

# Simple Computational Method of Predicting Electrical Characteristics in Solar Cells

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**Abstract**—A precise single-diode analytical model of Photovoltaic (PV) modules is generally nonlinear analytical equations with high computation complexity. This paper proposes a simple computational method for the PV model. Its exponential diode behaviour is simplified via Least Squares Approximation (LSA), which permits designers or engineers to predict the electrical characteristics of a PV module without using iterative numerical calculations. Based on experimental data, the methodology is validated by covering a wide range of operation conditions. Simulation and experimental results prove the effectiveness and accuracy of the proposed method.

**Index Terms**—Analytical models, photovoltaic cells, least square approximation.

## I. INTRODUCTION

Owing to the deterioration of the environmental pollution and the growth of fossil fuel price, solar energy has been encouraged to substitute conventional energy. A photovoltaic (PV) generator, known as one of the green alternative energy sources, converts solar energy into direct current electricity via the photovoltaic effect [1], [2].

Recently, modelling electrical characteristics of a shadowed PV generator has become a central issue since it was reported that the maximum power losses are magnified nonlinearly [3], [4]. Jewell and Unruh [5] demonstrated that the output power of a shadowed PV generator is much less than designed. Moreover, the shaded part of a solar PV array may cause “hot spot” and damage the solar cells [1]. Many measures therefore were proposed to lighten and overcome these undesired effects. The bypass diodes, which are usually connected across part of the series strings, are frequently used to prevent the shadowed cells under reverse bias. However, the electrical characteristic curve exhibits multiple peaks as long as the PV panels are under non-uniform insolation. With the emulated PV modules, Particle Swarm Optimization (PSO) algorithms are introduced in [6], [7] to extract the Global Maximum Power Point (GMPP). An alternate approach of reducing the effect of shadows on a solar PV

array’s output power is to adaptively reconfigure solar array by a real-time model-based algorithm [8]. All these intelligent PV systems drive scientists to develop a simple and accurate soft-computing method for predicting the electrical characteristics of solar PV modules.

Conventional PV models are generally analytical equations based on a physical description formulating photovoltaic generated current ( $I$ ) with the operating voltage ( $V$ ), the ambient temperature ( $T$ ) and the irradiance ( $G$ ). Saloux [9] reported that a nonlinear implicit relationship exists between  $I$  and  $V$ , but normally necessitates using tedious iterative numerical calculations. Moreover, estimation techniques are required to determine the parameters necessary to model an accurate PV cell or generator (e.g. five-parameter PV model [10]). These numerical and estimation methods significantly increase the computation complexity and thus are not feasible for real-time prediction of  $I$ - $V$  characteristics. In [6] and [8], series and/or shunt resistances are ignored to reduce computational effort. However, this kind of simplifications decreases the accuracy of analytical models.

This paper proposes a simple and accurate computational method for determining the electrical characteristics of an  $R_p$ -model. Least Squares Approximation (LSA) is applied to approximate the diode behaviour, which provides a way to obtain  $I$  by solving a closed-form analytical equation. It is worthwhile to remark that the principal motivation of this work is to develop a convenient computation approach for  $R_p$ -models. The sensing techniques, although crucial in practical implementation, thus we avoid the discussion here.

## II. EXISTING ANALYTICAL PV MODELS

Numerous single-diode analytical PV models have been developed to allow simulation for arbitrary time profiles of  $V$ ,  $T$  and  $G$ . As shown in Fig. 1, depending on the quantity of effects taken into account in modelling, PV module models can be classified into three types: a) ideal model, b)  $R_s$ -model and c)  $R_p$ -model. More accurate double-diode and three-diode models can be found in [1] and [11] respectively. However, these models are complicated and will not be discussed in this paper.

An ideal PV model is a linear independent current source in

parallel to a diode [1]. When the p-n junction of a PV module is exposed to incident light, a reverse current, which depends linearly on  $G$  and is also slightly influenced by the  $T$  [12], is generated across the junction. This current is known as photocurrent  $I_{pv}$  and it can be mathematically expressed as

$$I_{pv} = (I_{pvn} + K_i \Delta T) \frac{G}{G_n}, \quad (1)$$

where  $I_{pvn}$  is the photocurrent at Standard Test Conditions (STC) ( $T = 25^\circ\text{C}$ ,  $G = 1000 \text{ W/m}^2$ ) and usually assumed that  $I_{pvn} \approx I_{scn}$ .  $K_i$  is a constant named short circuit current coefficient,  $T = T - T_n$  ( $T_n$  is the nominal temperature), and  $G_n$  is the solar irradiance at STC.

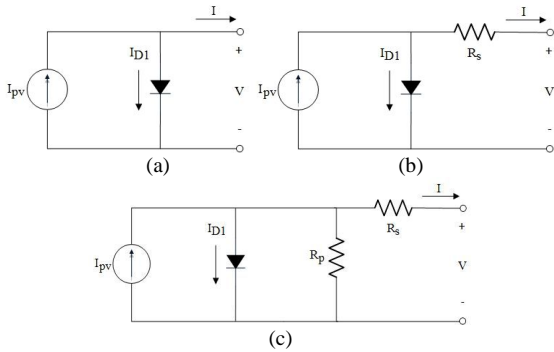


Fig. 1. Different mathematical models for a PV module: (a) ideal model, (b)  $R_s$ -model and (c)  $R_p$ -model.

Eliminated the effect of photocurrent, a PV module behaves like a conventional diode and the I-V characteristics of an ideal PV module can be derived by (2)

$$I = I_{pv} - I_o (e^{\frac{V}{nN_s V_T}} - 1). \quad (2)$$

In this equation,  $n$  is the diode ideality constant and varies between 1 and 2 depending on the material and the physical structure of the diode. The number of series connecting cells in the module is denoted by  $N_s$ .  $V_T$  is a constant called thermal voltage, whose value is a function of  $T$  that can be written as

$$V_T = \frac{kT}{q}, \quad (3)$$

where  $k$  is the Boltzmann constant ( $1.380650 \times 10^{-23} \text{ J/K}$ ) and  $q$  is the electron charge ( $1.602176 \times 10^{-19} \text{ C}$ ).  $I_o$  denotes the saturation current and its value can be described as a nonlinear function [9]

$$I_o = \frac{I_{scn} + K_i \Delta T}{e^{(V_{ocn} + K_i \Delta T)/(nN_s V_T)} - 1}. \quad (4)$$

In (4),  $K_i$  and the open circuit coefficient ( $K_v$ ), as well as the short circuit current and the open voltage at STC ( $I_{scn}$ ,  $V_{ocn}$ ), are normally available in the datasheet.

$R_s$ -model improves the ideal model by recognizing the series resistance ( $R_s$ ) in solar modules, which basically depends on the contact resistance of the metal base with the  $p$  semiconductor layer, the resistances of the  $p$  and  $n$  bodies, the contact resistance of the  $n$  layer with the top metal grid, and

the resistance of the grid [13]. The values of PV current and voltage forming I-V curve are given in (5)

$$I = I_{pv} - I_o (e^{\frac{V + IR_s}{nN_s V_T}} - 1). \quad (5)$$

$R_p$ -model further improves the modelling accuracy by identifying shunt resistance ( $R_p$ ) which exists mainly due to the leakage current of the  $p$ - $n$  junction and depends on the fabrication method of the PV cell [9]. Its I-V relationship is expressed as:

$$I = I_{pv} - I_d - \frac{V + IR_s}{R_p}, \quad (6)$$

$$I_d = I_o (e^{\frac{V + IR_s}{nN_s V_T}} - 1). \quad (7)$$

In the literature [14], (7) can be approximated by the exponential relationship

$$I_d \approx I_o e^{\frac{V + IR_s}{nN_s V_T}}. \quad (8)$$

The approximation values of  $R_s$  and  $R_p$  at STC ( $R_{sn}$ ,  $R_{pn}$ ) can be obtained by an iterative approach described in [8]. Since an illuminated PV module has different electrical behavior, Brano [10] further demonstrated that both  $R_s$  and  $R_p$  vary in almost inverse linear mode with the solar irradiance:

$$R_s(r_G) = \frac{R_{sn} \cdot G_n}{G}, \quad (9)$$

$$R_p(r_G) = \frac{R_{pn} \cdot G_n}{G}. \quad (10)$$

In brief, single-diode PV analytical models express the transcendental I-V characteristic with the sensed data of environment factors (e.g.  $T$ ,  $G$ ) and normally require iterative numerical methods to solve.

### III. ACCURATE MODEL APPROXIMATION

#### A. Least Squares Approximation

Function approximation provides an approach to represent a complicated function  $f(x)$  ( $f(x) \in C[a, b]$ ) by an easier form  $(x; a_0, a_1, \dots, a_n)$ , where  $a_0, \dots, a_n$  are parameters to be determined so as to achieve the best approximation of  $f(x)$ . The term least squares describes a frequently used approach to solving over-determined or inexact specified equations (e.g. transcendental functions, integrals and solutions of differential or algebraic equations) in an approximate sense [15]. Accordingly, LSA can be viewed as finding proper coefficients  $a_0, a_1, \dots, a_n$  so as to

$$\text{Minimize } \| f(x) - w(x; a_0, a_1, \dots, a_n) \|_2, \quad (11)$$

where  $(x; a_0, \dots, a_n)$  is usually a polynomial  $P_n(x)$  of degree at most  $n$ , and the approximation problem can be represented to minimize the error ( $E$ ):

$$E \equiv E(a_0, a_1, \dots, a_n) = \int_a^b (f(x) - P_n(x))^2 dx, \quad (12)$$

$$P_n(x) = a_0 + a_1x + \dots + a_{n-1}x^{n-1} + a_nx^n = \sum_{k=0}^n a_k x^k. \quad (13)$$

With the aim of finding real coefficient  $a_0, a_1, \dots, a_n$ , a necessary condition should be considered is that

$$\frac{\partial E}{\partial a_j} = 0, \text{ for each } j = 0, 1, \dots, n. \quad (14)$$

By applying derivative to (12), we get

$$\frac{\partial E}{\partial a_j} = -2 \int_a^b x^j f(x) dx + 2 \sum_{k=0}^n a_k \int_a^b x^{j+k} dx. \quad (15)$$

After substituting (14) into (15), the linear normal equations

$$\int_a^b x^j f(x) dx = \sum_{k=0}^n a_k \int_a^b x^{j+k} dx, \quad (16)$$

where for each  $j = 0, 1, \dots, n$ , can be derived to solve the unknown coefficients  $a_0, a_1, \dots, a_n$ . It has been proven that the normal equations always have a unique solution provided  $f(x) \in C[a, b]$ .

#### B. PV Model Approximation via LSA

As seen in (6), the analytical expression of the forward I-V characteristics of PV  $R_p$ -model contains a transcendental exponential function for estimating  $I_d$ . The primary aim of using LSA is to represent  $I_d$  by a simple polynomial function. Assuming the operating current varies in the range  $[0, I_{max}]$  ( $I_{max}$  is the upper limit of PV current in datasheet),  $I$  can be written in the following form

$$I = \frac{I_{max} + I_{max} \cdot t}{2}, \quad (17)$$

where  $t \in [-1, 1]$ . Equation (8) can be further expressed as a function with variable  $t$ , and  $I_d$  can be expressed as

$$I_d(t) = S \cdot e^{\Gamma \cdot t}, \quad (18)$$

where  $\Gamma = \frac{R_s \cdot I_{max}}{2n \cdot N_s \cdot V_T}$ ,  $S = I_o \cdot e^{\frac{2V + R_s \cdot I_{max}}{2n \cdot N_s \cdot V_T}}$ . In (18),

and are known as long as the environment factors  $T$  and  $G$  are sampled. In order to approximate (16) in a simple manner, polynomial  $P_n(x)$  is set to one degree as

$$P_1(t) = a_0 + a_1 t. \quad (19)$$

After substituting (19) and (9) into (6), the variable  $t$  can be solved and expressed as

$$t = \frac{R_p \cdot (2I_{pv} - 2a_0 - I_{max}) - 2V - I_{max} \cdot R_s}{2R_p \cdot a_1 + I_{max} \cdot (R_s + R_p)}. \quad (20)$$

$I$  can be calculated directly from (16), namely

$$I = \frac{I_{max}}{2} \cdot \left( 1 + \frac{R_p \cdot (2I_{pv} - 2a_0 - I_{max}) - 2V - I_{max} \cdot R_s}{2R_p \cdot a_1 + I_{max} \cdot (R_s + R_p)} \right). \quad (21)$$

As it has been described in the previous subsection, the normal equation of the  $R_p$ -model is

$$\int_{-1}^1 t^j \cdot S \cdot e^{\Gamma \cdot t} dt = a_0 \int_{-1}^1 t^j dt + a_1 \int_{-1}^1 t^{j+1} dt, \quad (22)$$

where  $j = 0, 1$ . It can be used to determine the coefficients  $a_0$  and  $a_1$ .

$$a_0 = \frac{S}{2\Gamma} (e^{\Gamma} - e^{-\Gamma}), \quad (23)$$

$$a_1 = \frac{3S}{2} \left[ \left( \frac{1}{\Gamma} - \frac{1}{\Gamma^2} \right) \cdot e^{\Gamma} + \left( \frac{1}{\Gamma} + \frac{1}{\Gamma^2} \right) \cdot e^{-\Gamma} \right]. \quad (24)$$

In conclusion, the analytical expression of  $I_d$  is approximated by LSA. Accordingly, the simplified I-V relationships can be formulated as a closed-form equation.  $I$  can be obtained via (19) and (21) after calculating the parameters  $I_{pv}$ ,  $R_s$ , and  $R_p$ , whose estimation methods are described in the previous section.

#### IV. VALIDATION OF THE PROPOSED PV MODEL APPROXIMATION METHOD

Aiming to validate the proposed approximation method, a comparison is made between the simulation results and the experimental data.  $R_s$ -,  $R_p$ -, and the approximated  $R_p$ - model were programmed in MATLAB and were simulated in a general-purpose computer.

TABLE I. THE RELATIVE ERROR OF  $I_{sc}$  AND  $V_{oc}$  UNDER DIFFERENT IRRADIANCE LEVELS ( $T = 25^\circ\text{C}$ ).

G (W/m <sup>2</sup> )	Relative Error at $I_{sc}$ (%)			Relative Error at $V_{oc}$ (%)		
	$R_s^1$	$R_p^2$	$AR_p^3$	$R_s^1$	$R_p^2$	$AR_p^3$
200	2.24	2.16	2.16	2.00	2.30	0.39
400	1.28	1.20	1.20	0.79	0.94	0.82
600	0.61	0.54	0.54	1.11	1.21	0.47
800	0.05	0.02	0.02	0.62	0.70	0.95
1000	0.10	0.17	0.17	0.92	0.98	0.64

Note: <sup>1</sup> $R_s$ -model; <sup>2</sup> $R_p$ -model; <sup>3</sup>Approximated  $R_p$ -model.

TABLE II. THE RELATIVE ERROR OF  $I_{sc}$  AND  $V_{oc}$  UNDER DIFFERENT TEMPERATURE CONDITIONS ( $G = 1000 \text{ W/m}^2$ ).

T (°C)	Relative Error at $I_{sc}$ (%)			Relative Error at $V_{oc}$ (%)		
	$R_s^1$	$R_p^2$	$AR_p^3$	$R_s^1$	$R_p^2$	$AR_p^3$
25	0.17	0.17	0.17	0.92	0.98	0.64
50	0.34	0.27	0.27	0.63	0.69	0.84
75	0.93	0.85	0.85	0.82	0.89	0.60

Note: <sup>1</sup> $R_s$ -model; <sup>2</sup> $R_p$ -model; <sup>3</sup>Approximated  $R_p$ -model.

Figure 2 shows the simulated I-V curves of a KC200GT solar module with the experimental data at five different irradiance levels and three different temperature conditions. The simulation results of  $R_s$ -model, which have been proven to be less accurate than  $R_p$ -model in [1], are not included in the plot to avoid overcrowding of traces. It is interesting to see that the simulated curves of the approximated model are closer to the experimental data in most case.

TABLE III.  $V_{mp}$  OF DIFFERENT MODULE MODELS UNDER STC.

Module Type	Module	$V_{mp}$ (V)			
		ED <sup>1</sup>	R <sub>s</sub> <sup>2</sup>	R <sub>p</sub> <sup>3</sup>	AR <sub>p</sub> <sup>4</sup>
Multi-crystal	MSX60	17.1	17.14	17.12	17.12
	SM55	17.4	17.48	17.45	17.42
	KG200GT	26.3	26.31	26.29	26.31
Monocrystal	SP-70	16.5	16.56	16.55	16.52
Thinfilm	ST-40	16.9	16.64	16.65	16.68

.Note: <sup>1</sup>Experimental data; <sup>2</sup>R<sub>s</sub>-model; <sup>3</sup>R<sub>p</sub>-model; <sup>4</sup>Approximated R<sub>p</sub>-model.

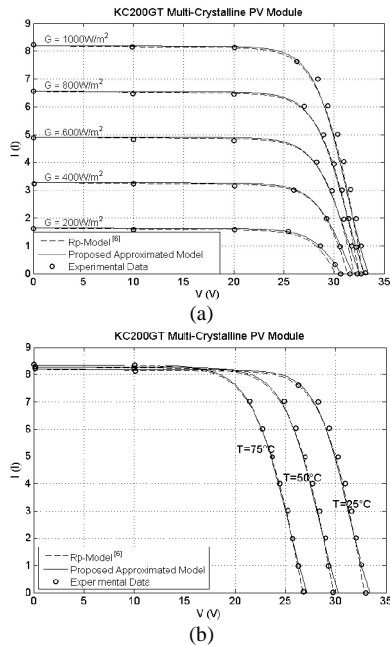


Fig. 2. I-V curves and experimental data of a KC200GT solar module: (a) for several irradiation levels at 1K W/m<sup>2</sup> and (b) for several temperature levels at 25°C.

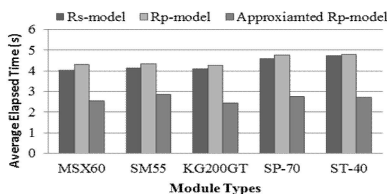


Fig. 3. The average elapsed time of different models (obtained from 10 tests of 2000000 operating points at STC).

Table I and Table II show the relative error of Short Circuit Current ( $I_{sc}$ ) and Open Circuit Voltage ( $V_{oc}$ ) under different environment conditions. Varying with  $T$  and  $G$ , the approximated R<sub>p</sub>-model has the same  $I_{sc}$  as R<sub>p</sub>-model, while it estimates  $V_{oc}$  with low relative errors (within 1 %) in simulation. The Maximum Power Point (MPP), at which PV modules deliver the maximum output power, is another important parameter of PV modules. Table III compares the voltage of MPP ( $V_{mp}$ ) for different modules, including mono-crystalline, multi-crystalline and thin-film types. The values of  $V_{mp}$  obtained by the approximated R<sub>p</sub>-model are close to the ones extracted from R<sub>p</sub>-model, and their relative errors are within 1 %.

Figure 3 further evaluates the computation speed of the proposed model approximation method. The average values of elapsed time are obtained from 10 tests of 2,000,000 operating points at STC. It can be seen that the mean elapsed time of R<sub>p</sub>-model can be reduced by around 40 % with the proposed model approximation method.

## V. CONCLUSIONS

This paper proposes a fast least-squares approximation method for determining electrical characteristics of a PV R<sub>p</sub>-model. This method is able to assess the electrical characteristics of an R<sub>p</sub>-model under any operating temperature and solar irradiance. The novelty about this method is that it uses least-squares regressions to approximate the analytical expression of the leakage current. The performance of the method is evaluated using practical data from the manufacturers and is compared to the simulation outcomes of the popular single diode R<sub>s</sub>- and R<sub>p</sub>- models. Test results have demonstrated that the proposed method reduces the computational effort of a single-diode PV model by around 40 % while its accuracy is similar to precise analytical models.

## REFERENCES

- [1] L. Castaner, S. Silvestre, *Modelling Photovoltaic System using PSpice*, John Wiley & Sons Inc, 2002. [Online]. Available: <http://dx.doi.org/10.1002/0470855541>
- [2] J. Ma, K. L. Man, T. O. Ting, N. Zhang, C. U. Lei, N. Wong, "A hybrid MPPT method for photovoltaic systems via estimation and revision method", *ISCAS*, 2013, in press.
- [3] H. Rauschenbach, "Electrical output of shadowed solar arrays", *IEEE Trans. Electron Devices*, vol. 18, no. 8, pp. 483–490, 1971. [Online]. Available: <http://dx.doi.org/10.1109/T-ED.1971.17231>
- [4] S. Rustemli, F. Dincer, "Modeling of Photovoltaic Panel and Examining Effects of Temperature in Matlab/Simulink", *Elektronika ir elektrotechnika (Electronics and Electrical Engineering)*, vol. 3, no. 109, pp. 35–40, 2011.
- [5] W. Jewell, T. Unruh, "Limits on cloud-induced fluctuation in photovoltaic generation", *IEEE Trans. Energy Conversion*, vol. 5, no. 1, pp. 8–14, 1990. [Online]. Available: <http://dx.doi.org/10.1109/60.50805>
- [6] L. R. Chen, C. H. Tsai, Y. L. Lin, Y. S. Lai, "A biological swarm chasing algorithm for tracking the PV maximum power point", *IEEE Trans. Energy Conversion*, vol. 25, no. 2, pp. 484–493, June 2010. [Online]. Available: <http://dx.doi.org/10.1109/TEC.2009.2038067>
- [7] K. Ishaque, Z. Salam, M. Amjad, S. Mekhilef, "An improved Particle Swarm Optimization (PSO)-based MPPT for PV with reduced steady-state oscillation", *IEEE Tran. Power Electronics*, vol. 27, no. 8, pp. 3627–3638, Aug. 2012. [Online]. Available: <http://dx.doi.org/10.1109/TPEL.2012.2185713>
- [8] D. Nguyen, B. Lehman, "An adaptive solar photovoltaic array using model-based reconfiguration algorithm", *IEEE Trans. Industrial Electronics*, vol. 55, no. 7, pp. 2644–2654, July 2008. [Online]. Available: <http://dx.doi.org/10.1109/TIE.2008.924169>
- [9] M. G. Villalva, J. R. Gazoli, E. R. Filho, "Comprehensive approach to modeling and simulation of photovoltaic arrays", *IEEE Trans. Power Electronics*, vol. 24, no. 5, pp. 1198–1208, 2009. [Online]. Available: <http://dx.doi.org/10.1109/TPEL.2009.2013862>
- [10] V. Lo Brano, A. Orioli, G. Ciulla, A. Gangi, "An improved five-parameter model for photovoltaic modules", *Solar Energy Materials and Solar Cells*, vol. 94, no. 8, pp. 1358–1370, 2010. [Online]. Available: <http://dx.doi.org/10.1016/j.solmat.2010.04.003>
- [11] K. Nishioka, N. Sakitani, Y. Uraoka, T. Fuyuki, "Analysis of multicrystalline silicon solar cells by modified 3-diode equivalent circuit model taking leakage current through periphery into consideration", *Solar Energy Materials and Solar Cells*, vol. 91, no. 13, pp. 1222–1227, 2007. [Online]. Available: <http://dx.doi.org/10.1016/j.solmat.2007.04.009>
- [12] W. D. Soto, S. A. Klein, W. A. Beckman "Improvement and validation of a model for photovoltaic array performance", *Solar Energy*, vol. 80, no. 1, pp. 78–88, 2006. [Online]. Available: <http://dx.doi.org/10.1016/j.solener.2005.06.010>
- [13] F. Lasnier, T. G. Ang, *Photovoltaic Engineering Handbook*, New York: Adam Hilger, 1990.
- [14] A. S. Sedra, K. C. Smith, *Microelectronics Circuit*, Oxford University Press, 2004.
- [15] C. Moler, *Numerical Computing with MATLAB*, SIAM, 2004. [Online]. Available: <http://dx.doi.org/10.1137/1.9780898717952>