters were identical with those applied for the shear-band simulation, except f_o=0.

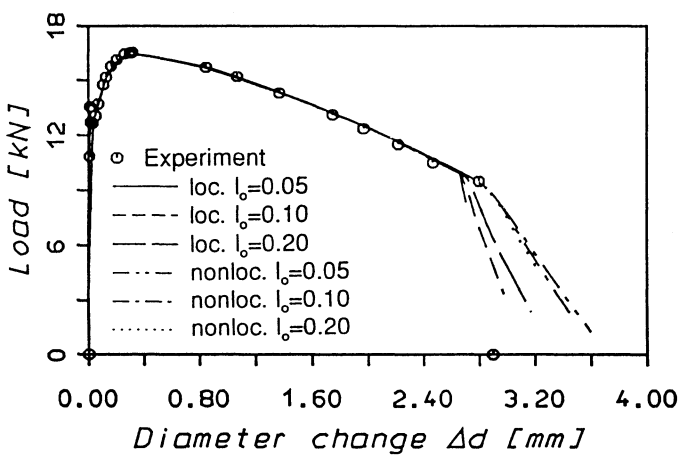

Figure 3: Measured and calculated load vs. displacement curves for a smooth bar

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Fig. 3 shows the load vs. diameter change curves from the simulations and the experiment for the smooth bar with a diameter of 6 mm. As expected, a deviation between different numerical results can be only observed in the final deformation stage, where the modelling of coalescence becomes active and the strain localization takes place. An important result is that the position for the sudden drop of the load (kink) at which f just exceeds f_c is practically independent of mesh size and therefore it can be used to determine the f_c-value. In the nonlocal model the f_c-value can be only reached at a further loading due to the average procedure and thus the sudden drop of the load is delayed. The slope of the load vs. displacement curve after the kink point is determined by the velocity of the crack extension initiated at the specimen center. Obviously, the slope calculated by the local model increases with refining element size, while that calculated by the nonlocal model (l_c=0.1 mm) does not depend on element size. This is a great advantage of nonlocal micromechanical models.

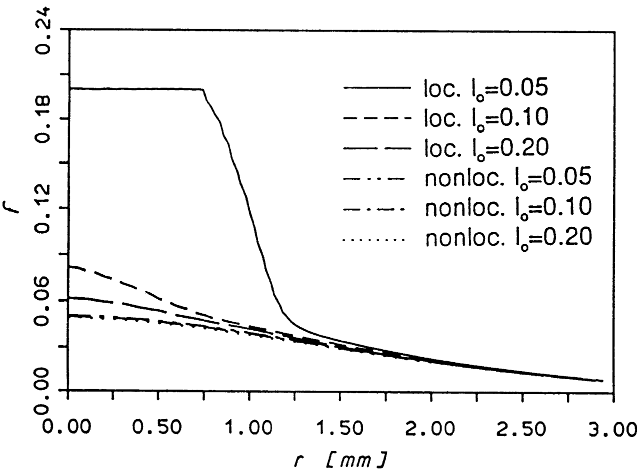

Figure 4: Distributions of void volume fraction at the cross-section of a smooth bar calculated with different models and element sizes

Fig. 4 shows for the same loading (about the kink point) the calculated distributions of void volume fraction at the cross-section of the smooth bar. It is quite clear that using the local Gurson model the damage variable, f, in the specimen center calculated with a fine mesh is much higher than that with a coarse mesh. The nonlocal model delivers for three different element sizes the same distribution of the f-value which is nearly identical with f^* at this load level. Fig. 3 indicates that the parameters obtained for the local model are not entirely applicable to the nonlocal model, because in comparison with

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the experimental data the kink point calculated with the parameters comes somewhat later and the final slope is too small. An additional analysis showed that a change of the f_f-value from 0.2 to 0.1 for the nonlocal model could result in a good agreement of the numerical prediction with the experimental founding. The following simulation of the CT specimen was carried out using the modified parameters.

VERIFICATION WITH A CRACKED SPECIMEN

A characteristic length is required for micromechanical modelling only when a large gradient exists in the stress and strain field. In order to study, whether the localization limiter can be used as a characteristic length for controlling the crack extension, a CT specimen with 20% side grooves was simulated with the nonlocal Gurson model. Quadratic elements with an edge size of 0.05 mm were generated in the near tip region and two different l_c-values (0.05 and 0.1 mm) were used for the same mesh. Fig. 5 compares the crack extension Δa vs. the displacement curves calculated with the different l_c-values where the experimental data are also plotted. It is obvious that the crack initiation occurs later if a larger l_c-value is used. However, it has to be noted that the size of the first crack extension calculated with a greater l_c-value is much larger than that from the calculation with a smaller l_c-value. An interesting effect shown in Fig. 5 is that the slope of the crack extension vs. displacement curve representing the velocity of

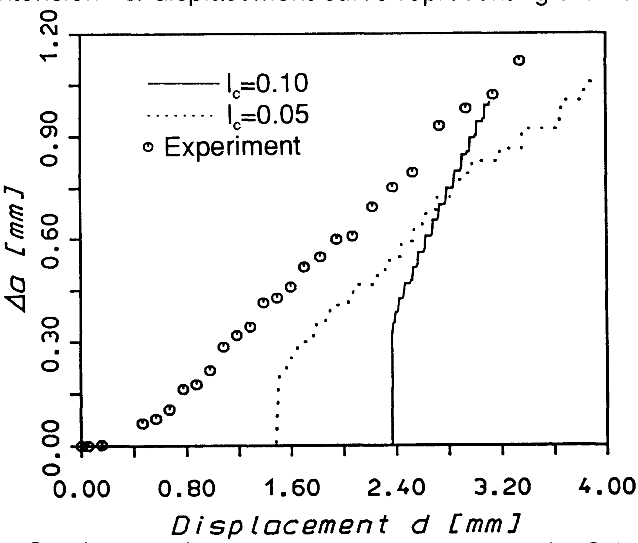

Figure 5: Crack extension vs. displacement curves of a CT specimen calculated using the nonlocal model with two different l_c-values in comparison with measured data

crack extension increases with increasing the l_c-value. The slope of the curve for l_c=0.10 mm is not as expected flatter, but steeper than that for l_c=0.05 mm.

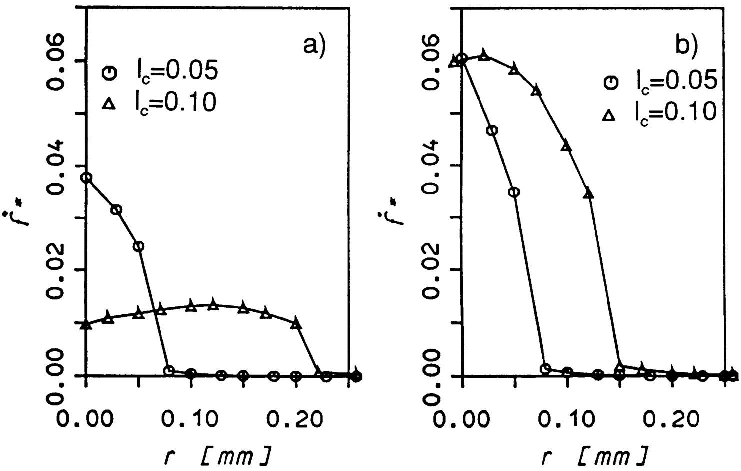

Figure 6: Calculated local increase rate of the damage variable f at the actual crack tip for two load levels a) d=2.23 mm, b) d=2.46 mm

To understand this effect the distributions of the local increase rate of f^* at the actual crack tip are compared in Fig. 6 for different l_c-values. The curves in Fig. 6a are calculated for the loading level d=2.23 mm at which the simulation with l_c=0.05 mm shows a crack extension of 0.45 mm while in the simulation with l_c=0.1 mm the crack initiation does not occur. Until this loading the local increase rate of f^* ahead of the crack tip is much higher for l_c=0.05 than for l_c=0.1, thus the crack extension takes place at first in the simulation with the smaller l_c-value. The curves shown in Fig. 6b are computed for the loading level d=2.46 mm where the calculated crack extension is 0.58 mm and 0.30 mm for l_c=0.05 and l_c=0.1, respectively. It is distinct that after the crack initiation the maximum of the local rate of f^* calculated with l_c=0.1 is nearly equal to that with l_c=0.05, but the size of the region with a great increase rate of damage for l_c=0.1 is twice as large as that for l_c=0.05. Consequently, in the following deformation stage the crack grows faster in the simulation with the larger l_c-value.

CONCLUSIONS

The nonlocal damage concept developed by Bažant has been introduced into the modified Gurson model by averaging the internal variable f^*. The analysis of the deformation and fracture behavior in uniaxial plane strain and axisymmetric tension indicates that the mesh-size dependency of numerical results can be eliminated by using the nonlocal micromechanical model. An interesting point found by simulating a cracked specimen is that an increase of the localization limiter l_c postpones the crack initiation, but accelerates the damage development after the initiation. Further experimental and numerical work is necessary to correlate the value of the localization limiter with the characteristic microstructural length which determines the resistance of materials to crack extension.

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