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Similarity and Compatibility in Fuzzy Set Theory

Assessment and Applications

With 27 Figures
and 46 Tables

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