

Similarity Solution of Hydro Magnetic Heat and Mass Transfer over a Vertical Plate with a Convective Surface Boundary Condition and Chemical Reaction

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Abstract: This paper examined the hydro magnetic boundary layer flow with heat and mass transfer over a vertical plate in the presence of magnetic field, chemical reaction and a convective heat exchange at the surface with the surrounding has been studied. The similarity solution is used to transform the system of partial differential equations and an efficient numerical technique is implemented to solve the reduced system by using the Runge-Kutta fourth order method with shooting technique. A comparison with the previous results shows a very good agreement. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters.

Keywords: vertical plate; convective boundary condition; chemical reaction; heat and mass transfer; magnetic field; similarity solution

1 Introduction

Magneto-hydrodynamic (MHD) boundary layers with heat and mass transfer over flat surfaces are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched into a cooling system. The fluid mechanical properties of the penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymer liquids like polyethylene oxide and polyisobutylene solution in cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. A comprehensive review on the subject to the above problem has been made by many researchers (Yang et al., [17]; Trevisan and Bejan., [15]; Sparrow et al., [14]; Evans, [7]). The similarity solution for hydromagnetic mixed convection of heat and mass transfer for hiemenz flow through a porous media as explained by Chamkha and Khaled [3]. Ghaly and Seddeek [8], investigated the effects of chemical reaction, heat and mass transfer on laminar flow along a semi infinite horizontal plate with temperature dependent viscosity. The effect of thermal radiation on heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field was reported in Makinde and Ogulu[11]. Recently Mikinde [12], reported a similarity solution of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition. The paper demonstrates that a similarity solution is possible if the convective heat transfer associated with the hot fluid on the lower surface of the plate is proportional to the inverse square root of the axial distance. Sakiadas [13], first presented boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al. [6] extended Sakiadas problem to include blowing or suction at the moving surface and investigated its effects on the heat and mass transfer in the boundary layer. Danberg and Fansber [4] investigated the nonsimilar solution for the flow in the boundary layer past a wall i.e stretched with a velocity proportional to distant along the wall. Gupta and Gupta [9] studied the heat and mass transfer corresponding to similarity solution for the boundary layer over an isothermal stretching sheet subject to blowing or suction. Chen and Char [2] investigated the effects of variable surface temperature and variable surface heat flux on the heat transfer characteristics of a linearly

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stretching sheet subject blowing or suction. Vajravelu and Hadjinicolaou [16] studied the convective heat transfer in an electrically conducting fluid near an isothermal stretching sheet and they studied the effect of internal heat generation or absorption. Recently, Elbasha [5] investigated heat transfer over a stretching surface with variable and uniform surface heat flux subject to injection and suction. Hence, the purpose of the present work is to extend the work of Makinde [12] to include hydromagnetic mixed convection heat and mass transfer over a vertical plate with a convective surface boundary condition and chemical reaction. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables, and these have been solved numerically. The effects of magnetic field, Prandtl number, Schmidt number, Grashof number, convective heat transfer parameter on fluid velocity and chemical reaction rate constant, temperature and concentration have been shown graphically. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2 Mathematics analysis

Let us consider a steady, laminar, hydromagnetic coupled heat and mass transfer by mixed convection flow over a vertical plate. The fluid is assumed to be Newtonian, electrically conducting and its property variations due to temperature and chemical species concentration are limited to fluid density. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation). In addition, there is no applied electric field and all of the Hall effects and Joule heating are neglected (Figure 1). Since the magnetic Reynolds number is very small for many fluids used in industrial applications, we assumed that the induced magnetic field is negligible.

Let the x -axis be taken along the direction of plate and y -axis normal to it. If u, v, T and C are the fluid x -component of

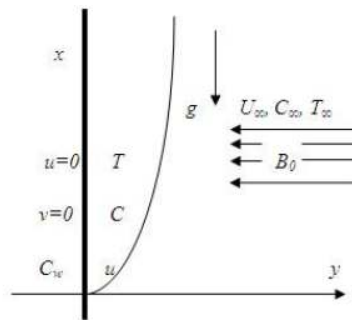


Figure 1: Flow configuration and coordinate system.

velocity, y -component of velocity, temperature and concentration respectively, then under the Boussinesq and boundary-layer approximations, the governing equations for this problem can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - U_\infty) + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \kappa r^2 (C - C_\infty) \tag{4}$$

where ν is the fluid kinematics viscosity, ρ - the density, σ - the electric conductivity of the fluid, β_T and β_C - the coefficients of thermal and concentration expansions respectively, k - the thermal conductivity, C_∞ - the free stream concentration, B_0 - the magnetic induction, U_∞ - the free stream velocity, κr - the Reaction rate, D - the mass diffusivity and g is the gravitational acceleration.

The boundary conditions at the plate surface and for into the cold fluid may be written as

$$u(x, 0) = v(x, 0) = 0, -K \frac{\partial T}{\partial y}(x, 0) = h[T - T_w(x, 0)], C_w(x, 0) = Ax^\lambda + C_\infty$$

$$u(x, \infty) = U_\infty, T(x, \infty) = T_\infty, C(x, \infty) = C_\infty \quad (5)$$

where h is the plate heat transfer coefficient, L is the plate characteristic length, C_w is the species concentration at the plate surface, λ is the plate surface concentration exponent and K is the thermal conductivity coefficient. The stream function ψ , satisfies the continuity equation (1) identically with

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

A similarity solution of Equations (1)-(6) is obtained by defining an independent variable η and a dependent variable f in terms of the stream function ψ as

$$\psi = (\nu x U_\infty)^{\frac{1}{2}} f(\eta), \eta = y \left(\frac{U_\infty}{\nu x} \right)^{\frac{1}{2}} \quad (7)$$

The dimensionless temperature and concentration are given as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (8)$$

where T_w is the temperature of the hot fluid at the left surface of the plate, Substituting the equations (6)-(8) in to Equations (1)-(5), we obtain

$$f''' + \frac{1}{2} f f'' - Ha(f' - 1) + Gr\theta + Gc\phi = 0 \quad (9)$$

$$\theta'' + \frac{1}{2} Pr f \theta' = 0 \quad (10)$$

$$\phi'' + \frac{1}{2} Sc f \phi' - \kappa r^2 Sc \phi = 0 \quad (11)$$

$$f(0) = 0, f'(0) = 0, \theta'(0) = Bi[\theta(0) - 1], \phi(0) = 0 \quad (12)$$

$$f'(\infty) = 1, \theta(\infty) = \phi(\infty) = 0 \quad (13)$$

where the prime symbol represents the derivative with respect to η and $Ha = \frac{\sigma B_0^2 x}{\rho U_\infty}$ (The Magnetic field parameter), $Gr = \frac{g \beta_T (T_w - T_\infty) x}{U_\infty^2}$ (The thermal Grashof number), $Gc = \frac{g \beta_C (C_w - C_\infty) x}{U_\infty^2}$ (The solute Grashof number), $Bi = \left(\frac{h}{k}\right) \left(\frac{\nu x}{U_\infty}\right)^{\frac{1}{2}}$ (The convective heat transfer parameter), $Pr = \frac{\nu}{\alpha}$ (The Prandtl number), $Sc = \frac{\nu}{D}$ (The Schmidt number), $\kappa r^2 = \frac{\kappa r^2 x}{U_\infty}$ (The chemical reaction rate constant).

It is noteworthy that the local parameters Bi , Ha , Gr and Gc in Equations (9)-(13) are functions of x . However, in order to have a similarity solution all the parameters Bi , Ha , Gr and Gc must be constant and we therefore assume

$$h = cx^{-\frac{1}{2}}, \sigma = ax^{-1}, \beta_T = bx^{-1} \text{ and } \beta_C = dx^{-1} \quad (14)$$

where a, b, c, d are constants.

Other physical quantities of interest in this problem such as the skin friction parameter $\tau = f'(0)$, the plate surface temperature $\theta(0)$, Nusselt number $Nu = -\theta'(0)$ and the Sherwood number $Sh = -\phi'(0)$ can be easily computed. For local similarity case, integration over the entire plate is necessary to obtain the total skin friction, total heat and mass transfer rate.

3 Solution of the problem

The equations (9)-(11) are coupled and non-linear ordinary differential equations and hence analytical solution is not possible. Hence the dimensionless governing equations (9)-(11) together with the boundary conditions (12 and 13) are solved numerically by using Runge-Kutta fourth order technique along with shooting technique (Jain et al.[10]). The step size $\Delta\eta = 0.05$ is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence. The shooting method for linear equations is based on replacing the boundary value problem by two initial value problems, and solution of the boundary value problem is a linear combination between the solutions of the two initial value problems. The shooting method for the nonlinear boundary value problem is similar to the linear case, except that the solution of the nonlinear problem can not be simply expressed as a linear combination between the solutions of the two initial value problems. The numerical computations have been done by the symbolic computation software Mathematica. The equation (11) being linear, solving it analytically, we directly get ϕ . The numerical approach is carried out in two stages. Solving the equation (10) by the nonlinear shooting method we obtain θ . Hence, equations (9) and (11) reduce to a system of linear equations with variable coefficients which could be solved by the linear shooting method to obtain f . The functions f' , θ and ϕ are shown in figures. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

4 Results and discussion

The governing equations (9)-(11) subject to the boundary conditions (12)-(13) are integrated as described in section 3. Numerical results are reported in the tables 1-2. The prandtl number was taken to be $Pr = 0.72$ which corresponds to air, the value of Schmidt number (Sc) were chosen to be $Sc = 0.24, 0.62, 0.78, 2.62$ representing diffusing chemical species of most common interest in air like H_2, H_2O, NH_3 and Propyl Benzene respectively. Attention is focused on positive value of the buoyancy parameters that is, Grashof number $Gr > 0$ (which corresponds to the cooling problem) and solutal Grashof number $Gc > 0$ (which indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface). In order to benchmark our numerical results, we have compared the plate surface temperature $\theta(0)$ and the local heat transfer rate at the plate surface $\theta'(0)$ in the absence of both magnetic field and buoyancy forces for various values of Bi with those of Aziz [1], Makinde [12] and found them in excellent agreement as demonstrated in table 1. From table 2, it is important to note that the local skin friction together with the local heat and mass transfer rate at the plate surface increases with increasing intensity of buoyancy forces (Gr, Gc), Magnetic field (Ha), Convective heat change parameter (Bi) and chemical reaction rate constant (κr). However, an increase in the Schmidt number (Sc) causes a decrease in both skin friction and surface heat transfer rate and an increase in the surface mass transfer rate.

4.1 Effects of parameter variation on velocity profiles

The effects of various parameters on velocity profiles in the boundary layer are depicted in Figures 2-7. It is observed from Figures 1-6, that the velocity starts from a zero value at the plate surface and increase to the free stream value far away from the plate surface satisfying the far field boundary condition for all parameter values. In Figure 1 the effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decrease. However, in this case an increase in the Ha only slightly slows down the motion of the fluid away from the vertical plate surface towards the free stream velocity, while the fluid velocity near the vertical plate surface increases. Similar trend of slight increase in the fluid velocity near the vertical plate is observed with an increase in convective heat transfer parameter (Bi). Figures 3 and 4 shows the variation of the boundary-layer velocity with the buoyancy forces parameters (Gr, Gc). In both cases an upward acceleration of the fluid in the vicinity of the vertical plate is observed with increasing intensity of buoyancy forces. Further downstream of the fluid motion decelerates to the free stream velocity. Figures 5 and 6 shows a slight decrease in the fluid velocity with an increase in Schmidt number (Sc) and chemical reaction rate constant (κr).

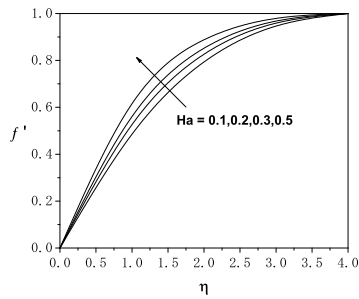


Figure 2: Variation of the velocity component f' with Ha for $Pr = 0.72, Sc = 0.62, Gr = Gc = Bi = 0.1, \kappa r = 0.5$.

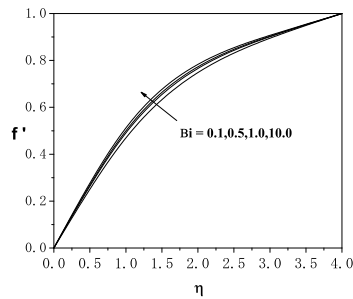


Figure 3: Variation of the velocity component f' with Bi for $Pr = 0.72, Sc = 0.62, Gr = Gc = Ha = 0.1, \kappa r = 0.5$.

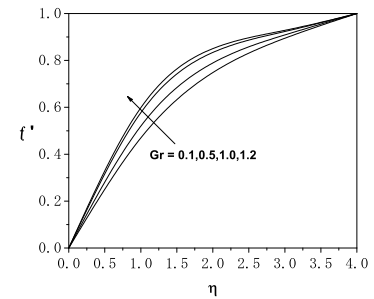


Figure 4: Variation of the velocity component f' with Gr for $Pr = 0.72, Sc = 0.62, Gr = Ha = Bi = 0.1, \kappa r = 0.5$.

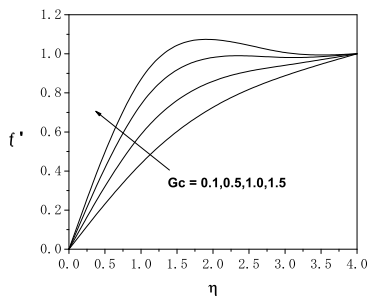


Figure 5: Variation of the velocity component f' with Gc for $Pr = 0.72, Sc = 0.62, Gr = Ha = Bi = 0.1, \kappa r = 0.5$.

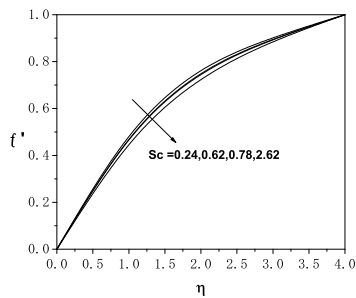


Figure 6: Variation of the velocity component f' with Sc for $Pr = 0.72, Gr = Gc = Ha = Bi = 0.1, \kappa r = 0.5$.

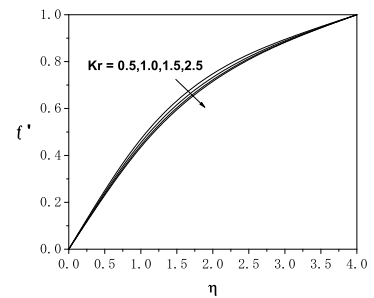


Figure 7: Variation of the velocity component f' with κr for $Pr = 0.72, Sc = 0.62, Gr = Gc = Ha = Bi = 0.1$.

4.2 Effects of parameter variation on temperature profiles

Generally, the fluid temperature attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary condition. This is observed in Figures 8-14. From these figures, it is interesting to note that the thermal boundary layer thickness decreases with an increase in the intensity of magnetic field Ha , the buoyancy forces (Gr, Gc), and permeability parameter (Pr). Moreover, the fluid temperature increases with an increase in the Schmidt number (Sc), the convective heat exchange at the plate surface (Bi) and chemical reaction rate constant (κr) leading to an increase in thermal boundary layer thickness.

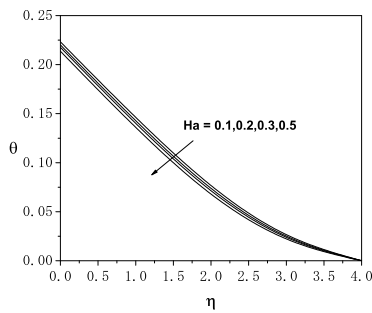


Figure 8: Variation of the temperature θ with Ha for $Pr = 0.72, Sc = 0.62, Gr = Gc = Bi = 0.1, \kappa r = 0.5$.

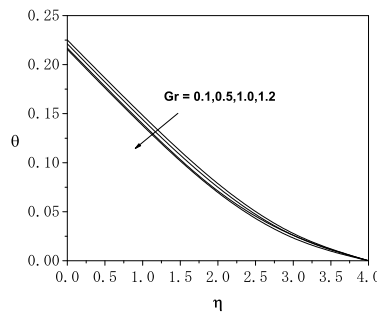


Figure 9: Variation of the temperature θ with Gr for $Pr = 0.72, Sc = 0.62, Gc = Ha = Bi = 0.1, \kappa r = 0.5$.

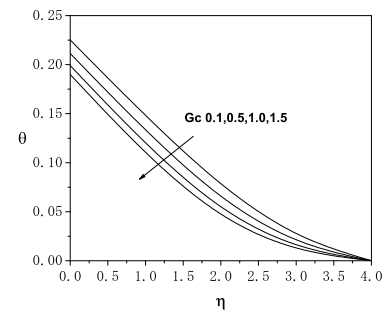


Figure 10: Variation of the temperature θ with Gc for $Pr = 0.72, Sc = 0.62, Gr = Bi = Ha = 0.1, \kappa r = 0.5$.

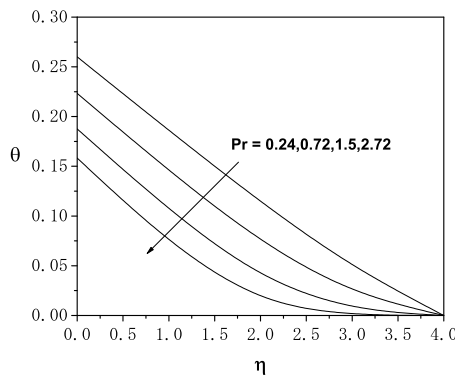


Figure 11: Variation of the temperature θ with Pr for $Sc = 0.62, Gr = Gc = Ha = Bi = 0.1, \kappa r = 0.5$.

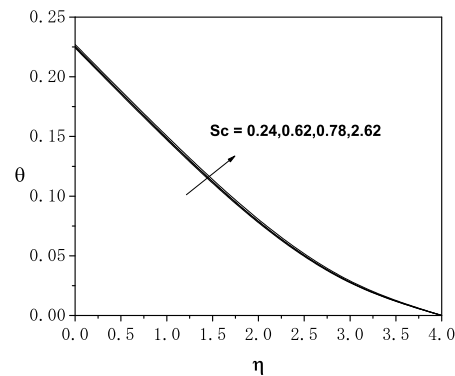


Figure 12: Variation of the temperature θ with Sc for $Pr = 0.72, Gr = Gc = Ha = Bi = 0.1, \kappa r = 0.5$.

4.3 Effects of parameter variation on concentration profiles

Figures 15-21 depict chemical species concentration profiles against span wise coordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the plate surface and decrease to zero far away from the plate satisfying the boundary condition. From these figures, it is noteworthy that the concentration boundary layer thickness decreases with an increase in the magnetic field intensity Ha , the buoyancy forces (Gr, Gc), Schmidt number

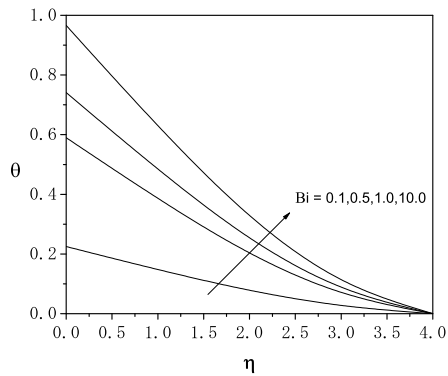


Figure 13: Variation of the temperature θ with Bi for $Pr = 0.72, Sc = 0.62, Gr = Gc = Ha = 0.1, \kappa r = 0.5$.

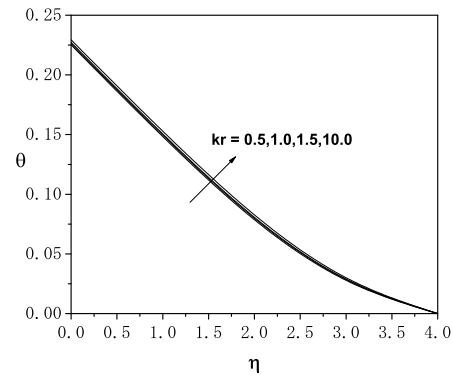


Figure 14: Variation of the temperature θ with κr for $Pr = 0.72, Sc = 0.62, Gr = Gc = Ha = Bi = 0.1$.

(Sc), the convective heat exchange at the plate surface (Bi) and chemical reaction rate constant (κr) and Moreover, the fluid concentration increases with an increase in the permeability parameter (Pr), leading to an increase in thermal boundary layer thickness.

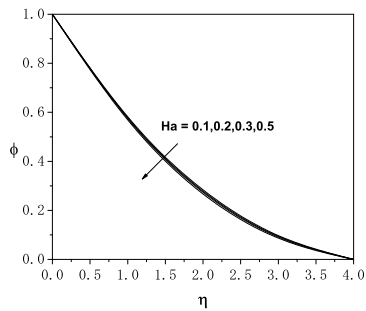


Figure 15: Variation of the concentration ϕ with Ha for $Pr = 0.72, Sc = 0.62, Gr = Gc = Bi = 0.1, \kappa r = 0.5$.

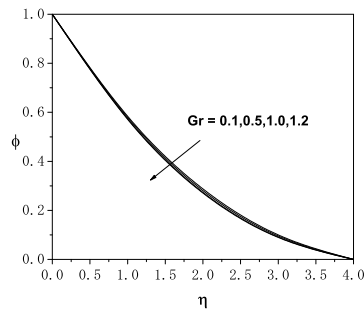


Figure 16: Variation of the concentration ϕ with Gr for $Pr = 0.72, Sc = 0.62, Bi = Gc = Ha = 0.1, \kappa r = 0.5$.

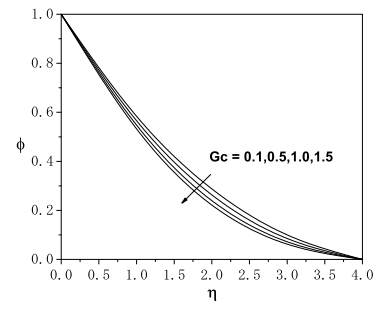


Figure 17: Variation of the concentration ϕ with Gc for $Pr = 0.72, Sc = 0.62, Gr = Bi = Ha = 0.1, \kappa r = 0.5$.

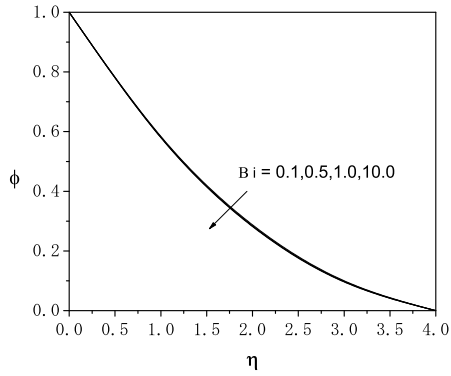


Figure 18: Variation of the concentration ϕ with Bi for $Pr = 0.72, Sc = 0.62, Gr = Gc = Ha = 0.1, \kappa r = 0.5$.

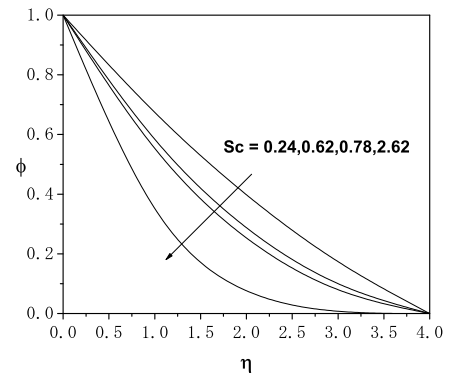


Figure 19: Variation of the concentration ϕ with Sc for $Pr = 0.72, Gr = Gc = Ha = Bi = 0.1, \kappa r = 0.5$.

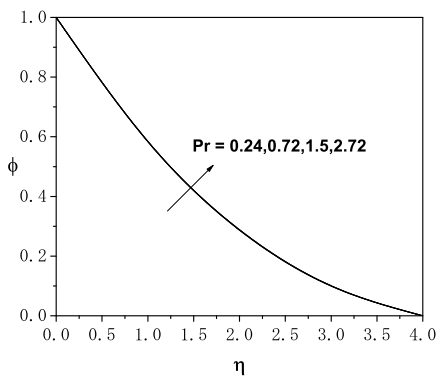


Figure 20: Variation of the concentration ϕ with Pr for $Pr = 0.72, \kappa r = 0.5, Gr = Gc = Ha = Bi = 0.1$.

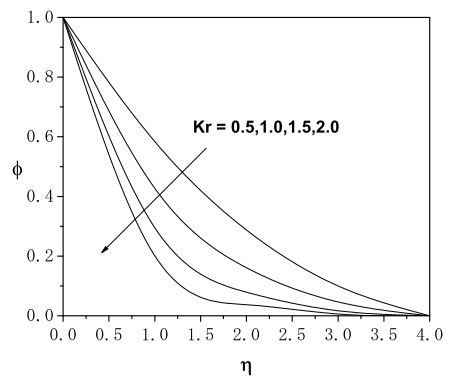


Figure 21: Variation of the concentration ϕ with κr for $Pr = 0.72, Sc = 0.62, Gr = Gc = Ha = Bi = 0.1$.

Table1: Computations showing comparison with Aziz[1] and Mikinde(2010) results for $Ha = 0, Gr = Gc = 0, Pr = 0.72, Sc = 0.63$ and $\kappa r = 0.5$

Bi	$\theta(0)$ Aziz[1]	$-\theta'(0)$ Aziz[1]	$\theta(0)$ Mikinde[11]	$-\theta'(0)$ Mikinde[11]	$\theta(0)$ Present	$-\theta'(0)$ present
0.05	0.1447	0.0428	0.14466	0.04276	0.132908	0.0433546
0.10	0.2528	0.0747	0.25275	0.07472	0.234631	0.0765369
0.20	0.4035	0.1193	0.40352	0.11929	0.380083	0.123983
0.40	0.5750	0.1700	0.57501	0.16999	0.550812	0.179675
0.60	0.6699	0.1981	0.66991	0.19805	0.647808	0.211315
0.80	0.7302	0.2159	0.73016	0.21586	0.710353	0.231718
1.0	0.7718	0.2282	0.77182	0.22817	0.754034	0.245966
5.0	0.9441	0.2791	0.94417	0.27913	0.938755	0.306223
10.0	0.9713	0.2871	0.97128	0.28714	0.96841	0.315896
20.0	0.9854	0.2913	0.98543	0.29132	0.983952	0.320966

Table2: Variation of $f''(0), -\theta'(0), \theta(0), -\phi'(0)$ at the plate with $Bi, Ha, gr, Gc, Sc, \kappa r$ for $Pr = 0.72$.

Bi	Gr	Gc	Ha	Sc	κr	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\phi'(0)$
0.1	0.1	0.1	0.1	0.24	0.5	0.597803	0.0777642	0.222358	0.354858
1.0	0.1	0.1	0.1	0.24	0.5	0.651686	0.261757	0.738243	0.356379
10	0.1	0.1	0.1	0.24	0.5	0.675083	0.344355	0.965564	0.357031
0.1	0.5	0.1	0.1	0.24	0.5	0.688034	0.0781551	0.218449	0.35739
0.1	1.0	0.1	0.1	0.24	0.5	0.793144	0.0785766	0.214234	0.360246
0.1	0.1	0.5	0.1	0.24	0.5	1.01776	0.0794505	0.205495	0.366654
0.1	0.1	1.0	0.1	0.24	0.5	1.50085	0.0809053	0.190947	0.378979
0.1	0.1	0.1	0.4	0.24	0.5	0.814486	0.0785162	0.214838	0.359809
0.1	0.1	0.1	0.6	0.24	0.5	0.931116	0.0788399	0.211601	0.362062
0.1	0.1	0.1	0.1	0.62	0.5	0.584136	0.0776696	0.223304	0.487137
0.1	0.1	0.1	0.1	0.78	0.5	0.579769	0.0776398	0.223602	0.53432

5 Conclusions

This paper studied hydromagnetic mixed convection heat and mass transfer over a vertical plate subjected to convective heat exchange with the surrounding in the presence of magnetic field and chemical reaction. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. A comparison with previously published work is performed and excellent agreement between the results is obtained. The results are presented graphically and the conclusions are drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. The results for the prescribed skin friction, local heat and mass transfer rate at the plate surface are presented and discussed. It was found that the local skin-friction coefficient, local heat and mass transfer rate at the plate surface increases with an increase in intensity of magnetic field, buoyancy forces, convective heat exchange parameter and chemical reaction rate constant.

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