

## Research Article

# Similarity Solution and Heat Transfer Characteristics for a Class of Nonlinear Convection-Diffusion Equation with Initial Value Conditions

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A class of nonlinear convection-diffusion equation is studied in this paper. The partial differential equation is converted into nonlinear ordinary differential equation by introducing a similarity transformation. The asymptotic analytical solutions are obtained by using double-parameter transformation perturbation expansion method (DPTPEM). The influences of convection functional coefficient  $k(z)$  and power law index  $n$  on the heat transport characteristics are discussed and shown graphically. The comparison with the numerical results is presented and it is found to be in excellent agreement. The method and technique used in this paper have the significance in studying other engineering problems.

## 1. Introduction

Convection-diffusion equations arose from various fields of applied sciences such as heat transfer problems in a draining film [1] or a nanofluid filled enclosure [2], radial transport in a porous medium [3], and water transport in soils [4] and have received extensive attentions during the past several decades. For convection-diffusion equations, only few cases with special initial or boundary value conditions have analytical solutions. Therefore, most of the main concerns were the study of the qualitative properties of the solutions [5–13] and numerical study [14–23] for convection-diffusion equations. However, the very important approximate solution of convection-diffusion equation has not been well solved. In this paper we present similarity solutions for a class of convection-diffusion equation, which are then solved by DPTPEM. The method was first proposed by Yan Zhang et al. [24, 25]. In this paper, we can find that the DPTPEM can be successfully used to solve the studied problem, and the approximate solutions obtained by DPTPEM agree very well the numerical solutions obtained by bvp4c with Matlab [26].

## 2. Mathematical Formulation

In this paper, we consider a class of nonlinear convection-diffusion equation as follows [20]:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( u^n \left| \frac{\partial u}{\partial x} \right|^{n-1} \frac{\partial u}{\partial x} \right) + t^{-n/(n+1)} k(u) \frac{\partial u}{\partial x}, \quad (1)$$

$(x, t) \in G,$

subject to the following initial value conditions:

$$\begin{aligned} u(0, t) &= 0, \quad t > 0; \\ u(x, 0) &= 1, \quad x > 0. \end{aligned} \quad (2)$$

where  $G = \{(x, t); x > 0, t > 0\}$ ,  $n$  is positive power law exponent. Convection functional coefficient  $k(z)$  is assumed to be a real-valued continuous differential function defined on  $[0, 1]$ ,  $k(0) = 0$ ,  $k(z) > 0$  for  $z > 0$ .  $D(u) = -u^n |u_x|^{n-1} u_x$  is the heat density per unit area.

### 3. Converting into Nonlinear Ordinary Differential Equation Boundary Value Problems

Introduce the following similarity variables [20]:

$$\begin{aligned} u(x, t) &= u(\eta), \\ \eta &= xt^{-1/(n+1)}. \end{aligned} \quad (3)$$

The transformed convection-diffusion equation together with the initial value conditions given by (1)-(2) can be written as

$$\begin{aligned} & \left( (u(\eta))^n |u'(\eta)|^{n-1} u'(\eta) \right)' \\ & + \left( \frac{\eta}{n+1} + k(u(\eta)) \right) u'(\eta) = 0 \end{aligned} \quad (4)$$

$$\begin{aligned} u(0) &= 0, \\ u(+\infty) &= 1. \end{aligned} \quad (5)$$

Conversely, if  $u(\eta)$  is a solution of (4)-(5), then  $u(x, t)$  must be a similarity solution to (1)-(2).

Let  $u = u(\eta)$  be a solution to (4)-(5). If  $u(\eta)$  is strictly increasing in  $[0, +\infty)$ , then the function  $\eta = \eta(u)$  inverse to  $u = u(\eta)$  exists. Here  $\eta'(u) = (u'(\eta))^{-1}$  holds on  $(0, 1)$ . Then (4) can be changed into the following nonlinear ordinary differential equation:

$$\left( u^n (\eta'(u))^{-n} \right)' = - \left( \frac{1}{n+1} \eta(u) + k(u) \right), \quad (6)$$

and integration from  $u$  to 1 yields

$$\eta'(u) = u \left( \int_u^1 \left( \frac{1}{n+1} \eta(s) + k(s) \right) ds \right)^{-1/n}. \quad (7)$$

Set

$$h(z) = \int_z^1 \left( \frac{1}{n+1} \eta(s) + k(s) \right) ds. \quad (8)$$

Combine (7) and (8) to get

$$h''(z) = -\frac{1}{n+1} zh^{-1/n}(z) - k'(z). \quad (9)$$

The corresponding boundary conditions are

$$\begin{aligned} h'(0) &= 0, \\ h(1) &= 0. \end{aligned} \quad (10)$$

where  $h(z)$  is heat diffusion flux. The derivation process indicates that only the positive solutions of (9)-(10) are physically significant.

### 4. A Brief Introduction to DPTPEM

The method was first proposed by Yan Zhang and Liancun Zheng [24, 25].

We consider a class of ordinary differential equation initial or boundary value problem without small parameter  $\varepsilon$  as follows:

$$P: \begin{cases} L[u(x)] = f(x), & x = (x_1, x_2, \dots, x_m) \in \Omega, \\ u^{(j)}(x) = a_j, & j = 0, 1, \dots, n-1, x \in (\partial\Omega \cap \Omega). \end{cases} \quad (11)$$

where  $L$  is  $n$  order differential operator without small parameter  $\varepsilon$ ,  $a_j$  is a constant, and  $\Omega$  is the region containing the origin.

Introduce an embedded parameter transformation as follows:

$$u(x) = \varepsilon^a g(\xi) + \sum_{k=0}^{n-1} \frac{u^{(k)}(0)}{k!} x^k, \quad x = \varepsilon^b \xi, \quad (12)$$

where  $\varepsilon$  is an artificial small parameter,  $\xi$  is independent variable parameter,  $g$  is dependent variable parameter, and  $a$  and  $b$  are undetermined constants. Substituting this transformation (12) into (11), we can obtain the following nonlinear initial value problem:

$$P_0: \begin{cases} L_\varepsilon[g(\varepsilon, \xi)] = h(\xi), & \xi = (\xi_1, \xi_2, \dots, \xi_m) \in \Omega_1, \\ g^{(j)}(0) = 0, & j = 0, 1, \dots, n-1. \end{cases} \quad (13)$$

where  $h(\xi)$  is a function of  $\xi$ ,  $\Omega_1$  is the region containing the origin. The solution of  $P_0$  is expressed in form of power series by applying perturbation expansion method

$$g(\xi, \varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k g_k \xi^k. \quad (14)$$

Combine (12) and (14) to get the solution  $u$  of (11) in terms of power series. The undetermined parameters are determined according to the boundary conditions of the original problem.

The basic idea of the DPTPEM consists of three steps. Firstly, introducing an artificial small parameter  $\varepsilon$ , the independent variable  $x$  and the unknown function  $u(x)$  are transformed simultaneously, and the problem is transformed into a new one related to small parameter  $\varepsilon$ . Secondly, the transformed new differential equation is expanded in the form of power series of  $\varepsilon$  and decomposed into the sum of several solution components. Then, by using known initial or boundary value conditions, we try to find out the solution components of each order separately and combine them to get the solution  $g(\xi)$  of the new problem. Thirdly, by substituting the solution of the new equation into the transformation equation (12), the small parameter  $\varepsilon$  is eliminated and the solution of the original problem is obtained.

### 5. Approximate Analytical Results of (9)-(10) and Discussion

The initial condition is assumed as follows:

$$h(0) = \alpha. \quad (15)$$

In order to solve (9), we transform the dependent variable and independent variable as follows according to formula (12):

$$\begin{aligned} h(z) &= \varepsilon^2 g(\xi) + \alpha, \\ z &= \varepsilon \xi, \end{aligned} \quad (16)$$

where  $\varepsilon$  is an artificial small parameter.

Let  $k(z) = z$  and substituting this transformation (16) into (9)-(10), we can obtain the following nonlinear initial value problems:

$$g''(\xi) + \frac{1}{n+1} \varepsilon \xi [\varepsilon^2 g(\xi) + \alpha]^{-1/n} + 1 = 0, \quad (17)$$

$$\begin{aligned} g(0) &= 0, \\ g'(0) &= 0. \end{aligned} \quad (18)$$

Expanding  $[\varepsilon^2 g(\xi) + \alpha]^{-1/n}$  in a power series development, we can get

$$\begin{aligned} g''(\xi) &+ \frac{1}{n+1} \varepsilon \xi \left[ \frac{1}{\alpha^{1/n}} - \frac{1}{n\alpha^{(1/n)+1}} \varepsilon^2 g(\xi) \right. \\ &+ \frac{n+1}{2n^2 \alpha^{1/n+2}} \varepsilon^4 g^2(\xi) - \frac{(n+1)(2n+1)}{6n^3 \alpha^{1/n+3}} \varepsilon^6 g^3(\xi) \\ &\left. + \frac{(n+1)(2n+1)(3n+1)}{24n^4 \alpha^{1/n+4}} \varepsilon^8 g^4(\xi) + \dots \right] + 1 = 0 \end{aligned} \quad (19)$$

The solution of (19) can be obtained by expanding  $g(\xi)$  in a power series development near  $\varepsilon = 0$  as follows:

$$\begin{aligned} g(\xi) &= g_0(\xi) + \varepsilon g_1(\xi) + \varepsilon^2 g_2(\xi) + \varepsilon^3 g_3(\xi) \\ &+ \varepsilon^4 g_4(\xi) + \varepsilon^5 g_5(\xi) + \varepsilon^6 g_6(\xi) + \varepsilon^7 g_7(\xi) \\ &+ \varepsilon^8 g_8(\xi) + \varepsilon^9 g_9(\xi) + \varepsilon^{10} g_{10}(\xi) + \dots \end{aligned} \quad (20)$$

Substituting (20) into (19) and equating the coefficients of  $\varepsilon^i$ , we can get the following expressions:

$$\begin{aligned} \varepsilon^0 : g_0''(\xi) + 1 &= 0, \\ \varepsilon^1 : g_1''(\xi) + \frac{\xi}{(n+1)\alpha^{1/n}} &= 0, \\ \varepsilon^2 : g_2''(\xi) &= 0, \\ \varepsilon^3 : g_3''(\xi) - \frac{\xi g_0(\xi)}{n(n+1)\alpha^{1/n+1}} &= 0, \\ \varepsilon^4 : g_4''(\xi) - \frac{\xi g_1(\xi)}{n(n+1)\alpha^{1/n+1}} &= 0, \\ \varepsilon^5 : g_5''(\xi) - \frac{\xi g_2(\xi)}{n(n+1)\alpha^{1/n+1}} + \frac{\xi g_0^2(\xi)}{2n^2 \alpha^{1/n+2}} &= 0, \\ \varepsilon^6 : g_6''(\xi) - \frac{\xi g_3(\xi)}{n(n+1)\alpha^{1/n+1}} + \frac{\xi g_0(\xi) g_1(\xi)}{n^2 \alpha^{1/n+2}} &= 0, \end{aligned}$$

$$\begin{aligned} \varepsilon^7 : g_7''(\xi) - \frac{\xi g_4(\xi)}{n(n+1)\alpha^{1/n+1}} + \frac{\xi [2g_0(\xi) g_2(\xi) + g_1^2(\xi)]}{2n^2 \alpha^{1/n+2}} \\ - \frac{(2n+1)\xi g_0^3(\xi)}{6n^3 \alpha^{1/n+3}} &= 0, \\ \varepsilon^8 : g_8''(\xi) - \frac{\xi g_5(\xi)}{n(n+1)\alpha^{1/n+1}} \\ + \frac{\xi [g_0(\xi) g_3(\xi) + g_1(\xi) g_2(\xi)]}{n^2 \alpha^{1/n+2}} \\ - \frac{(2n+1)\xi g_0^2(\xi) g_1(\xi)}{2n^3 \alpha^{1/n+3}} &= 0, \\ \varepsilon^9 : g_9''(\xi) - \frac{\xi g_6(\xi)}{n(n+1)\alpha^{1/n+1}} \\ + \frac{\xi [2g_0(\xi) g_4(\xi) + 2g_1(\xi) g_3(\xi) + g_2^2(\xi)]}{2n^2 \alpha^{1/n+2}} \\ - \frac{(2n+1)\xi [g_0^2(\xi) g_2(\xi) + g_0(\xi) g_1^2(\xi)]}{2n^3 \alpha^{1/n+3}} \\ + \frac{(2n+1)(3n+1)\xi g_0^4(\xi)}{24n^4 \alpha^{1/n+4}} &= 0, \\ \varepsilon^{10} : g_{10}''(\xi) - \frac{\xi g_7(\xi)}{n(n+1)\alpha^{1/n+1}} \\ + \frac{\xi [g_0(\xi) g_5(\xi) + g_1(\xi) g_4(\xi) + g_2(\xi) g_3(\xi)]}{n^2 \alpha^{1/n+2}} \\ - \frac{(2n+1)\xi [3g_0^2(\xi) g_3(\xi) + 6g_0(\xi) g_1(\xi) g_2(\xi) + g_1^3(\xi)]}{6n^3 \alpha^{1/n+3}} \\ + \frac{(2n+1)(3n+1)\xi g_0^3(\xi) g_1(\xi)}{6n^4 \alpha^{1/n+4}} &= 0, \\ \dots \dots \dots \end{aligned} \quad (21)$$

Then, we can obtain  $g_i(\xi)$  ( $i = 0, 1, 2, \dots$ ) in the following.

$$\begin{aligned} g_0(\xi) &= -\frac{1}{2} \xi^2, \\ g_1(\xi) &= -\frac{1}{6(n+1)\alpha^{1/n}} \xi^3, \\ g_2(\xi) &= 0, \\ g_3(\xi) &= -\frac{1}{40n(n+1)\alpha^{1/n+1}} \xi^5, \\ g_4(\xi) &= -\frac{1}{180n(n+1)^2 \alpha^{2/n+1}} \xi^6, \\ g_5(\xi) &= -\frac{1}{336n^2 \alpha^{1/n+2}} \xi^7, \\ g_6(\xi) &= -\frac{10n+13}{6720n^2(n+1)^2 \alpha^{2/n+2}} \xi^8, \\ g_7(\xi) &= -\left[ \frac{5n+7}{25920n^2(n+1)^3 \alpha^{3/n+2}} \right] \xi^9 \end{aligned}$$

$$\begin{aligned}
& + \frac{2n+1}{3456n^3\alpha^{1/n+3}} \Big] \xi^9, \\
g_8(\xi) &= -\frac{70n+61}{151200(n+1)n^3\alpha^{2/n+3}} \xi^{10}, \\
g_9(\xi) &= -\left[ \frac{280n^2+590n+319}{2217600n^3(n+1)^3\alpha^{3/n+3}} \right. \\
& \left. + \frac{6n^2+5n+1}{42240n^4\alpha^{1/n+4}} \right] \xi^{11}, \\
g_{10}(\xi) &= -\left[ \frac{15120n^2+18636n+6078}{95800320(n+1)n^4\alpha^{2/n+4}} \right. \\
& \left. + \frac{1200n^2+2670n+1530}{102643200n^3(n+1)^4\alpha^{4/n+3}} \right] \xi^{12}, \\
& \dots\dots\dots
\end{aligned} \tag{22}$$

Combine (16), (20), and the above  $g_i(\xi)$  ( $i = 0, 1, 2, \dots$ ) to get solution  $h(z)$  of (9)-(10).

$$\begin{aligned}
h &= \alpha - \frac{1}{2}z^2 - \frac{1}{6(n+1)\alpha^{1/n}}z^3 - \frac{1}{40n(n+1)\alpha^{1/n+1}} \\
& \cdot z^5 - \frac{1}{180n(n+1)^2\alpha^{2/n+1}}z^6 - \frac{1}{336n^2\alpha^{1/n+2}}z^7 \\
& - \frac{10n+13}{6720n^2(n+1)^2\alpha^{2/n+2}}z^8 \\
& - \left[ \frac{5n+7}{25920n^2(n+1)^3\alpha^{3/n+2}} + \frac{2n+1}{3456n^3\alpha^{1/n+3}} \right] z^9 \\
& - \frac{70n+61}{151200(n+1)n^3\alpha^{2/n+3}}z^{10} \\
& - \left[ \frac{280n^2+590n+319}{2217600n^3(n+1)^3\alpha^{3/n+3}} + \frac{6n^2+5n+1}{42240n^4\alpha^{1/n+4}} \right] \\
& \cdot z^{11} - \left[ \frac{15120n^2+18636n+6078}{95800320(n+1)n^4\alpha^{2/n+4}} \right. \\
& \left. + \frac{1200n^2+2670n+1530}{102643200n^3(n+1)^4\alpha^{4/n+3}} \right] z^{12} + \dots
\end{aligned} \tag{23}$$

Similarly, let  $k(z) = z^2$ ; we obtain approximate analytical solution of (9)-(10) as follows:

$$\begin{aligned}
h &= \alpha - \left[ \frac{1}{3} + \frac{1}{6(n+1)\alpha^{1/n}} \right] z^3 \\
& - \left[ \frac{1}{90n(n+1)\alpha^{1/n+1}} + \frac{1}{180n(n+1)^2\alpha^{2/n+1}} \right] z^6
\end{aligned}$$

TABLE 1: Values of  $\alpha$  for  $n = 0.2, 0.6, 1.0, 2.5, 5.0, 10.0$ , and  $k(z) = z$ .

$n$	$k(z)$	$\alpha$
0.2	$z$	1.013
0.6	$z$	0.7815
1.0	$z$	0.6792
2.5	$z$	0.5701
5.0	$z$	0.5342
10.0	$z$	0.5169

$$\begin{aligned}
& - \left[ \frac{1}{1296n^2\alpha^{1/n+2}} + \frac{5n+6}{6480n^2(n+1)^2\alpha^{2/n+2}} \right. \\
& \left. + \frac{5n+7}{25920n^2(n+1)^3\alpha^{3/n+2}} \right] z^9 - \left[ \frac{2n+1}{21384n^3\alpha^{1/n+3}} \right. \\
& \left. + \frac{120n+89}{855360(n+1)n^3\alpha^{2/n+3}} \right. \\
& \left. + \frac{60n^2+119n+60}{855360n^3(n+1)^3\alpha^{3/n+3}} \right. \\
& \left. + \frac{40n^2+89n+51}{3421440n^3(n+1)^4\alpha^{4/n+3}} \right] z^{12} + \dots
\end{aligned} \tag{24}$$

Let  $k(z) = z^3$ ; approximate analytical solution of (9)-(10) is presented as follows:

$$\begin{aligned}
h &= \alpha - \frac{1}{6(n+1)\alpha^{1/n}}z^3 - \frac{1}{4}z^4 \\
& - \frac{1}{180n(n+1)^2\alpha^{2/n+1}}z^6 \\
& - \frac{1}{168n(n+1)\alpha^{1/n+1}}z^7 \\
& - \frac{5n+7}{25920n^2(n+1)^3\alpha^{3/n+2}}z^9 \\
& - \frac{7n+8}{15120n^2(n+1)^2\alpha^{2/n+2}}z^{10} - \frac{1}{3520n^2\alpha^{1/n+2}}z^{11} \\
& - \frac{40n^2+89n+51}{3421440n^3(n+1)^4\alpha^{4/n+3}}z^{12} + \dots
\end{aligned} \tag{25}$$

It is obvious that we can promptly obtain the value of  $\alpha$  by applying  $h(1) = 0$  in (23)-(25) for each fixed  $n$ . Based on (23)-(25) and the corresponding  $\alpha$  we can easily obtain the graph of the heat diffusion flux distribution for different  $n$  and  $k(z)$ . The results are presented in Tables 1-5 and Figures 1-5.

It can be seen from Figures 1-3 that the heat diffusion flux  $h(z)$  decreases with increase of  $n$  for specific  $k(z)$ , the physical meaning is that heat diffusion flux  $h(z)$  is a decreasing function of  $n$ , which means that the profiles exhibited by a

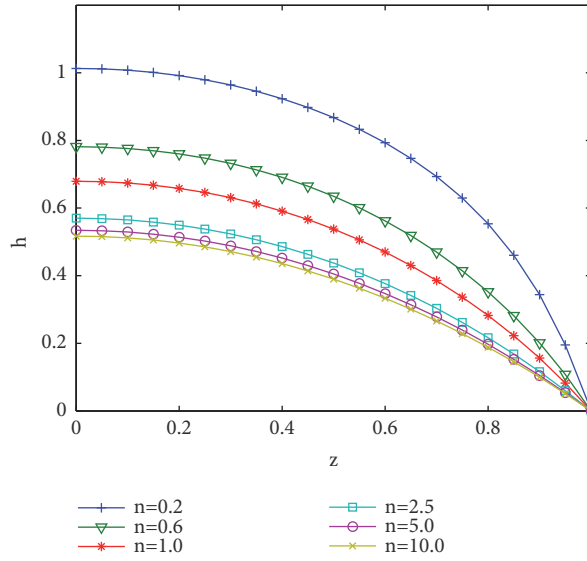


FIGURE 1: Flux distribution for  $n = 0.2, 0.6, 1.0, 2.5, 5.0, 10.0, k(z) = z$ .

TABLE 2: Values of  $\alpha$  for  $n = 0.2, 0.6, 1.0, 2.5, 5.0, 10.0$ , and  $k(z) = z^2$ .

$n$	$k(z)$	$\alpha$
0.2	$z^2$	0.9122
0.6	$z^2$	0.6532
1.0	$z^2$	0.5352
2.5	$z^2$	0.4090
5.0	$z^2$	0.3691
10.0	$z^2$	0.3506

TABLE 3: Values of  $\alpha$  for  $n = 0.2, 0.6, 1.0, 2.5, 5.0, 10.0$ , and  $k(z) = z^3$ .

$n$	$k(z)$	$\alpha$
0.2	$z^3$	0.8643
0.6	$z^3$	0.5927
1.0	$z^3$	0.4670
2.5	$z^3$	0.3300
5.0	$z^3$	0.2870
10.0	$z^3$	0.2676

TABLE 4: Values of  $\alpha$  for  $n = 2.0$  and  $k(z) = z, z^2, z^3$ .

$n$	$k(z)$	$\alpha$
2.0	$z$	0.5885
2.0	$z^2$	0.4300
2.0	$z^3$	0.3529

TABLE 5: Values of  $\alpha$  for  $n = 8.0$  and  $k(z) = z, z^2, z^3$ .

$n$	$k(z)$	$\alpha$
8.0	$z$	0.5212
8.0	$z^2$	0.3551
8.0	$z^3$	0.2723

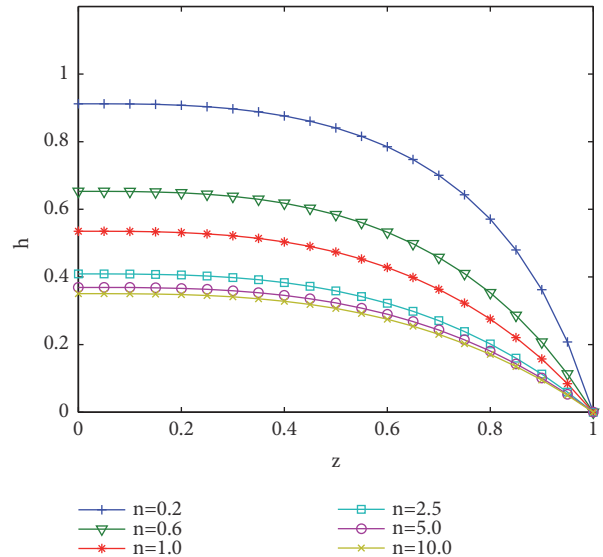


FIGURE 2: Flux distribution for  $n = 0.2, 0.6, 1.0, 2.5, 5.0, 10.0, k(z) = z^2$ .

big power law index  $n$  possess a smaller diffusion. Figures 4 and 5 indicate the heat diffusion flux  $h(z)$  decrease with the decrease of  $k(z)$ .

In order to verify the efficiency and reliability of approximate analytical solution obtained by using DPTPEM, a

comparison of approximate analytical solution and numerical one obtained by bvp4c with Matlab is presented in Tables 6 and 7 and Figures 6 and 7. It is obvious that excellent

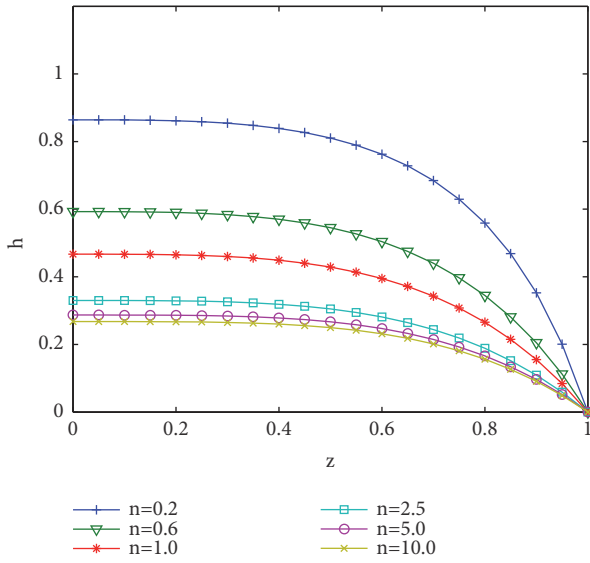


FIGURE 3: Flux distribution for  $n = 0.2, 0.6, 1.0, 2.5, 5.0, 10.0, k(z) = z^3$ .

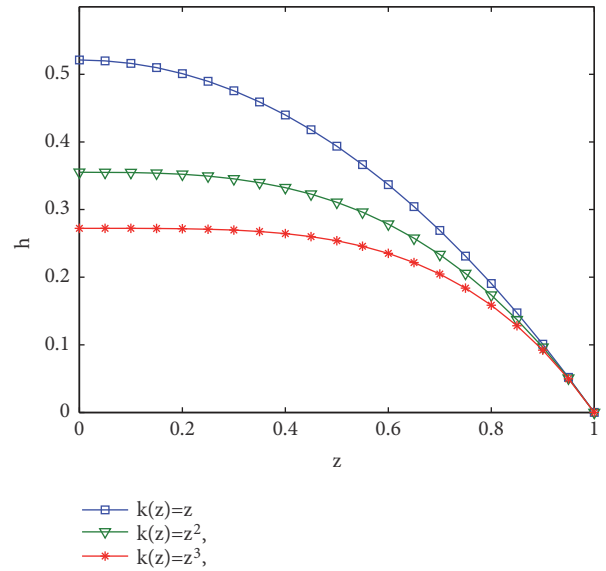


FIGURE 5: Flux distribution for  $n = 8.0, k(z) = z, z^2, z^3$ .

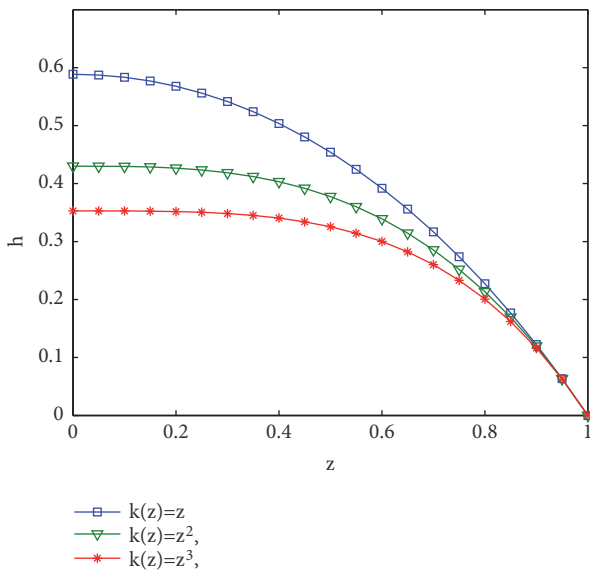


FIGURE 4: Flux distribution for  $n = 2.0, k(z) = z, z^2, z^3$ .

agreement exists for approximate analytical solution and numerical one.

### 6. Conclusions

In the paper, a class of nonlinear convection-diffusion was studied. The partial differential equation and corresponding initial value conditions were transformed into a class of singular nonlinear boundary value problems of ordinary differential equation when similarity variables were introduced. An efficient approximate analytical method named DPTPEM was applied to solve these nonlinear problems. The reliability and effectiveness of the DPTPEM were verified by comparing approximate results with the numerical solutions. The effects

TABLE 6: Comparison of  $h(z)$  for  $n = 10$  and  $k(z) = z$ .

$z$	DPTPEM	<i>bvp4c</i>
0.0	0.5169	0.5169
0.1	0.5119	0.5119
0.2	0.4968	0.4968
0.3	0.4715	0.4715
0.4	0.4359	0.4359
0.5	0.3899	0.3899
0.6	0.3334	0.3334
0.7	0.2663	0.2663
0.8	0.1884	0.1884
0.9	0.0997	0.0997
1.0	0	0

TABLE 7: Comparison of  $h(z)$  for  $n = 2$  and  $k(z) = z^3$ .

$z$	DPTPEM	<i>bvp4c</i>
0.0	0.3529	0.3551
0.1	0.3528	0.3550
0.2	0.3518	0.3540
0.3	0.3483	0.3506
0.4	0.3405	0.3427
0.5	0.3255	0.3278
0.6	0.3000	0.3023
0.7	0.2601	0.2624
0.8	0.2008	0.2031
0.9	0.1164	0.1186
1.0	0	0

of convection functional coefficient  $k(z)$  and power law index  $n$  on transfer behavior were presented. The results show that heat diffusion flux  $h(z)$  is an increasing function

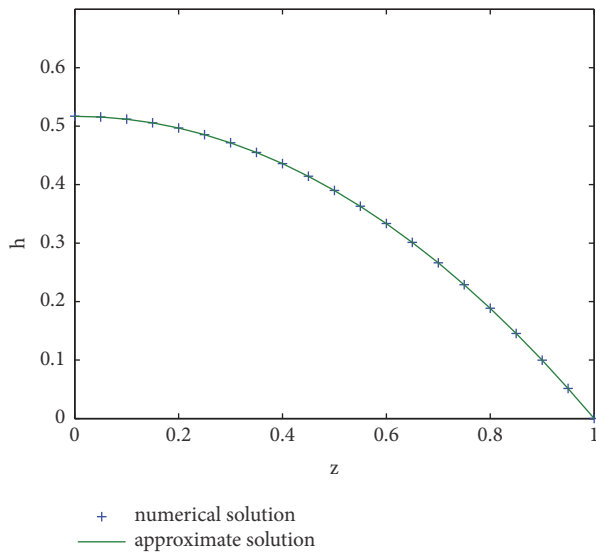


FIGURE 6: The comparison of the results of approximate analytical solution and numerical solution for  $n = 10.0, k(z) = z$ .

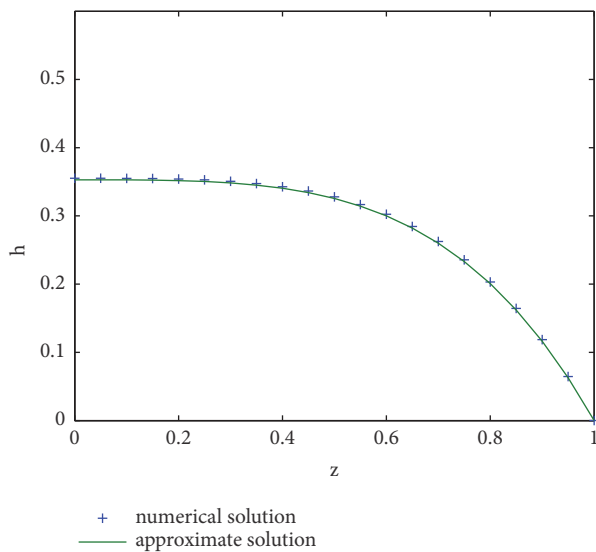


FIGURE 7: The comparison of the results of approximate analytical solution and numerical solution for  $n = 2, k(z) = z^3$ .

of convection functional coefficient  $k(z)$  and a decreasing function of power law index  $n$ .

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The author declares that there are no conflicts of interest.

**Acknowledgments**

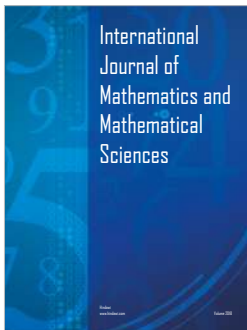
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