CHAPTER 142

SIMILARITY CONDITIONS FOR THERMAL-HYDRAULIC MODEL TESTS OF TIDAL ESTUARIES

by

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INTRODUCTION

In connection with the design of proposed nuclear power plants on the Lower Weser River and on the Lower Elbe River in Germany, tidal models were constructed and tests carried out at the Franzius-Institute of the Technical University of Hannover for the purpose of studying the mixing and spreading of the hot water discharge. Thermal-hydraulic models are today still the most reliable method for the prediction of temperature distributions in the total area of temperature increase, especially in tidal regions.

Hydraulic models with free surface flow are in general based on the FROUDE scaling law. The flow fields in both the hydraulic model and in the prototype show a dynamic similitude when the FROUDE numbers of the flow in the model and in the nature are equal at corresponding points, i.e.:

$$\mathbb{F}_M = \mathbb{F}_N$$

or: $$\frac{v_M}{\sqrt{g \cdot h_M}} = \frac{v_N}{\sqrt{g \cdot h_N}}$$

In case differences of density exist in zones, in which there is not yet a complete mixing of the heated cooling water with the ambient river water, one must apply the densimetric FROUDE scaling law:

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$$F_{dM} = F_{dN}$$

or:
$$\frac{v_M}{\sqrt{g \cdot h_M \cdot \left(\frac{\Delta\rho}{\rho}\right)_M}} = \frac{v_N}{\sqrt{g \cdot h_N \cdot \left(\frac{\Delta\rho}{\rho}\right)_N}}$$

with v = mean velocity

h = mean water depth

g = acceleration of gravity

$\frac{\Delta\rho}{\rho}$ = relative density difference between the cooling water and the ambient river water

In thermal-hydraulic models the same fluid (water) as in nature will be normally used. The physical values of the density ρ and of the specific heat c, as well as temperature differences ΔT, too, must therefore be equal in the model and in the nature, i.e.

$$\frac{\rho_N}{\rho_M} = 1 \; ; \; \frac{c_N}{c_M} = 1 \; ; \; \frac{\Delta T_N}{\Delta T_M} = 1 \; .$$

By applying the FROUDE scaling law, friction forces cannot be considered, so that the REYNOLDS number depends on the depth scale of the model as follows:

$$\frac{\mathbb{R}_N}{\mathbb{R}_M} = \frac{\frac{v_N \cdot h_N}{\nu_N}}{\frac{v_M \cdot h_M}{\nu_M}} = \lambda_V^{1/2} \cdot \lambda_V = \lambda_V^{3/2}$$

with λ_V = vertical scale of the water depths,

$\lambda_V^{1/2}$ = scale of the velocities

$\nu_N : \nu_M \doteq 1$

Turbulent flow under prototype conditions can only be reproduced in the model, when the vertical model scale is sufficiently large to guarantee turbulent flow in the model too. The choice of the depth scale must therefore depend on a selected turbulence level.

The length scale for a model of the tidal area of an estuary as for example the Lower Weser River or the Lower Elbe River, depends on the availability of space for the model in the laboratory.

From these two conditions follows, that the length scale must be greater than the depth scale; i.e.

$$\lambda_H > \lambda_V$$

This means that a tidal model is in general distorted:

$$\kappa = \frac{\lambda_H}{\lambda_V} .$$

However, in as much as the spreading components of a model should not be distorted, the horizontal dissipation of heat in a distorted model is exaggerated with respect to the vertical dissipation. For this reason, only a limited scale distortion may be chosen for thermal-hydraulic models.

As a consequence of the comments made above, a distortion of $\kappa = 3$ as a maximum was chosen for the thermal-hydraulic tidal model studies in the Franzius-Institute.

But there is another problem in handling a thermal-hydraulic model:

Temperature distributions not only depend on the degree of turbulence in the flow, they depend in a high degree on the heat exchange between the water surface and the adjacent atmosphere. This heat exchange at the water surface does not depend on the hydrodynamic conditions of the river flow or of the cooling water discharge. It will be rather influenced by the increase of temperature at the water surface compared with the equilibrium temperature of the river and by the meteorological conditions, such as evaporation, conduction and solar radiation. The heat exchange is characterized by the heat exchange coefficient.

The phenomenon of heat exchange at the water surface exists as well as in nature as in a thermal-hydraulic model.

Therefore special attention must be paid to the similitude of the heat exchange in the model.

The physical processes concerning the heat exchange at the water surface are the same in the prototype and in the model. It is therefore not possible to determine a scale for the heat exchange coefficient on the base of the FROUDE scaling law.

Beyond that, it is too expensive to reproduce in the laboratory the different meteorological conditions, such as relative humidity, atmospheric pressure, temperature, wind speed and wind direction.

Therefore, one must find a special relationship between the temperature distribution in the model, depending on the meteorological conditions of the laboratory and that climate in the nature, for which temperature distributions in the nature would be the same as measured in the model. By comparison of the scales for the heat input into the model and for the heat, transferred at the water surface into the atmosphere of the laboratory, one can find a factor for the heat exchange coefficient.

SCALE FOR THE HEAT INPUT

The waste heat of a power plant is injected with the cooling water discharge. The heat input is

$$H_i = Q \cdot \Delta T_C \cdot c \tag{1}$$

with Q = cooling water discharge

ΔT_C = temperature increase at the condenser

c = specific heat of water

With the FROUDE scaling law, the scale of the heat input is

$$\frac{H_{i_N}}{H_{i_M}} = \frac{Q_N}{Q_M} \cdot \frac{T_{CN}}{T_{cM}} \cdot \frac{c_N}{c_M} = \frac{Q_N}{Q_M} \cdot 1 \cdot 1 \tag{2}$$

With the scale for the horizontal lengths

$$\frac{\ell_{HN}}{\ell_{HM}} = \lambda_H$$

and for the vertical lengths

$$\frac{\ell_{VN}}{\ell_{VM}} = \lambda_V$$

the scale for the heat input becomes

$$\frac{H_{i_N}}{H_{i_M}} = \lambda_H \cdot \lambda_V^{3/2} \quad \text{(=scale for the discharge)} \qquad (2a)$$

for a distorted model

and $$\frac{H_{i_N}}{H_{i_M}} = \lambda^{5/2}$$

for an undistorted model with $\lambda_H = \lambda_V = \lambda$.

SCALE FOR THE HEAT EXCHANGE

It is supposed, that the heat input into the river flow is essentially given off by a heat exchange between the water surface and the adjacent atmosphere. The heat transfer at the bottom of the model is negligable.

The heat exchange at the water surface is given by:

$$H_e = K \cdot A \cdot \Delta T \qquad (3)$$

with K = heat exchange coefficient
A = water surface, available for the heat exchange
ΔT = temperature increase over the equilibrium temperature.

The scale of the heat exchange is ($K_N = K_M$)

$$\frac{H_{e_N}}{H_{e_M}} = \frac{K_N}{K_M} \cdot \frac{A_N}{A_M} \cdot \frac{T_N}{T_M} = \frac{A_N}{A_M} \cdot 1 \cdot 1 \qquad (4)$$

and with

$$\frac{\ell_{HN}}{\ell_{HM}} = \lambda_H$$

$$\frac{H_{e_N}}{H_{e_M}} = \lambda_H^{\,2} \qquad \text{(=scale for the water surface)} \qquad (4a)$$

for a distorted and for an undistorted model.

REDUCTION FACTOR

The scale of the heat input (i.e. discharge) into a model is given by $\lambda^{5/2}$ and the scale of the heat exchange (i.e. the water surface) is given by λ^2.

This means that in thermal-hydraulic models the heat transferred at the water surface is less reduced than the heat input. The heat exchange depends linearly on the available water surface A and on the heat exchange coefficient K (equ. 3). The ratio of the scales for the heat input and for the heat exchange can be used in order to determine that heat exchange coefficient, which corresponds to the temperature distributions in prototype, which were obtained from the model on the basis of the FROUDE scaling law.

The reduction factor is given by:

$$\frac{\dfrac{H_{i_N}}{H_{i_M}}}{\dfrac{H_{e_N}}{H_{e_M}}} = \frac{\lambda_H \cdot \lambda_V^{\,3/2}}{\lambda_H^{\,2}} = \frac{\lambda_V^{\,3/2}}{\lambda_H} = \frac{K_N}{K_M} \qquad (5)$$

in a distorted model, or

$$\frac{\dfrac{H_{i_N}}{H_{i_M}}}{\dfrac{H_{e_N}}{H_{e_M}}} = \frac{\lambda^{5/2}}{\lambda^{2}} = \lambda^{1/2} = \frac{K_N}{K_M} \qquad (5)$$

in an undistorted model. It depends on the geometrical scales of the model, as shown in figure 1.

HEAT EXCHANGE COEFFICIENT IN THE LABORATORY

For the critical judgement of temperature distributions obtained in thermal-hydraulic models, the heat exchange coefficient of the laboratory must be taken into account. Therefore, in the model the heat exchange between the water surfacc and the adjacent atmosphere is determined by means of systematic measurements in a special isolated basin with a free surface, which should be located directly beside the model.

In these tests the time-dependent cooling of the artificially heated water volume with a wellknown water surface and water depth of the basin leads to the calculation of a heat exchange coefficient (Fig. 2):

$$K = \frac{H_e}{A \cdot \Delta T_o}$$

with K = heat exchange coefficient in $W/m^2 \cdot {}^oC$

H_e = heat exchange at the water surface

$= \dfrac{V \cdot \Delta T \cdot c}{\Delta t} \cdot 4{,}18 \cdot 10^6$ in W

c = specific heat of water

$= 1\ Mcal/m^3 \cdot {}^oC$

V = water volumc of the basin in m^3

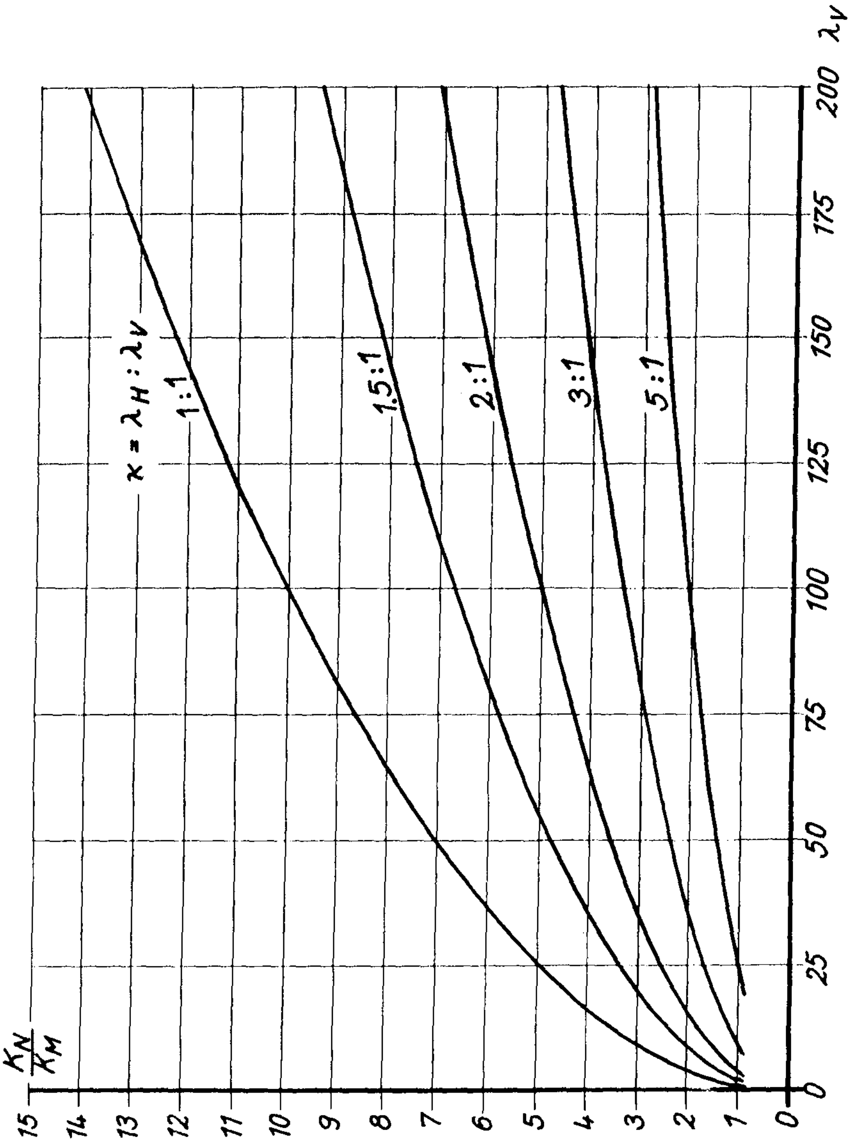

Fig. 1 Vertical scale λ_V versus reduction factor $K_N : K_M$ for different distortions κ

ΔT	=	decrease of the water temperature in the time interval Δt in ^{o}C
Δt	=	time interval in sec
$4{,}18 \cdot 10^{6} W$	=	1 Mcal/sec
A	=	water surface of the basin in m^2
ΔT_o	=	$T - T_E$ in ^{o}C
T	=	mean water temperature in the time interval Δt in ^{o}C
T_E	=	equilibrium temperature of the water in the basin in ^{o}C

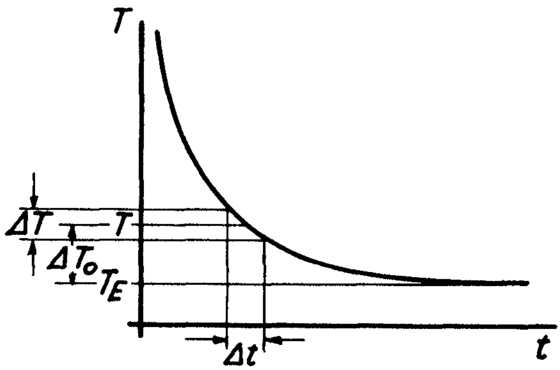

Fig. 2 Time dependent cooling in the basin

For example, for laboratory conditions during the tests, heat exchange coefficients K_M = 10 to 15 $W/m^2 . ^oC$ (42 to 63 BTU/s . $ft^2 . ^oF$) were found. In nature, however, it is much more difficult to determine the heat exchange coefficient K_N. In the German estuarine areas one assumes heat exchange coefficients of K = 50 to 100 $W/m^2 . ^oC$ for the summer months (210 to 420 BTU/s) and K = 20 to 50 $W/m^2 . ^oC$ (84 to 210 BTU/s . $ft^2 . ^oF$) for the winter period.

In the Franzius-Institute in Hannover, systematic field measurement programs in the Lower Weser River and in the Elbe River near Hamburg are planned in order to obtain better knowledge of the heat exchange coefficients and of the spreading of the heated water discharges in the special areas of the German estuaries.

CONCLUSIONS

With the present state of knowledge one can carry out thermal-hydraulic model tests in such a way, that one can expect reliable results about temperature distributions in the total area of temperature increase by cooling water injections. The choice of suitable scales for the model and especially the consideration of the heat exchange at the water surface guarantee the similarity of model and prototype flow conditions in thermal-hydraulic model studies.