Open access • Journal Article • DOI:10.1007/S00607-008-0017-4

Simple a posteriori error estimators for the h -version of the boundary element method — Source link

Samuel Ferraz-Leite, Dirk Praetorius

Institutions: Vienna University of Technology

Published on: 08 Dec 2008 - Computing (Springer Vienna)

Topics: Estimator, Galerkin method, Sobolev space, Boundary element method and Norm (mathematics)

8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Simple A Posteriori Error Estimators for the h-Version of the Boundary Element Method

*** Samuel Ferraz-Leite[1] and Dirk Praetorius[2]**

[1] Vienna University of Technology
Institute for Analysis and Scientific Computing,
Wiedner Hauptstraße 8–10, 1040 Wien, Austria
samuel.ferraz-leite@tuwien.ac.at
http://www.asc.tuwien.ac.at/∼samuel

[2] Vienna University of Technology
Institute for Analysis and Scientific Computing,
Wiedner Hauptstraße 8–10, 1040 Wien, Austria
dirk.praetorius@tuwien.ac.at
http://www.math.tuwien.ac.at/∼dirk

Key Words: *BEM, a posteriori error estimation, adaptive mesh refinement, Symm's integral equation.*

ABSTRACT

The h-$h/2$-strategy is one very basic and well-known technique for the a posteriori error estimation for Galerkin discretizations of energy minimization problems. Let ϕ denote the exact solution. One then considers

$$\eta_H := \|\phi_h - \phi_{h/2}\|$$

to estimate the error $\|\phi - \phi_h\|$, where ϕ_h is a Galerkin solution with respect to a mesh $\mathcal{T}_h$ and $\phi_{h/2}$ is a Galerkin solution for a mesh $\mathcal{T}_{h/2}$ obtained from uniform refinement of $\mathcal{T}_h$. We stress that η_H is always efficient – even with known efficiency constant $C_{\text{eff}} = 1$, i.e.

$$\eta_H \leq \|\phi - \phi_h\|.$$

Reliability of η_H follows immediately from the assumption $\|\phi - \phi_{h/2}\| \leq \sigma \|\phi - \phi_h\|$ with some saturation constant $\sigma \in (0,1)$. Under this assumption, there holds

$$\|\phi - \phi_h\| \leq \frac{1}{\sqrt{1-\sigma^2}}\, \eta_H.$$

However, for boundary element methods, the energy norm $\|\cdot\|$ is non-local and thus the error estimator η_H does not provide information for a local mesh-refinement. Recent localization techniques from [1] for $\widetilde{H}^{-\alpha}$-norms and [3] for $\widetilde{H}^{\alpha}$-norms allow to replace the energy norm in this case by h-weighted L^2-norms resp. H^1-norms, where h denotes the local mesh-size. In particular, this very basic error estimation strategy is also applicable to steer an h-adaptive mesh-refinement. For instance, for Symm's integral equation, the L^2-norm based estimator

$$\mu_H := \|h^{1/2}(\phi_h - \phi_{h/2})\|_{L^2(\Gamma)}$$

is equivalent to η_H. We thus may use μ_H to steer the mesh and η_H to estimate the error.

Further simplifications of the proposed error estimators η_H and μ_H consist of replacing ϕ_h by some appropriate projection $\Pi_h \phi_{h/2}$, for instance, by use of the L^2-projection onto the discrete space corresponding to $\mathcal{T}_h$.

Moreover, the error estimator η_H is proven to be equivalent to the averaging estimator in [4] and the two level estimator from [5].

Numerical experiments in 2D and 3D for first-kind integral equations with weakly-singular integral operator conclude the talk.

REFERENCES

[1] C. Carstensen, D. Praetorius. *Averaging techniques for the effective numerical solution of Symm's integral equation of the first kind*, SIAM J.Sci.Comp. **27** (2006), 1226–1260.

[2] S. Ferraz-Leite, D. Praetorius. *Simple A Posteriori Error Estimators for the h-Version of the Boundary Element Method*, ASC Report **01/2007**, Institute for Analysis and Scientific Computing, Vienna University of Technology, Wien, 2007

[3] S. Funken, S. Ferraz-Leite, D. Praetorius. *Averaging on large patches for integral equations in 3D*, work in progress (2007)

[4] C. Erath, S. Funken, S. Ferraz-Leite, D. Praetorius. *Energy norm based a posteriori error estimation for boundary element methods in two dimensions*, ASC Report **07/2007**, Institute for Analysis and Scientific Computing, Vienna University of Technology, Wien, 2007

[5] P. Mund, E. P. Stephan, J. Weisse. *Two-level methods for the single layer potential in* $\mathbb{R}^3$, Computing **60** (1998), 243–266.