Similarity solution for MHD plane free jet as boundary layer flow induced by impermeable stretching plane

Anuj Kumar Jhankal[1], Manoj Kumar[2]

[1]Birla Institute of Technology, Extension Center-Jaipur, Jaipur-302 017 (INDIA). anujjhankal@yahoo.com
[2]Arya College of Engineering & Research Center, Kukas, Jaipur-303 101 (INDIA). manoj_maths78@yahoo.com

Abstract
This paper considers the problem of a laminar electrically conducting fluid in a free jet as boundary layer flow induced by impermeable plane. The governing boundary layer equations are transformed into ordinary differential equations using similarity transformation which are than solved using maple software. The effects of various values of magnetic parameter on velocity profiles and skin friction coefficient are presented and discussed.

Keywords: Boundary layer, Plane free jet, Magnetic field, Numerical study.

Mathematics Subject Classification: 76D10, 76D25, 76W05.

List of symbols

A	Strength of stretching velocity
B_0	Constant applied magnetic field
f	Dimensionless stream function
f_w	Dimensionless suction velocity
J	Momentum flow
l	Length of the slit
m	Stretching exponent
M	Dimensionless magnetic field parameter
u	Downstream velocity
U_w	Stretching velocity
v	Transversal velocity
V_w	Suction velocity
x	Coordinate in direction of surface motion
y	Coordinate in direction normal to surface motion

Greek symbols

η	dimensionless similarity variable
ρ	density of fluid
σ	current density
ν	kinematic viscosity

Superscript

$'$	derivative with respect to η

Subscript

w	condition at the wall

1. Introduction

Study of MHD flow play an important role in various industrial applications. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The problem of heat and mass transfer in the boundary layers on continuous stretching surfaces moving in a quiescent fluid medium have attracted considerable attention during the last few decades. It is importance in connection with many engineering problems, such as wire drawing, glass-fiber and
paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and simultaneous heating or cooling during such processes have a decisive influence on the quality of the final products.

The pioneering works of Sakiadis [1] a rapidly increasing number of papers investigating different aspects of this problem have been published. Most of the studies in this field describe the heat and mass flow in the vicinity of the continuous stretching surface with the aid of similarity solutions of the boundary layer equations. Except for exponentially stretching plane surfaces [2], the kinematic driving conditions of the real processes are modeled in most cases by different power-law variations of the stretching velocity, $U_w(x) = A.x^m$. For a recent review of a vast literature see, e.g. [3,4].

The analysis of laminar incompressible non-magnetic free jet flows of Schlichting [5] and Bickley [6] was extended to the magnetic case by a number of authors such as Peskin [7], Smith and Camble [8], Pozzi and Bianchini [9], Bansal [10], Jat, Jhankal and Kulhari [11] etc. Magyari and Keller [12] describe the intrinsic connection between the stretching induced boundary layer flows on the one hand and free laminar jets of the classical fluid dynamics on the other.

The aim of the present paper is to describe the problem of a laminar electrically conducting fluid in a plane free jet as boundary layer flows induced by impermeable stretching plane.

2. Mathematical formulation

Let an electrically conducting, viscous, incompressible fluid be discharged through a narrow slit, in the presence of transverse magnetic field of uniform strength B_0 . The induced magnetic field and viscous dissipation is assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small. The x-axis is directed along the continuous stretching surface and points in the direction of motion. The y-axis is perpendicular to x axis and to the direction of slot of length $l \to \infty$ (z-axis) where the continuous stretching plane issues. The boundary layer equations governing the steady flow induced by a plane wall stretching with velocity $u(x,0) = U_w(x)$ are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

Where u and v are the x and y components of the velocity filed of the steady plane boundary layer flow, respectively. Equations (1) and (2) are subject to boundary conditions in the presence of a possible lateral mass flow of suction/injection $V_w(x)$ are:

$$u(x,0) = U_w(x), \qquad v(x,0) = V_w(x)$$
$$u(x,\infty) = 0 \tag{3}$$

For power law boundary conditions the present problem bears a close resemblance to the classical wedge-flow problem described by the Falkner-Skan equation [I3,14] with two essential differences:

(a) The absence of pressure gradient and magnetic field term in the momentum equation
(b) The boundary conditions. Nevertheless, the general functional structure of the similarity solution in both cases is the same and reads:

$$u(x,y) = U_w(x)\, f'(\eta), \qquad \eta = \left[\frac{(m+1)A}{2\nu}\right]^{\frac{1}{2}} y x^{(m-1)/2}, A > 0, m > -1$$

$$v(x,y) = V_w(x)\left[\frac{f(\eta)}{f(0)} + \frac{m-1}{m+1}\frac{\eta f'(\eta)}{f(0)}\right], \quad U_w(x) = Ax^m,$$

$$V_{w(x)} = -\left[\frac{(m+1)\nu A}{2}\right]^{\frac{1}{2}} x^{\frac{m-1}{2}} f(0) \tag{4}$$

The dimensionless stream function $f(n)$ satisfies the ordinary differential equation

$$f'''(\eta) + f(\eta).f''(\eta) - \frac{2m}{m+1}[f'(\eta)]^2 - \frac{2}{m+1}\frac{\sigma B_0^2 x}{\rho U_w(x)}f'(\eta) = 0 \tag{5}$$

subject to the boundary conditions

$$f'(0) = 1, f(0) = f_w$$
$$f'(\infty) = 0 \tag{6}$$

Here f_w denotes the dimensionless suction/injection velocity. In this way $f_w = 0$ corresponds to an impermeable wall, $f_w > 0$ to the suction $(i.e. V_w(x) < 0)$ and $f_w < 0$ to the lateral injection $(i.e. V_w(x) < 0)$ of the fluid through a permeable wall. We are interested in the stretching mechanism of an impermeable $(f_w = 0)$ plane surface so that the momentum flow in the absence of the magnetic field

$$J = \rho l \int_{-\infty}^{\infty} u^2(x,y)dy = \rho l \left(\frac{2\nu A^3}{m+1}\right)^{\frac{1}{2}} x^{(3m+1)/2} \int_{-\infty}^{\infty} f'^2(\eta)\, d\eta \tag{7}$$

of the induced velocity boundary layer renders a prescribed constant value J. this requirement is obviously satisfied if and only if m=-1/3. Then the equation (5) can be written as

$$f''' + ff'' + f'^2 - 3Mf' = 0 \quad (8)$$

Where $M = \frac{\sigma B_0^2 x}{\rho U_W(x)}$ (magnetic parameter) (9)

Subject to the boundary conditions

$$\eta = 0, f'(0) = 1, f(0) = 0$$
$$\eta \to \infty, f'(\infty) = 0 \quad (10)$$

3. Numerical solutions

Our aim is to find out the value of velocity function $f'(\eta)$ at different points , by solving non-linear differential equation (8) together with boundary conditions (10) . The equation (8) and (10) constitute a non-linear boundary value problem. In this paper, the symbolic algebra software maple was used to solve these equations. The boundary condition (10) at $\eta = \infty$ were replaced by those at η=8 in accordance with standard practice in the boundary layer analysis. Maple uses the well known Runge-Kutta-Fehlberg fourth-fifth order (RFK45) method to generate the numerical solution of boundary value problem. The RKF45 algorithm in maple has been well tested for its accuracy and robustness.

The effect of magnetic parameter M on the velocity is presented in figure 1. Application of a transverse magnetic field to an electrically conducting flow gives rise to a resistive type of force called Lorentz force. This force has the tendency to slow down the motion of fluid in the boundary layer. As expected, as M increases the velocity decreases.

On the other hand, table 1 presents the values of skin friction $f''(0)$ for the various values of magnetic parameter M.

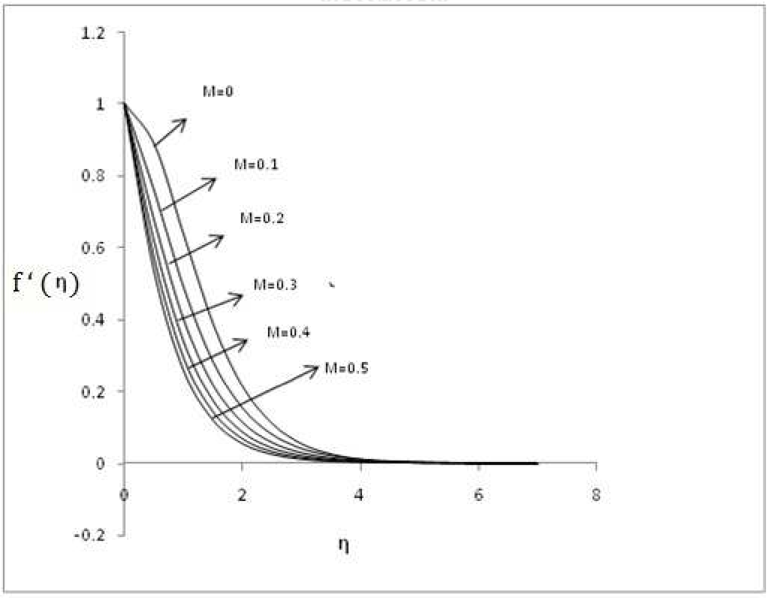

Figure 1. Velocity profiles for various values of magnetic parameter M.

Table 1. Values of $f''(0)$ for various values of M.

Magnetic Parameter M	**Skin Friction $f''(0)$**
0	-0.000284641
0.1	-0.350112543
0.2	-0.599358964
0.3	-0.794074073
0.4	-0.956519435
0.5	-1.097850474

4. Conclusions

A mathematical model has been presented for the laminar electrically conducting fluid in a free jet as boundary layer flows impermeable stretching plane. From the study, following conclusions can be drawn:

1. As the magnetic parameter (M) increases we can find the decrease in the velocity distribution in the flow region (the effect of M decrease the momentum boundary layer thickness). Thus we conclude that we can control the velocity field by introducing magnetic field.
2. The skin friction coefficient decrease as magnetic parameter increases.

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