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Simple BRS Gauge-Fixing

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It is pointed out that the BRS gauge-fixing procedure has wide applicability beyond what it is as the usual gauge-fixing method. The examples we discuss are the equivalence theorem, generalized field transformations and a natural derivation of the gaugeon formalism. We also propose a direct way of BRS gauge-fixing for open gauge theories.

§ 1. Introduction

The simple BRS gauge-fixing procedure[1] is an elegant way of introducing gauge-fixing and FP ghost terms in a Lagrangian. It is widely used in order to perform Lagrangian gauge-fixing of gauge-invariant systems (with closed gauge algebras). However, it seems not fully recognized that the BRS procedure itself possesses much larger applicability beyond what it is as the usual gauge-fixing method.

In this paper, we point out that the BRS procedure provides a generic and useful framework for introducing new fields into a Lagrangian which are absent at the beginning, without altering the dynamical content of the system. We mention such examples as the equivalence theorem,[2] generalized field transformations including spectrum-changing ones,[3] and a natural derivation of the gaugeon formalism.[4] We also propose a direct way of BRS gauge-fixing for general gauge theories. It enables us to perform simple BRS gauge-fixing of open (and/or reducible) gauge theories. We deal with the Siegel superpoint particle[5] for definiteness, which we take as one of the simplest systems with open gauge algebras.

§ 2. Equivalence theorem

For simplicity, let us adopt a Lagrangian (in n-dimensional spacetime)

$$\mathcal{L}(\phi)=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\,, \tag{1}$$

where ϕ is a real bosonic field. (Extensions to cases including a number of bosonic and/or fermionic fields are straightforward.) We can realize a local point transformation*) $\phi=f(\phi')$ in the Lagrangian as follows: We first introduce a new field ϕ' and regard the original Lagrangian (1) as a function of the two fields ϕ and ϕ' but actually independent of the latter. Then (1) is trivially invariant under an arbitrary deformation of the field ϕ', namely, ϕ' is a gauge degree of freedom. Hence we are led to introduce a BRS transformation

*) In the quantum-mechanical case, the notion of operator-ordering should be properly taken into account.[6]

$$\delta\phi=0\,;\quad \delta\phi'=c\,,\quad \delta\bar{c}=ib\,, \tag{2}$$

and perform BRS gauge-fixing by adding

$$\mathcal{L}_B=-i\delta[\bar{c}\{\phi-f(\phi')\}]=b\{\phi-f(\phi')\}-i\bar{c}\frac{\delta f}{\delta\phi'}c$$

to the Lagrangian (1). Integrating out the fields b and ϕ successively, we are left with the desired Lagrangian

$$\mathcal{L}_T=\mathcal{L}(f(\phi'))-i\bar{c}\frac{\delta f}{\delta\phi'}c\,, \tag{3}$$

endowed with the reduced BRS transformation

$$\delta\phi'=c\,,\quad \delta\bar{c}=-i\frac{\delta\mathcal{L}}{\delta\phi}(f(\phi'))\,. \tag{4}$$

This transformation law is obtained from the BRS transformation (2) by eliminating the fields b and ϕ with the aid of their equations of motion $b=-\delta\mathcal{L}/\delta\phi$ and $\phi=f(\phi')$. The theory (3) is clearly equivalent[7] to the original one (1) by construction.

On the other hand, we can start from the naively transformed Lagrangian

$$\mathcal{L}'(\phi')=\mathcal{L}(f(\phi'))=\frac{1}{2}f'^2\partial_\mu\phi'\partial^\mu\phi'-V(f(\phi'))\,;\quad f'=\frac{\delta f}{\delta\phi'}\,. \tag{5}$$

Then, unitarity of the theory necessitates[8] adding a Lee-Yang term[9]

$$\mathcal{L}_{\mathrm{LY}}=-i\delta^n(0)\ln\det\frac{\delta f}{\delta\phi'} \tag{6}$$

to the Lagrangian (5). The resultant Lagrangian (5)+(6) coincides with the Lagrangian (3) with the ghosts $\bar{c}$ and c integrated out. This is in accord with the equivalence theorem[2] which states that two Lagrangians which are naively transformed to each other by a point transformation yield the same theory after canonical quantization.

§ 3. Generalized field transformation

The BRS procedure for field transformations exposed in the previous section can be applied not only to point transformations but also to spectrum-changing ones[3] including spacetime derivatives of the field ϕ'. For example, let us consider the transformation $\phi=f(\phi')=(\partial_\mu\partial^\mu+m^2)\phi'$. This is not a one-to-one correspondence,*) and the spectrum of the naively transformed theory $\mathcal{L}(f(\phi'))$ is different from that of the original one (1). Hence they are not equivalent to each other on the contrary to the case of point transformations. Nevertheless, the equivalence between the two theories (1) and (3) still holds (if the latter theory exists at all), because the ghosts compensate[10] the extra modes introduced by the spectrum-changing transformation.

*) This multi-valuedness of $\phi\mapsto\phi'$ is a continuous one, which can be compensated by introducing FP ghosts, as is stated shortly. Transformations containing discrete multi-valuedness such as $\phi=\phi'-2\phi'^2+\phi'^3$ might be problematic.

Furthermore, such generalized transformations as $\phi=f(\phi', \phi'')$ can also be treated in a similar manner. As an example, we investigate gauge transformations in this framework. Let us consider the transition from Coulomb to Landau gauge in the pure abelian gauge theory of a gauge field A_μ, whose Lagrangian is denoted by $\mathcal{L}_A$. This transition can be made by a gauge transformation

$$A_\mu=A'_\mu-\partial_\mu\Lambda\,, \quad \partial_i\partial^i\Lambda+\partial_0 A'^0=0\,, \tag{7}$$

which we regard as a change of the variables A_μ into A'_μ and Λ, and apply the BRS procedure to it. Note here that contrary to the case of usual spectrum-changing transformations,[3] it is not necessary in the present scheme to use an explicit form of Λ in terms of A'_μ. The need for an explicit form of Λ would cause difficulty when one treats the corresponding nonabelian case.

The choice of Coulomb gauge leads to the Lagrangian $\mathcal{L}=\mathcal{L}_A+\mathcal{L}_C$ where $\mathcal{L}_C$ is the following gauge-fixing and FP ghost terms:

$$\mathcal{L}_C=-i\delta(\bar{c}\partial_i A^i)=b\partial_i A^i+i\bar{c}\partial_i\partial^i c\,.$$

Here we have used the BRS transformation law

$$\delta A_\mu=\partial_\mu c\,, \quad \delta\bar{c}=ib\,. \tag{8}$$

The additional BRS transformation for the change of variables (7) is given by

$$\delta A'_\mu=c_\mu\,, \quad \delta\bar{c}^\mu=ib^\mu\,; \quad \delta\Lambda=C-c\,, \quad \delta\bar{C}=i(B+b)\,. \tag{9}$$

The FP ghost and NL field corresponding to the new variable Λ were denoted by $C-c$ and $B+b$, respectively, for later convenience. In order to impose the relation (7), we add a BRS gauge-fixing term

$$\begin{aligned}\mathcal{L}_B&=-i\delta[\bar{c}^\mu(A_\mu-A'_\mu+\partial_\mu\Lambda)+\bar{C}(\partial_i\partial^i\Lambda+\partial_0 A'^0)]\\ &=b^\mu(A_\mu-A'_\mu+\partial_\mu\Lambda)-i\bar{c}^\mu(c_\mu-\partial_\mu C)\\ &\quad+(B+b)(\partial_i\partial^i\Lambda+\partial_0 A'^0)+i\bar{C}(\partial_i\partial^i\{C-c\}+\partial_0 c^0)\end{aligned} \tag{10}$$

to the original Lagrangian $\mathcal{L}$. Integrating out the fields b^μ, A_μ, $\bar{c}^\mu$ and c_μ successively, we are left with the transformed Lagrangian

$$\mathcal{L}_T=\mathcal{L}_{A'}+b\partial_\mu A'^\mu+i(\bar{c}-\bar{C})\partial_i\partial^i c+B(\partial_i\partial^i\Lambda+\partial_0 A'^0)+i\bar{C}\partial_\mu\partial^\mu C\,, \tag{11}$$

where $\mathcal{L}_{A'}$ is the same as $\mathcal{L}_A$ except for A_μ replaced by A'_μ. Moreover we can integrate out c, $\bar{c}$, B and Λ sequentially to obtain the Lagrangian

$$\mathcal{L}'_T=\mathcal{L}_{A'}+b\partial_\mu A'^\mu+i\bar{C}\partial_\mu\partial^\mu C$$

with the reduced BRS transformation law (corresponding to (4) derived in the previous section)

$$\delta A'_\mu=\partial_\mu C\,, \quad \delta\bar{C}=ib\,. \tag{12}$$

This exactly corresponds to the choice of Landau gauge, where the gauge-fixing and FP ghost terms are given by $\mathcal{L}_L=-i\delta(\bar{C}\partial_\mu A'^\mu)$ in terms of the BRS transformation

(12).

Inclusion of matter fields and/or extensions to nonabelian cases are straightforward, and we omit to present them here. (See the treatment of nonabelian gauge theory in § 5.)

§ 4. Gaugeon formalism

In this section, we consider an application of our scheme to a subject other than field transformation. This gives an example which shows the utility of the method as a book-keeping device for introducing new fields into a Lagrangian without affecting the dynamical content of the system. We provide a natural derivation of the gaugeon formalism[4] with a slight improvement.

The gaugeon formalism was proposed in order to accommodate gauge transformations to gauge theories with linear covariant gauge-fixings. Quantum gauge theories with different gauge choices are described in totally different Hilbert spaces. Hence one cannot perform quantum gauge transformations within one theory subject to one particular gauge-fixing. Nevertheless, the gaugeon formalism enables one to make quantum gauge transformations among covariant gauges with the help of a gaugeon field G which behaves as a gauge-transformation function.

For simplicity, we also deal with the case of pure abelian gauge theory considered in the previous section. The gauge-fixing and FP ghost terms to be added to $\mathcal{L}_A$ in a covariant gauge are given by

$$\mathcal{L}_\alpha=-i\delta\left[\bar{c}\left(\partial_\mu A^\mu+\frac{1}{2}\alpha b\right)\right]=b\partial_\mu A^\mu+\frac{1}{2}\alpha b^2+i\bar{c}\partial_\mu\partial^\mu c\,,$$

in terms of the BRS transformation (8), where α is a gauge parameter. Introducing a new field G and its BRS transformation law

$$\delta G=C\,,\quad \delta\bar{C}=iB\,,$$

we can add another term

$$\mathcal{L}_G=-i\delta\left[\bar{C}\left(\partial_\mu\partial^\mu G+\frac{1}{2}b\right)\right]=B\left(\partial_\mu\partial^\mu G+\frac{1}{2}b\right)+i\bar{C}\partial_\mu\partial^\mu C$$

to the Lagrangian $\mathcal{L}_A+\mathcal{L}_\alpha$. The total Lagrangian

$$\mathcal{L}_T=\mathcal{L}_A+\mathcal{L}_\alpha+\mathcal{L}_G \qquad (13)$$

is easily seen to be covariant under a (quantum) gauge transformation

$$A_\mu=A'_\mu-\beta\partial_\mu G\,,\quad \bar{C}=\bar{C}'+\beta\bar{c}$$

with their BRS partners $c=c'-\beta C$, $B=B'+\beta b$ (and the other fields are intact), where β is an arbitrary constant. That is to say, $\mathcal{L}_T$ changes to

$$\mathcal{L}'_T=\mathcal{L}_{A'}+\mathcal{L}_{\alpha+\beta}+\mathcal{L}_G$$

under the above transformation. Therefore the transition between two different gauges, $\mathcal{L}_\alpha$ and $\mathcal{L}_{\alpha+\beta}$, can be made directly by a quantum gauge transformation in the gaugeon formalism (13). We note that the original formalism given ad hoc in Ref. 4) needs old-fashioned subsidiary conditions à la Gupta-Bleuler,[10] while the present form (13) is clearly capable of being implemented by Kugo-Ojima's subsidiary condition[1),10)] to insure physical unitarity.

§ 5. BRS gauge-fixing by field transformation

In the preceding sections, we mainly concern ourselves with the way of performing field transformations by means of BRS gauge-fixing. In this section on the other hand, we reconsider gauge-fixing of (nonabelian) gauge theories from the viewpoint of field transformation.

Gauge-fixing in a gauge theory is a gauge transformation in nature. Let G be a compact Lie group and $\mathcal{G}$ its Lie algebra. We consider the Yang-Mills Lagrangian $\mathcal{L}_A$ of a gauge field A_μ taking values in $\mathcal{G}$. The choice of covariant gauge means that one performs a field transformation*) from A_μ into A'_μ and g such that

$$A_\mu = gA'_\mu g^{-1} + i(\partial_\mu g)g^{-1}, \quad \partial_\mu A'^\mu = 0, \tag{14}$$

where g is a field taking values in G, and that one uses the transformed field A'_μ instead of the original one A_μ.

We can make this transformation along the lines of the BRS procedure proposed in § 3. We first introduce the following BRS transformation:

$$\delta A'_\mu = g^{-1}c_\mu g, \quad \delta\bar{c}^\mu = ib^\mu;$$

$$-ig^{-1}\delta g = c, \quad \delta\bar{c} = ib.$$

Here we have written the ghost corresponding to g as igc so as to make c be $\mathcal{G}$-valued. Then, to impose the relation (14), we add a term

$$\begin{aligned}\mathcal{L}_B &= -i\delta[\bar{c}^\mu(A_\mu - gA'_\mu g^{-1} - i(\partial_\mu g)g^{-1}) + \bar{c}\partial_\mu A'^\mu] \\ &= b^\mu(A_\mu - gA'_\mu g^{-1} - i(\partial_\mu g)g^{-1}) - i\bar{c}^\mu(c_\mu - g(D_\mu c)g^{-1}) - i\delta[\bar{c}\partial_\mu A'^\mu]\end{aligned}$$

to the original Lagrangian $\mathcal{L}_A$, where $D_\mu c = \partial_\mu c + i[A'_\mu, c]$.

Integrating out the fields b^μ, A_μ, $\bar{c}^\mu$ and c_μ successively, we are left with

$$\mathcal{L}_T = \mathcal{L}_{A'} - i\delta(\bar{c}\partial_\mu A'^\mu) = \mathcal{L}_{A'} + b\partial_\mu A'^\mu + i\bar{c}\partial_\mu D^\mu c. \tag{15}$$

The reduced BRS transformation (obtained in an analogous way to the cases (4) and (12)) is given by

$$\delta A'_\mu = D_\mu c, \quad -ig^{-1}\delta g = c, \quad \delta\bar{c} = ib. \tag{16}$$

The expression (15) is nothing other than the total Lagrangian of Yang-Mills theory in the Landau gauge. We note that the form of the BRS transformation $-ig^{-1}\delta g = c$ clearly indicates that the FP ghost c is a Maurer-Cartan form on the group of gauge transformations.[10] It immediately leads to the following BRS transformation

*) If G is nonabelian, this gauge transformation seems to have discrete multi-valuedness, which corresponds to the Gribov ambiguity[11] present in the usual gauge-fixing.

law: $\delta c=-(i/2)[c, c]$. This is automatically consistent with the nilpotency of δ and the transformation law $\delta A'_\mu = D_\mu c$.

It is remarkable that we have never fixed the original gauge degree of freedom of the field A_μ in the above procedure of 'gauge-fixing', as is clear from $\delta A_\mu=0$. The Lagrangian (15) still has the field g as one of its variables, though g does not appear in it explicitly because of the original gauge invariance of $\mathcal{L}_A$. We only make a change of field variables A_μ into A'_μ and separate the gauge degree of freedom g.

The gauge-fixing procedure proposed above can be applied in an analogous manner to open gauge theories, which we study in the next section.

§ 6. BRS gauge-fixing of the Siegel superpoint

Respecting locality and covariance which play important roles in relativistic field theories, gauge-invariant systems generally have open gauge-transformation generator algebras.[12] Unfortunately, open gauge theories are beyond the scope of the ordinary Lagrangian BRS gauge-fixing procedure,[1] which is a transparent method for covariant treatment of closed gauge theories.*)

The Lagrangian extended BRS formalism of Batalin and Vilkovisky[14] is generic enough to provide gauge-fixed actions for open gauge theories with on-shell nilpotent BRS transformations. However, in contrast to the simple BRS gauge-fixing procedure, it is an implicit method in the point that the gauge-fixed action is obtained through solving the master equation.**)

In this section, we consider non-extended BRS gauge-fixing method for open gauge theories in the spirit of Ref. 1). Namely, we perform a direct and explicit construction of the gauge-fixed action by defining off-shell nilpotent BRS transformations and adding BRS-exact terms to the action. We apply the gauge-fixing procedure proposed in the previous section to the Siegel superpoint particle[5] as a toy model which has open gauge algebras of simple forms. Off-shell nilpotency is built in the procedure itself. Gauge-fixing and ghost terms are incorporated into BRS-exact form, as is the case for theories with closed algebras. We see that the present scheme provides a clear-cut understanding on the origin of higher-ghost terms[16] appearing in the resultant gauge-fixed theory.

Let us consider the Siegel superpoint in four dimensions, for definiteness, which is a supersymmetric extension of a relativistic point particle. Its action reads***)

$$S=\int_0^1 d\tau L\,;\quad L=\dot{x}p-\frac{1}{2}ep^2+\bar{\pi}\dot{\theta}-i\bar{\pi}\not{p}\psi\,, \tag{17}$$

where π, θ and ψ are Majorana fermions, dots denote the τ-derivative, $\bar{\pi}$ is the Dirac

*) Algebra closure is a necessary and sufficient condition for the existence of off-shell nilpotent BRS transformation.[13]

**) Roughly speaking, the Batalin-Vilkovisky master equation is the Ward-Takahashi identity stemming from the BRS symmetry for the effective action with the source fields in the latter corresponding to the antifields in the former.

***) The expression given in Ref. 5) is obtained from the Lagrangian below through a field redefinition $e\to e-2\bar{\theta}\psi$ and $\pi\to\pi+i\not{p}\theta$.

conjugate of π, and $\not{p}=\gamma p$. This action is invariant*) under the following gauge transformation:

$$x=x'-\xi p-i\bar{\pi}'\gamma\kappa\,,\quad e=e'-\dot{\xi}-2a\bar{\kappa}(\dot{\theta}'-i\not{p}\psi')\,,$$
$$\pi=\pi'-ap^2\kappa\,,\quad \theta=\theta'-i\not{p}\kappa\,,\quad \psi=\psi'-\dot{\kappa}\,;$$
$$\kappa(0)=\kappa(1)=\xi(0)=\xi(1)=0\,, \tag{18}$$

where a is an arbitrary real constant. We note that the terms proportional to a in this transformation constitute an on-shell trivial symmetry by themselves. The generator algebra of the above gauge transformation is closed only when $a=0$. When $a\neq 0$, the algebra is open and it is impossible[5] to apply straightforwardly the scheme described in Ref. 1). But in the new setting below, necessary procedure is much the same as that in the closed algebra case considered in the previous section.

We introduce the following BRS transformation:

$$\delta\xi=C\,,\quad \delta C_*=iB\,;$$
$$\delta x'=C_x\,,\quad \delta C_{x*}-iB_x\,;\quad \delta e'=C_e\,,\quad \delta C_{e*}=iB_e\,;$$
$$\delta\kappa=ic\,,\quad \delta c_*=b\,;\quad \delta\pi'=ic_\pi\,,\quad \delta c_{\pi*}=b_\pi\,;$$
$$\delta\theta'=ic_\theta\,,\quad \delta c_{\theta*}=b_\theta\,;\quad \delta\psi'=ic_\psi\,,\quad \delta c_{\psi*}=b_\psi\,.$$

Notice that the antighosts are denoted by C_*, c_* and so forth throughout this section. ($\bar{c}$ means the Dirac conjugate of c.) Then we add a term

$$L_B=-i\delta[C_{x*}(x-x'+\xi p+i\bar{\pi}'\gamma\kappa)+C_{e*}(e-e'+\dot{\xi}+2a\bar{\kappa}(\dot{\theta}'-i\not{p}\psi'))$$
$$+\bar{c}_{\pi*}(\pi-\pi'+ap^2\kappa)+\bar{c}_{\theta*}(\theta-\theta'+i\not{p}\kappa)+\bar{c}_{\psi*}(\psi-\psi'+\dot{\kappa})+\dot{C}_*e'+i\dot{\bar{c}}_*\psi']$$

to the original Lagrangian L to implement the relation (18). Integrating out the appropriate variables, we are left with

$$S_T=\int_0^1 d\tau L_T\,;\quad L_T=L'-i\delta(\dot{C}_*e'+i\dot{\bar{c}}_*\psi')\,,$$

where L' is the same as L except for x, e, π, θ and ψ replaced by the corresponding primed variables.

Difference from the closed algebra case lies in the reduced BRS transformation law obtained in a similar manner to the one (16) in the previous section:

$$\delta x'=Cp-ap^2\bar{c}\gamma\kappa+\bar{\pi}'\gamma c\,,\quad \delta e'=\dot{C}+2ia\bar{c}(\dot{\theta}'-i\not{p}\psi')+2a\bar{\kappa}\not{p}c\,,$$
$$\delta\pi'=iap^2c\,,\quad \delta\theta'=-\not{p}c\,,\quad \delta\psi'=i\dot{c}\,,\quad \delta\kappa=ic\,,\quad \delta C_*=iB\,,\quad \delta c_*=b\,. \tag{19}$$

The BRS transform of the variable e' explicitly contains the gauge-transformation function κ due to the algebra nonclosure. This results in troublesome non-decoupling

*) There are some subtleties concerning actions due to surface terms and boundary conditions in connection with gauge choices to be made.[15] We impose the vanishing boundary conditions on the gauge-transformation functions to freely discard total derivatives containing them which appear in Lagrangians.

of the gauge degree of freedom κ from the Lagrangian L_T:

$$L_T=L'+\dot{B}e'+i\dot{C}_*[\dot{C}+2ia\bar{c}(\dot{\theta}'-i\not{p}\psi')+2a\bar{\kappa}\not{p}c]+\dot{\bar{b}}\psi'+i\dot{\bar{c}}_*\dot{c}\ .$$

As a matter of fact, the form of the above Lagrangian allows us to 'sweep away' the variable κ from it. Namely, we make a field redefinition

$$x''=x'-2ia\dot{C}_*\bar{\kappa}\gamma c \tag{20}$$

to get the expression

$$L_T=L''+\dot{B}e'+i\dot{C}_*[\dot{C}+2ia\bar{c}(\dot{\theta}'-i\not{p}\psi')]+\dot{\bar{b}}\psi'+i\dot{\bar{c}}_*\dot{c}\ , \tag{21}$$

where L'' is the same as L' except for x' replaced by x''. The BRS transformation law (19) yields that of the variable x'' as follows:

$$\delta x''=Cp-a(p^2-2\dot{B})\bar{c}\gamma\kappa+\bar{\pi}'\gamma c-2a\dot{C}_*\bar{c}\gamma c\ .$$

Note that the variable κ still appears in the BRS transformation, and it maintains the off-shell nilpotency.

Because the variable κ, which is absent from the Lagrangian (21), is a pure gauge degree of freedom, we can set $\kappa=0$ without affecting the dynamics on shell. Then we obtain the total theory (21) with the following BRS transformation law:

$$\delta x''=Cp+\bar{\pi}'\gamma c-2a\dot{C}_*\bar{c}\gamma c\ ,\quad \delta e'=\dot{C}+2ia\bar{c}(\dot{\theta}'-i\not{p}\psi')\ ,$$

$$\delta\pi'=iap^2c\ ,\quad \delta\theta'=-\not{p}c\ ,\quad \delta\psi'=i\dot{c}\ ,\quad \delta C_*=iB\ ,\quad \delta c_*=b\ . \tag{22}$$

This is a symmetry of the Lagrangian (21) by construction. However, it has lost the off-shell nilpotency and is nilpotent only on shell. It is clear that the cubic ghost term in $\delta x''$ just stems from the field redefinition (20) we performed.

We can further simplify the form of the Lagrangian by means of a field redefinition

$$\pi''=\pi'-2a\dot{C}_*c\ .$$

We then arrive at the following expression that is independent of the parameter a introduced in the transformation (18):

$$L_T=L'''+\dot{B}e'+i\dot{C}_*\dot{C}+\dot{\bar{b}}\psi'+i\dot{\bar{c}}_*\dot{c}\ , \tag{23}$$

where L''' is the same as L'' except for π' replaced by π''. The BRS transformation law (22) now provides a simple one

$$\delta x''=Cp+\bar{\pi}''\gamma c\ ,\quad \delta e'=\dot{C}+2ia\bar{c}(\dot{\theta}'-i\not{p}\psi')\ ,$$

$$\delta\pi''=ia(p^2-2\dot{B})c\ ,\quad \delta\theta'=-\not{p}c\ ,\quad \delta\psi'=i\dot{c}\ ,\quad \delta C_*=iB\ ,\quad \delta c_*=b\ .$$

We note that this looks like a naive modification of the original transformation (18) with the terms proportional to a also constituting an on-shell trivial symmetry by themselves.

The above procedure can be applied to the on-shell reducible case in a similar way.[17] This is no accident, since open algebras are none other than on-shell closed algebras.[12)]

§ 7. Conclusion

We investigated generalized field transformations in Lagrangians by means of the BRS procedure. That was shown to provide a useful book-keeping device for introducing new fields into a Lagrangian without changing the dynamics of the system. It led, for example, to a natural derivation of the gaugeon formalism (13) straightforwardly. Gauge-fixing was reconsidered in this framework, and the simple BRS gauge-fixing method[1)] was generalized to be applicable to generic gauge theories including those with open gauge algebras. We performed simple BRS gauge-fixing of the Siegel superpoint (17) as an example which has open gauge algebras. The present procedure clarified the origin of higher-ghost terms[16)] in the gauge-fixed theory.

In addition to the examples we have considered so far, there remain various possible applications of the method explored in this paper, since generalized field transformation is quite a generic tool to investigate Lagrangian field theories in a kinematical manner. For example, the collective-coordinate method[18)] can be systematically developed in this framework, and the superfield formulation of stochastic quantization[19)] is nothing other than that of the BRS symmetry arising in the Lagrangian formalism of stochastic quantization. We hope that the BRS framework provides a simple and unified view on field-theoretical 'identities' derived separately in many other approaches.

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