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Similarity Solutions for the Stagnation-Point Flow and Heat Transfer Over a Nonlinearly Stretching/Shrinking Sheet

(Penyelesaian Keserupaan bagi Aliran Titik Genangan dan Pemindahan Haba terhadap Helaian Meregang/Mengecut)

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ABSTRACT

This paper presents a numerical analysis of a stagnation-point flow towards a nonlinearly stretching/shrinking sheet immersed in a viscous fluid. The stretching/shrinking velocity and the external flow velocity impinges normal to the stretching/shrinking sheet are assumed to be in the form $U \sim x^m$, where m is a constant and x is the distance from the stagnation point. The governing partial differential equations are converted into ordinary ones by a similarity transformation, before being solved numerically. The variations of the skin friction coefficient and the heat transfer rate at the surface with the governing parameters are graphed and tabulated. Different from a stretching sheet, it is found that the solutions for a shrinking sheet are non-unique for $m > 1/3$.

Keywords: Boundary layer; dual solutions; nonlinear stretching/shrinking, similarity solution

ABSTRAK

Makalah ini melaporkan analisis berangka aliran titik genangan terhadap helaian meregang/mengecut tak linear yang terendam dalam bendalir likat. Halaju regangan/pengecutan dan halaju aliran bebas normal ke arah helaian meregang/mengecut diandaikan dalam bentuk $U \sim x^m$, dengan m adalah pemalar dan x adalah jarak dari titik genangan. Persamaan-persamaan menakluk dalam bentuk persamaan pembezaan separa ditukar kepada persamaan pembezaan biasa menggunakan penjelmaan keserupaan, sebelum diselesaikan secara berangka. Perubahan-perubahan pekali geseran kulit dan kadar pemindahan haba pada permukaan terhadap parameter-parameter terlibat dilaporkan dalam bentuk graf dan jadual. Berbeza dengan kes regangan, didapati bahawa penyelesaian bagi kes pengecutan adalah tidak unik bagi $m > 1/3$.

Kata kunci: Lapisan sempadan; penyelesaian dual; penyelesaian keserupaan; regangan/pengecutan tak linear

INTRODUCTION

The problem of laminar boundary-layer flow resulting from the flow of an incompressible viscous fluid past a stretching sheet where the velocity on the boundary is away and proportional to the distance from a fixed point, was considered by Crane (1970). Gupta and Gupta (1977) added suction or injection effect on the surface. Wang (1984) obtained similarity solutions to the axisymmetric case. The combination of both stagnation flow and stretching surface was considered by Mahapatra and Gupta (2002, 2003) and was extended to oblique stagnation flow by Lok et al. (2006). The boundary layer flow due to a shrinking sheet has attracted considerable interest recently. There are two conditions that the flow towards the shrinking sheet likely to exist, whether an adequate suction on the boundary is imposed (Miklavčič & Wang 2006) or a stagnation flow is considered (Wang 2008), so that the velocity of the shrinking sheet is confined in the boundary layer. Similar problems with various boundary conditions and in different situations have been considered by Bachok et al. (2010), Fang and Zhang (2009), Ishak et al. (2010), Nadeem and Awais (2008), Sajid and Hayat (2009)).

Vajravelu (2001) studied the flow and heat transfer aspects for the viscous fluid flow over a nonlinearly stretching sheet, but the heat transfer characteristics in this flow was analyzed only in the case when the sheet is kept at a constant temperature. Bataller (2008) extended this problem to two different types of thermal boundary conditions on the sheet, i.e. prescribed surface temperature and prescribed surface heat flux. Different from Vajravelu (2001) and Bataller (2008), we study in this paper the stagnation-point flow toward a nonlinearly stretching/shrinking sheet with velocity $U_w = ax^m$ and free stream velocity $U_\infty = bx^m$, where a, b and m are constants. To the best of the authors' knowledge, this problem has not been studied before and therefore the results obtained are novel. Examples of practical applications of this problem include the polymer melts and solutions, heat treated materials traveling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheets or electronic chips, crystal growing, on a rising, shrinking balloon and many others. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching/shrinking.

PROBLEM FORMULATION

Consider a laminar two-dimensional stagnation flow of a viscous and incompressible fluid normal to a stretching/shrinking sheet of constant temperature T_w. The boundary layer equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+U_\infty\frac{dU_\infty}{dx}+\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions:

$$u=U_w,\ v=0,\ T=T_w \text{ at } y=0$$
$$u\to U_\infty,\ T\to T_\infty \quad \text{as} \quad y\to\infty, \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively, T is the fluid temperature in the boundary layer, T_∞ the ambient temperature, ν the kinematic viscosity and α is the thermal diffusivity.

The governing partial differential equations (1)–(3) can be reduced to ordinary differential equations by introducing the following transformation:

$$\eta=\left(\frac{U_\infty}{\nu x}\right)^{1/2} y, \psi=\left(\nu x U_\infty\right)^{1/2} f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{5}$$

where η is the similarity variable and ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfies (1).

The transformed ordinary differential equations are:

$$f'''+\frac{m+1}{2}ff''+m\left(1-f'^2\right)=0, \tag{6}$$

$$\frac{1}{\Pr}\theta''+\frac{m+1}{2}f\theta'=0, \tag{7}$$

where primes denote differentiation with respect to η. The boundary conditions (4) now become:

$$f(0)=0,\ f'(0)=\varepsilon,\ \theta(0)=1, \tag{8}$$
$$f'(\eta)\to 1,\ \theta(\eta)\to 0 \text{ as } \eta\to\infty,$$

where $\varepsilon = a/b$ is the stretching/shrinking parameter with $\varepsilon > 0$ for stretching and $\varepsilon < 0$ for shrinking.

RESULTS AND DISCUSSION

Numerical solutions to the transformed ordinary differential equations (6) and (7) subjected to the boundary conditions (8) were obtained using a shooting method. The equations were solved for several values of the stretching/shrinking parameter ε and the velocity exponent parameter m, for two values of Prandtl number Pr, namely Pr = 0.7 (air) and Pr = 7 (water). Figures 1 and 2 show the variations of the reduced skin friction coefficient $f''(0)$ and the reduced heat transfer rate at the surface $-\theta'(0)$ with ε. It is seen that there are regions of unique solutions for $\varepsilon \geq -1.0$, dual solutions for $\varepsilon_c < \varepsilon \leq -1.0$ and no solutions for $\varepsilon < \varepsilon_c < -1.0$. Therefore, the solutions exist up to the critical value $\varepsilon_c (\leq -1.0)$, beyond which no solution exists. These values of ε_c are given in Table 1. Moreover, as presented in Figures 1 and 2, dual solutions exist for $m > 1/3$.

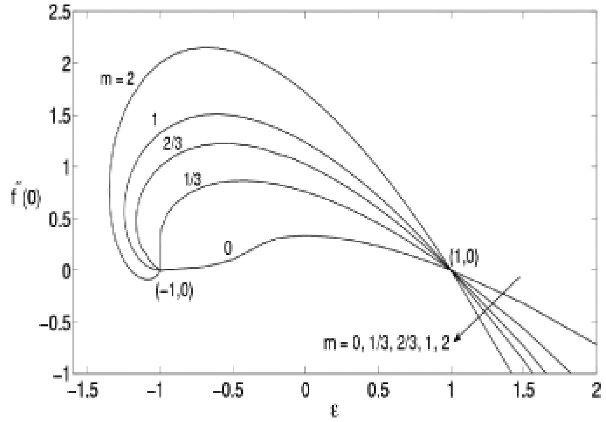

FIGURE 1. Skin friction coefficient $f''(0)$ as a function of ε for various values of m

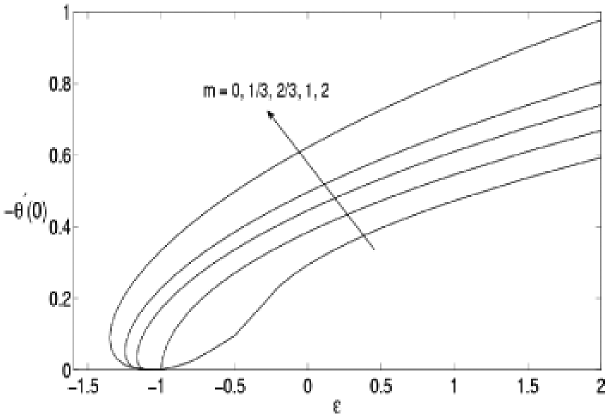

FIGURE 2. Variation of $-\theta'(0)$ with ε for various values of m when Pr = 0.7

TABLE 1. The minimum values of ε (i.e. ε_c) for which solution to the system of ordinary differential equations (6) - (8) exists for Pr = 0.7

m	ε_c
2/3	-1.1652
1	-1.2465
2	-1.3498

Table 2 presents the values of the upper and lower branch of $f''(0)$ and $-\theta'(0)$, which respectively denoted by $f_u''(0), f_l''(0)$ and $-\theta_u'(0), -\theta_l'(0)$ for m = 2/3, 1 and 2 when ε = -1.1 (shrinking case). As discussed by Merkin

(1985), Harris et al. (2009) and Weidman et al. (2006), who reported the existence of dual solutions, the upper branch solutions are stable, while the lower branch solutions are not. For the stable solutions, the heat transfer rate at the surface is higher for the stretching case compared to that of shrinking. It is seen in Figure 1 that all curves intersect at a point (1, 0). This is not surprising since there is no shear stress at the surface when the stretching velocity equals the free stream velocity, regardless of the values of *m*. However, as seen in Figure 2, the heat transfer at the solid-fluid interface still occurs, i.e. $-\theta'(0) \neq 0$, even there is no shear stress for this case ($\varepsilon = 1$), since they are at different temperatures. Moreover, all the solution curves have (-1, 0) as their limit point.

Figures 3 to 6 display the samples of velocity and temperature profiles for different values of *m*, Pr and ε. These profiles satisfy the far field boundary conditions (8) asymptotically, which support the numerical results, besides supporting the dual nature of the solutions presented in Figures 1 and 2.

TABLE 2. Values of $f_u''(0)$, $f_l''(0)$ and $-\theta_u'(0)$, $-\theta_l'(0)$ for $m = 2/3$, 1 and 2 when $\varepsilon = -1.1$ (shrinking case) and Pr = 0.7

m	$f_u''(0)$	$f_l''(0)$	$-\theta_u'(0)$	$-\theta_l'(0)$
2/3	0.8213	0.1455	0.1149	0.0003
1	1.1867	0.0492	0.1828	0.0003
2	1.8719	-0.0898	0.2903	0.0004

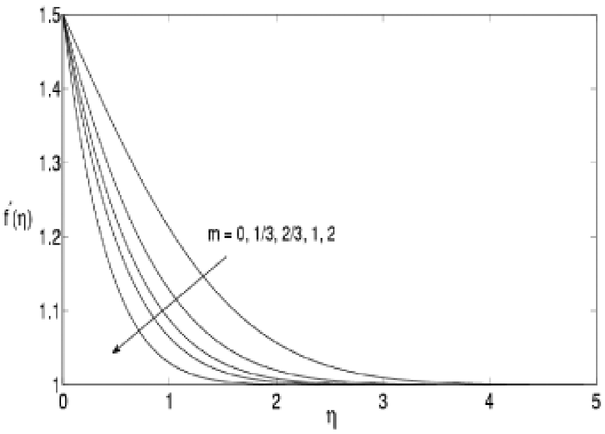

FIGURE 3. Velocity profiles $f'(\eta)$ for various values of *m* when ε = 1.5

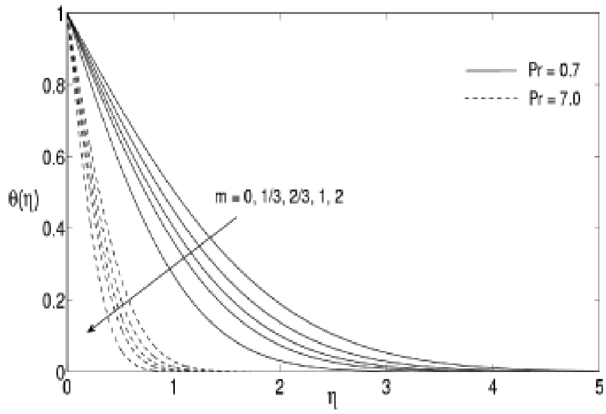

FIGURE 4. Temperature profiles θ(η) for various values of *m* for Pr = 0.7 and 7.0 when ε = 1.5

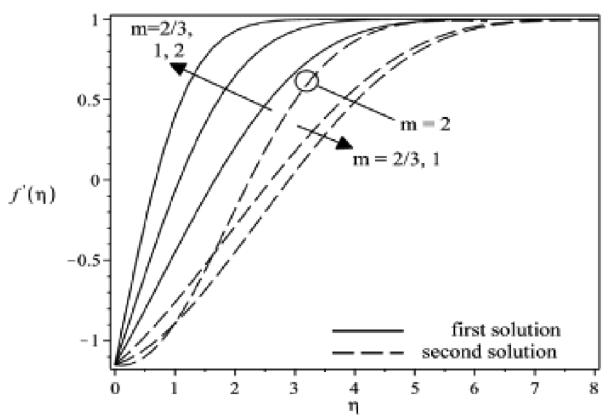

FIGURE 5. Velocity profiles $f'(\eta)$ for various values of *m* when = -1.15

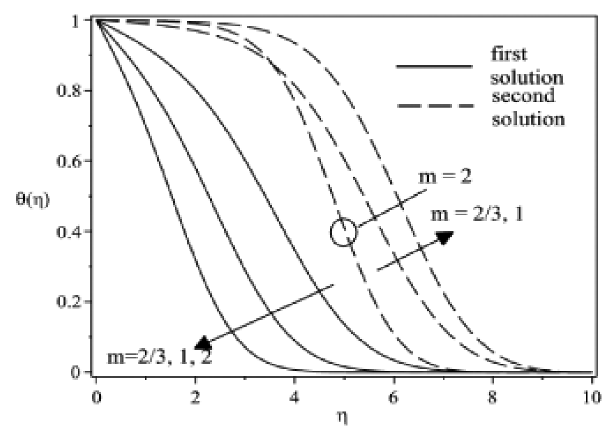

FIGURE 6. Temperature profiles θ(η) for various values of *m* for Pr = 0.7 when $\varepsilon = -1.15$

Conclusion

Similarity solutions for the problem of stagnation-point flow towards a nonlinearly stretching/shrinking sheet with constant surface temperature were investigated numerically. The governing partial differential equations were converted into ordinary ones by a similarity transformation, before being solved numerically by a shooting method. The variations of the skin friction coefficient and the heat transfer rate at the surface with the governing parameters were obtained and discussed. Different from a stretching sheet, it was found that the solutions for a shrinking sheet are non-unique for $m > 1/3$. Moreover, for the stable solution, the heat transfer rate at the surface is higher for the stretching case compared to those of shrinking.

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