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Brief note

SIMILARITY SOLUTIONS ON MIXED CONVECTION HEAT TRANSFER FROM A HORIZONTAL SURFACE SATURATED IN A POROUS MEDIUM WITH INTERNAL HEAT GENERATION

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The aim of this work is to study the mixed convection boundary layer flow from a horizontal surface embedded in a porous medium with exponential decaying internal heat generation (IHG). Boundary layer equations are reduced to two ordinary differential equations for the dimensionless stream function and temperature with two parameters: ε , the mixed convection parameter, and λ , the exponent of x. This problem is numerically solved with a system of parameters using built-in codes in Maple. The influences of these parameters on velocity and temperature profiles, and the Nusselt number, are thoroughly compared and discussed.

Key words: mixed convection, horizontal porous surface, internal heat generation.

1. Introduction

A couple of recent papers have been devoted to the subject of similarity solutions for free with internal heat generation in porous media for several geometric configurations, see Postelnicu *et al.* [3, 4]. Several recent papers, such as Magyari *et al*. [5, 6], Mealey and Merkin [7], Merkin [8] throw a new light on the internal heat generation in porous media, remaining in the frame of similar solutions. Boundary layer analysis was performed for mixed convection about a horizontal plate in saturated porous media with aiding external flows [9].

This paper studies the mixed convection boundary layer flow along horizontal surfaces embedded in porous media driven by internal heat generation. It is found that a similarity solution exists when both the wall temperature distribution and the velocity outside the boundary layer vary according to the power law fluid and constant defined in Eq.(2.4b).

2. Basic equations

Consider the mixed convection boundary layer flow over horizontal surfaces in porous media with internal heat generation. The x-coordinate is measured along the surface and the y-coordinate normal to it. The basic equations are continuity equation, Darcy's law and energy equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{2.1}$$

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$$\frac{\partial u}{\partial y}=-\frac{g\beta K}{\upsilon}\frac{\partial T}{\partial x}, \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+q''' . \tag{2.3}$$

The boundary conditions for the model are

$$v=0, \quad T=T_w, \quad \text{at} \quad y=0, \tag{2.4a}$$

$$u=U_\infty, \quad T=T_\infty, \quad \text{as} \quad y\to\infty, \tag{2.4b}$$

K, as in Eq.(2.2), is the average value of permeability. We consider $T_w=T_\infty+Ax^\lambda$ and $U_\infty=Bx^m$.

We now introduce the following dimensionless variables

$$\eta=\frac{y}{x}\left(\mathrm{Pe}_x\right)^{1/2}, \qquad \psi=\alpha\left(\mathrm{Pe}_x\right)^{1/2}f(\eta), \qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \tag{2.5}$$

We then consider the internal heat generation (IHG) of the form

$$q'''=\frac{\alpha\left(T_w-T_\infty\right)}{x^2}\mathrm{Pe}_x e^{-\eta} . \tag{2.6}$$

Introducing Eqs (2.5) and (2.6) into Eqs (2.2) - (2.3) we respectively have

$$f''=\varepsilon\left(\frac{m-1}{2}\eta\theta'+\lambda\theta\right), \tag{2.7}$$

$$\theta''+\frac{m+1}{2}f\theta'-\lambda f'\theta+ce^{-\eta}=0 \tag{2.8}$$

where $\varepsilon=\mathrm{Ra}_x\big/\left(\mathrm{Pe}_x\right)^{3/2}$ is the parameter for mixed convection, $\mathrm{Pe}_x=U_\infty x/\alpha$ is the local Peclet number, and $\mathrm{Ra}_x=\dfrac{g\beta K\left(T_w-T_\infty\right)x}{\alpha\upsilon}$ is the local Darcy-Rayleigh number.

In order to let ε be independent of x, we must impose $\lambda=\left(3m+1\right)/2$.

The corresponding boundary conditions are

$$f(0)=0, \quad \theta(0)=1, \quad \text{at} \quad \eta=0, \tag{2.9a}$$

$$f(\infty)=1, \quad \theta(\infty)=0, \quad \text{as} \quad \eta\to\infty. \tag{2.9b}$$

Two cases of interest have been studied in the literature (without IHG)

a) $m=$ constant heat flux

b) $m =$ stagnation-point flow.

A quantity of engineering interest is

$$\mathrm{Nu}_x / \left(\mathrm{Pe}_x\right)^{1/2} = -\theta'(0). \tag{2.10}$$

3. Results and discussion

The two sets of boundary value problems Eqs (2.7), (2.8) and (2.9) were solved by using the *dsolve* routine from MAPLE [10].

Figures 1-2 show the dimensionless velocity and temperature for the mixed convection parameter with stagnation point flow. It is seen from Fig.1 that the thickness of the hydrodynamic boundary layer decreases and increases as $\varepsilon \prec 0$ or $\varepsilon \succ 0$.

The flow is constant for $\varepsilon = 0$. It is noticed that the flow is more significant for IHG than without IHG.

On the other hand, the thickness of the thermal boundary layer increases for both $\varepsilon \prec 0$ or $\varepsilon \succ 0$. It is noticed that the heat transfer phenomena are reverse as seen in Fig.1 with and without IHG.

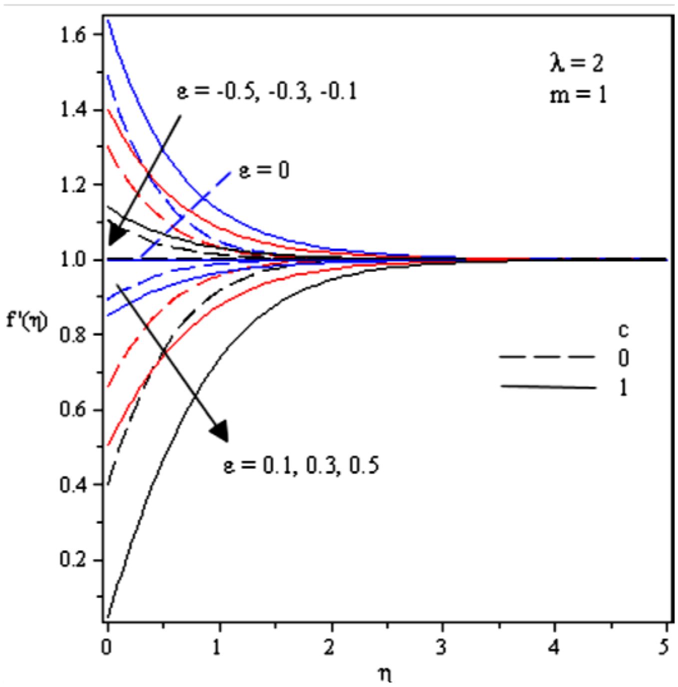

Fig.1. Velocity profiles for the mixed convection parameter with and without IHG (Stagnation point flow).

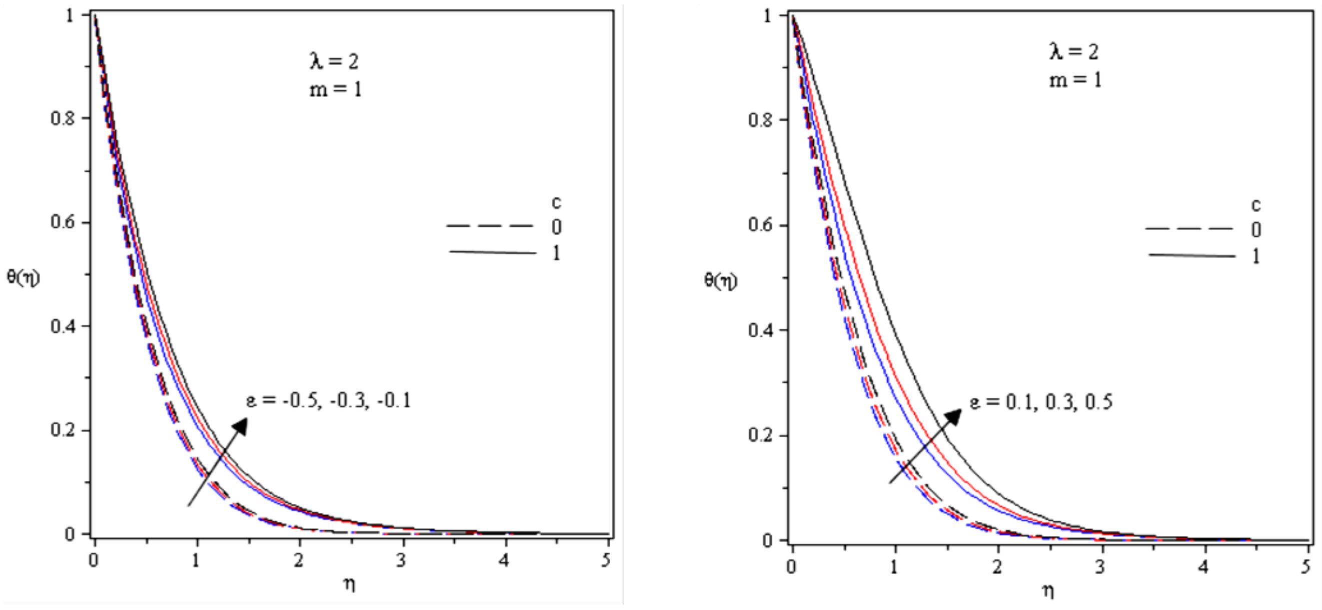

Fig.2. Temperature profiles for mixed convection parameter with and without IHG (Stagnation point flow).

Figures 3-4 show the dimensionless velocity and temperature for the mixed convection parameter with constant heat flux. We can draw the same conclusions as in the case of stagnation point flow. Table 1 presents the local Nusselt number for the mixed convection parameter with and without IHG. It is seen that the Nusselt number decreases as ε increases.

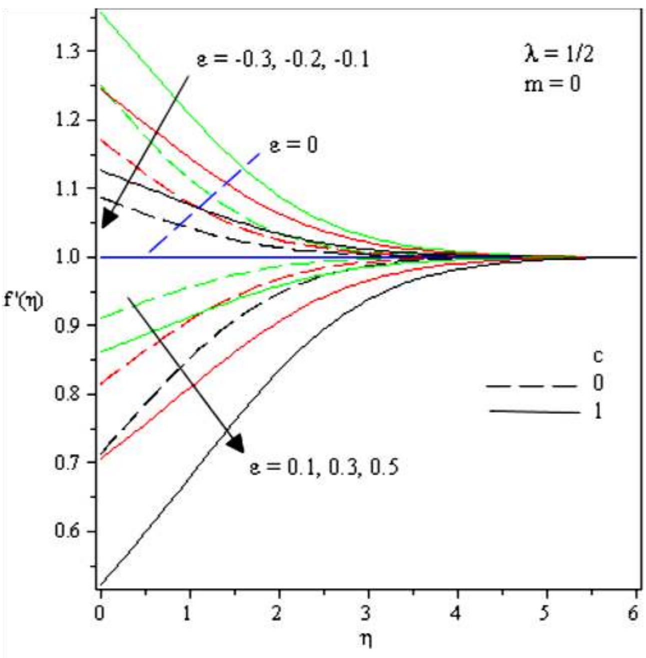

Fig.3. Velocity profiles for the mixed convection parameters with and without IHG.

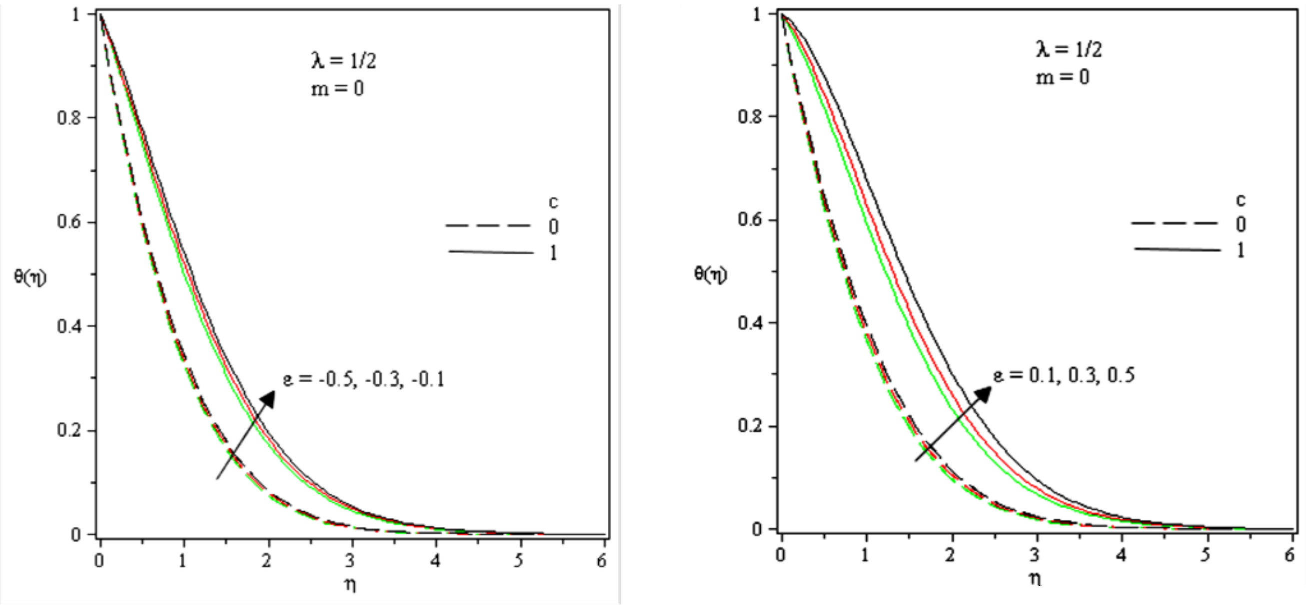

Fig.4. Temperature profiles for the mixed convection parameter with and without IHG (Constant heat flux).

Table 1. Values of $-\theta'(0)$ for different values of ε, c for $m=0$, $\lambda=1/2$.

ε	*c = 0*	*c = 0.5*	*c = 1*
-0.3	*0.96338379*	*0.71157759*	*0.46259282*
-0.2	*0.93933313*	*0.67913285*	*0.42126587*
-0.1	*0.91371283*	*0.64390991*	*0.37558694*
0.1	*0.85648129*	*0.56201643*	*0.26490351*
0.2	*0.82391671*	*0.51268443*	*0.19369201*
0.3	*0.78771412*	*0.45433429*	*0.10168781*

4. Conclusions

A laminar and steady-state model has been developed for the mixed convective heat transfer flow from a horizontal surface embedded in a porous medium with exponential decaying internal heat generation. Numerical solutions for the normalized two-point boundary value problems have been obtained using quadrature functions in Maple. We conclude that increasing the mixed convection parameter decelerates the momemtum boundary layer whereas it strongly increases thermal boundary values. On the other hand, increasing the mixed convection parameter substantially reduces the Nusselt number. Also, it has been found that in both cases internal heat generation induced more flow than that of without internal heat generation.

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