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Keywords

computational complexity, continuous phase modulation, least mean squares methods, maximum likelihood sequence estimation, receivers, signal representation

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Simple CPM Receivers Based on a Switched Linear Modulation Model

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***Abstract*—Based on a switched linear modulation model recently developed for continuous-phase modulated (CPM) signal representation and approximation, and incorporated with new phase state symbol definitions, three simple CPM receivers are proposed in this letter. Their performance simulation results and complexity comparison are given using a quaternary 2RC (raised cosine frequency pulse) CPM scheme.**

***Index Terms*—Continuous-phase modulation (CPM), maximum-likelihood sequence estimation (MLSE), Viterbi algorithm.**

I. Introduction

LAURENT's approximation of continuous-phase modulated (CPM) signals as a sum of pulse-amplitude modulated (PAM) pulses [1]–[3] has been an effective tool to construct simplified CPM receivers [5]–[7] with complexity much lower than that of the optimal maximum-likelihood sequence estimation (MLSE) receiver [10]–[12]. Unlike other reduced-complexity receivers based on CPM signal-space dimension reduction [13], [14], both the number of matched filters (MFs) and trellis states are reduced simultaneously in the simplified receivers under Laurent PAM decomposition. However, in the original Laurent PAM decomposition, the data symbols to be demodulated are hidden in a number of pseudosymbols [2] (the coefficients associated with the Laurent PAM pulses). Thus, the metric calculations in the Viterbi algorithms used by these simplified receivers are still complicated. For some rational modulation index CPM signals, there is room for further trellis-state reduction. The noncoherent sequence detection (SD) receivers available in the literature [8], [9] still require a large number of trellis states, which are not computationally efficient, especially for multilevel CPM signals.

By exploiting the switched linear modulation model [4], which removes the pseudosymbols in the CPM signal approximation and minimizes the approximation error in the minimum mean-square error (MMSE) sense, and defining new phase state symbols, simpler SD and symbol-by-symbol detection receivers are proposed in this letter.

II. Switched Linear Modulation Model and Phase State Symbols

Assuming unity signal power, the lowpass-equivalent complex envelope of an M-ary CPM signal can be expressed as $S(t) = e^{j\sum_{n=-\infty}^{\infty} a_n \varphi(t-nT)}$, where the data symbol a_n with symbol interval T belongs to the M-ary alphabet $[\pm 1, \pm 3, \ldots, \pm(M-1)]$, and the phase-shift function $\varphi(t)$ can be any monotonic function for $0 \le t \le LT$, where L is a positive integer, with the property

$$\varphi(t) = \begin{cases} 0, & \text{for } t \le 0 \\ h\pi, & \text{for } t \ge LT \end{cases}$$

where h denotes the modulation index. Using the first-order switched linear modulation model [4], the CPM signal can be approximated by

$$S(t) \approx \sum_{n=-\infty}^{+\infty} a_{0,n-1} h_{a_n}(t-nT) \tag{1}$$

where

$$h_{a_n}(t) = \overline{w}_1(t)\left(e^{ja_n\varphi(t)} - \frac{\sin M\varphi(t)}{M\sin\varphi(t)}\right) \tag{2}$$

is the first-order MMSE incremental pulse

$$\overline{w}_1(t) = \prod_{i=-\infty}^{-1} \frac{\sin M\left[h\pi - \varphi(t-iT)\right]}{M\sin\left[h\pi - \varphi(t-iT)\right]} \prod_{i=1}^{\infty} \frac{\sin M\varphi(t-iT)}{M\sin\varphi(t-iT)} \tag{3}$$

is an MMSE window function, and $a_{0,n}$ is the phase state symbol defined recursively by

$$a_{0,n} = a_{0,n-1} e^{jh\pi a_n}. \tag{4}$$

In the new CPM signal model (1), there is no pseudosymbol any more, and the signal approximation error is minimized under the MMSE criterion [4].

If h is rational, i.e., it can be expressed as $h = q/p$, where p and q are relatively prime integers, then $a_{0,n}$ will have p different values $e^{jh\pi k}$, $k = 0, 1, \ldots, p-1$, when q is even, or $2p$ different values $e^{jh\pi k}$, $k = 0, 1, \ldots, 2p-1$, when q is odd. To reduce the number of trellis states required for SD, we introduce a new phase state symbol b_n, defined recursively by $b_n = b_{n-1} e^{-jh\pi(M-1-a_n)}$, so that it is related to $a_{0,n}$ by

$$a_{0,n} = e^{j(M-1)h\pi n} b_n. \tag{5}$$

It is easily shown that b_n has only p possible values $e^{-j2h\pi k}$, $k = 0, 1, \ldots, p-1$, no matter whether q is even or odd. We refer to b_n as the *relative phase state symbol* since it has a phase difference $(M-1)h\pi n$ relative to the phase state symbol $a_{0,n}$.

If h is irrational, $a_{0,n}$ will have infinite number of values. However, from (4), we see that the phase difference $h\pi a_n$ between two successive phase state symbols $a_{0,n}$ and $a_{0,n-1}$ will have only M possible values. Hence, we define

$$e^{jh\pi a_n} = a_{0,n} a_{0,n-1}^* \tag{6}$$

as the *differential phase state symbol*.

III. Simple CPM Receivers

The received CPM signal over an additive white Gaussian noise (AWGN) channel can be expressed as $r(t) = e^{j\varphi_0} S(t) +$

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$z(t)$, where φ_0 is the carrier phase and $z(t)$ is a zero-mean complex Gaussian noise with double-sided power spectral density N_0. To detect the transmitted data from $r(t)$, three simple receivers are proposed, which are described in the following subsections.

A. Simplified SD Receiver

The first simple CPM receiver is derived from the optimal MLSE receiver implemented using the Viterbi algorithm. Since this receiver searches the minimum Euclidean path through the state trellis, it is only valid for a rational modulation index CPM signal with finite trellis states. Following the well-developed procedure [11], [12] and using the CPM signal approximation (1), the metrics for the surviving sequences up to time $t = NT$ in the Viterbi algorithm are derived as

$$J_N = J_{N-1} + e^{-j\varphi_0} a_{0,N-1}^* \int_{-\infty}^{\infty} r(t) h_{a_N}^*(t - NT)dt. \quad (7)$$

The last term on the right-hand side of (7) represents the metric increment contributed by symbol a_N, which can be further expressed in terms of the relative phase state symbol as $e^{-j\varphi_0} e^{-j(M-1)h\pi(N-1)} b_{N-1}^* \int_{-\infty}^{\infty} r(t) h_{a_N}^*(t - NT)dt$ for trellis-state reduction. With this metric increment expression, the receiver can be constructed as a bank of M MFs with impulse responses $h_{a_N}^*(-t)$, followed by a Viterbi processor with p trellis states (since b_{N-1} has p possible values). If the carrier phase is known, the coherent SD is realized by maximizing the real part of J_N. Otherwise, the noncoherent SD is realized by maximizing the envelope of J_N, so that the factor $e^{-j\varphi_0}$ in the metrics has no effect on the decision. We call this receiver the simplified SD receiver.

B. Simplified Differential SD (DSD) Receiver

The second simple CPM receiver is a noncoherent SD receiver for an irrational modulation index CPM signal. It also serves as an alternative to the noncoherent simplified SD receiver if p is too large. As discussed in Section II, although the phase state symbol $a_{0,n}$ has an infinite number of different values, the differential phase state symbol defined by (6) will only have M different values. Since $a_{0,N-1} = a_{0,N-2} e^{jh\pi a_{N-1}}$ from (4), the metric increment in (7) can be rewritten as $e^{-j\varphi_0} a_{0,N-2}^* e^{-jh\pi a_{N-1}} \int_{-\infty}^{\infty} r(t) h_{a_N}^*(t - NT)dt$. We see that, if $a_{0,N-2}$ is used as a phase state reference and the decision is made on the differential phase state symbol $e^{jh\pi a_{N-1}}$ (so that a_{N-1} is detected at time $t = NT$ with a latency of one symbol interval), it is still possible to have a finite-state SD receiver. This receiver can be constructed as a bank of M MFs with impulse responses $h_{a_N}^*(-t)$, followed by a Viterbi processor with M trellis states. The SD can be realized by maximizing the envelope of J_N so that the factor $e^{-j\varphi_0}$ and any initial value of the phase state symbol $a_{0,N-2}$ will not affect the decision. More specifically, the algorithm can proceed as follows.

- Step 1) Choose any initial value of $a_{0,N-2}$ satisfying $|a_{0,N-2}| = 1$, and initialize all M survival metrics to zero at $N = 0$.
- Step 2) Calculate the outputs of the M MFs at $t = NT$.
- Step 3) Calculate the metric increments according to $a_{0,N-2}$, M possible trellis state values $e^{jh\pi a_{N-1}}$, and M MF outputs, and add the calculated metric increments to the respective survival metrics to have M^2 possible metrics J_N.
- Step 4) Decide a_{N-1} according to the trellis state which leads to the maximum magnitude of J_N.
- Step 5) Decide the M survival metrics which stem from the trellis state associated with the decided a_{N-1}.
- Step 6) Update $a_{0,N-2}$ to $a_{0,N-1}$ using the decided a_{N-1}, then increase N by one and go to Step 2 for the next iteration.

We call this SD technique, which makes decisions on the differential phase state, the differential SD (DSD). It is obviously noncoherent. Accordingly, we call this receiver the simplified DSD receiver.

C. Simplified MF Receiver

The third simple CPM receiver is a symbol-by-symbol detection receiver which performs only matched filtering for demodulation (no SD is required). To determine the received data symbol a_N at time $t = NT$, the main MMSE signal component of a_N, defined by $\bar{S}(t, a_N) = a_{0,N-1} \bar{S}_{a_N}(t - NT)$, where $\bar{S}_{a_N}(t) = \overline{w}_1(t) e^{ja_N\varphi(t)}$ is called the main MMSE complex pulse [4], is used to match the received signal (i.e., to perform correlation). Since the correlation between $\bar{S}(t, a_N)$ and the received signal $r(t)$ after compensating the carrier phase is $\int_{-\infty}^{\infty} e^{-j\varphi_0} r(t) \bar{S}^*(t, a_N)dt = e^{-j\varphi_0} a_{0,N-1}^* \int_{-\infty}^{\infty} r(t) \bar{S}_{a_N}^*(t - NT)dt$, this MF receiver is constructed as a bank of MFs with impulse responses $\bar{S}_{a_N}^*(-t)$, which produce outputs $\int_{-\infty}^{\infty} r(t) \bar{S}_{a_N}^*(t - NT)dt$ at time $t = NT$, followed by a decision maker, which makes decisions on a_N according to the largest correlation value. For coherent receiving, since the carrier phase is known and, according to (5), the phase state symbol $a_{0,N-1}$ has p different values $e^{j(M-1)h\pi(N-1)} e^{-j2h\pi k}$, $k = 0, 1, \ldots, p-1$, the decision variables are defined as $U(k, a_N) = \Re[e^{-j\varphi_0} e^{-j(M-1)h\pi(N-1)} e^{j2h\pi k} \int_{-\infty}^{\infty} r(t) \bar{S}_{a_N}^*(t - NT)dt]$. For noncoherent receiving, since no phase information is required, the decision variables are defined as $U(a_N) = |\int_{-\infty}^{\infty} r(t) \bar{S}_{a_N}^*(t - NT)dt|$. The output symbol a_N is decided based on the largest among the pM decision variables $U(k, a_N)$ for coherent receiving, or the M decision variables $U(a_N)$ for noncoherent receiving. Defining a new window function $\overline{w}_0(t) = \prod_{i=-\infty}^{0} (\sin M[h\pi - \varphi(t - iT)]/M \sin[h\pi - \varphi(t - iT)]) \prod_{i=1}^{\infty} (\sin M\varphi(t - iT)/M \sin \varphi(t - iT))$, we have $\bar{S}_{a_n}(t) = h_{a_n}(t) + \overline{w}_1(t)(\sin M\varphi(t)/M \sin \varphi(t)) = h_{a_n}(t) + \overline{w}_0(t+T)$ from (2). The decision variables can be alternatively calculated using M MFs with impulse responses $h_{a_N}^*(-t)$ for coherent receiving and one additional real-valued MF $\overline{w}_0(-t)$ for noncoherent receiving. This simplified MF receiver is the noninteger modulation index version of the receiver proposed for the integer modulation index CPM signal in [3]. Because of the symmetry property $h_{a_N}(t) = h_{-a_N}^*(t)$ and the constraint $(1/M) \sum_{a_N=-M+1}^{M-1} h_{a_N}(t) = 0$, which can be derived directly from (2), only $M - 1$ real-valued MFs are actually needed to implement the M complex-valued MFs with impulse responses $h_{a_N}^*(-t)$.

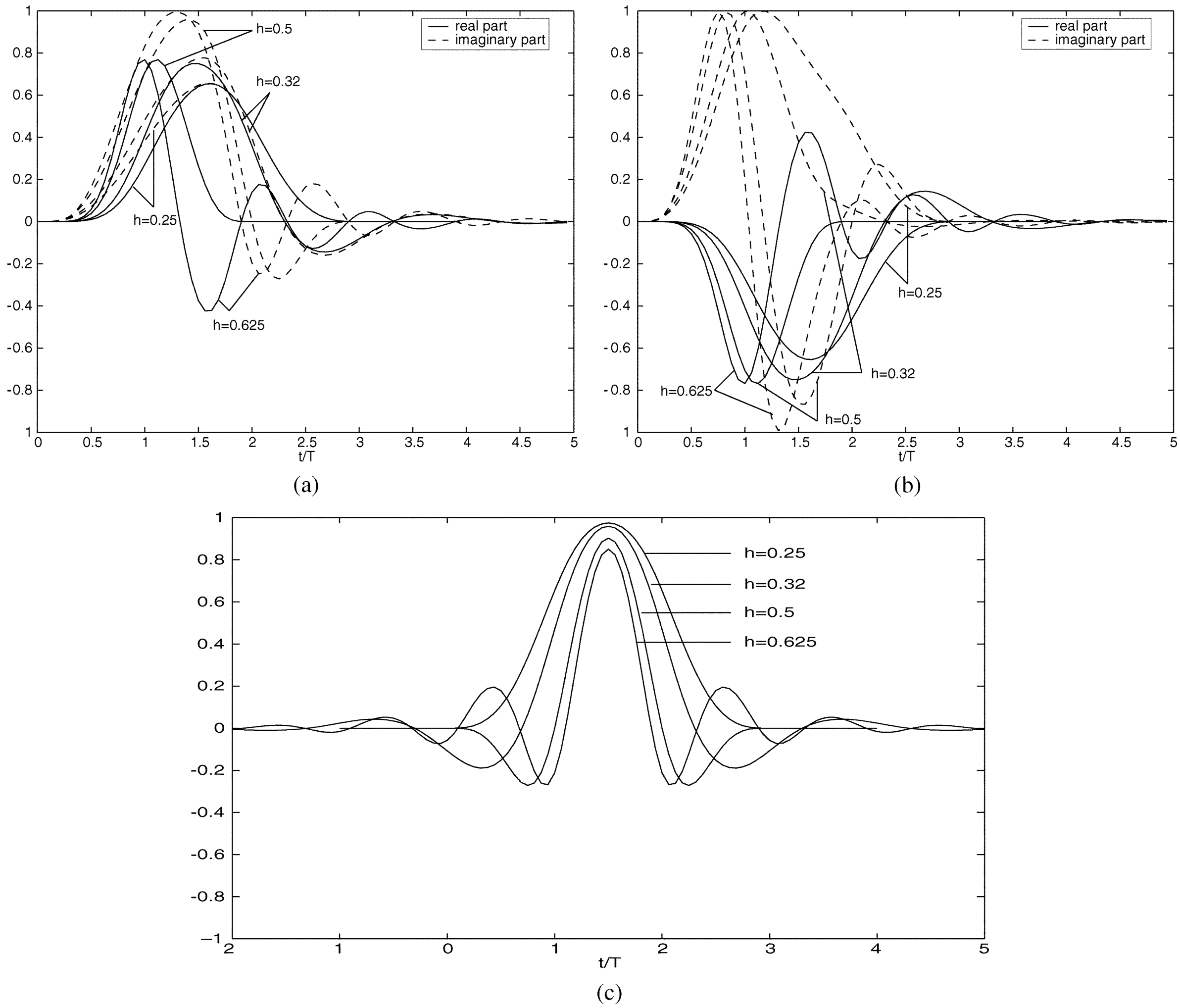

Fig. 1. MMSE incremental pulses and window functions of quaternary 2RC scheme with h =0.25, 0.5, 0.32, and 0.625. (a) $h_{+1}(t)$. (b) $h_{+3}(t)$. (c) $\overline{w}_0(t)$.

IV. Simulation Results and Comparison

The quaternary 2RC scheme (raised cosine frequency pulse with $L = 2$) with different modulation indexes $0.25(= 1/4)$, $0.32(= 8/25)$, $0.5(= 1/2)$, and $0.625(= 5/8)$ are used to evaluate the performance of these simplified receivers. The respective MMSE incremental pulses $h_{a_N}(t)$, $a_N \in [\pm 1, \pm 3]$, and $\overline{w}_0(t)$ are shown in Fig. 1 ($h_{-1}(t)$ and $h_{-3}(t)$ are not displayed because of the symmetry property). For $h = 0.25$ and 0.5, these pulses have finite duration $3T$. For $h = 0.32$ and 0.625, these pulses are of infinite duration, but are truncated to finite duration $4T$ in the simulation. Since $\Re[h_{+3}(t)] = -\Re[h_{+1}(t)]$ for the quaternary CPM signal, only three real-valued pulses, i.e., $\Re[h_{+1}(t)]$, $\Im[h_{+1}(t)]$, and $\Im[h_{+3}(t)]$ are sufficient to represent the four complex-valued MMSE incremental pulses.

Fig. 2 shows the performance of the simplified SD receiver for the quaternary 2RC scheme with $h = 0.25$ and 0.5. The Viterbi algorithms with decision latencies of one symbol $(d = 1)$ and two symbols $(d = 2)$ are tested, respectively. Significant performance improvement is achieved only for $h = 0.25$ by increasing the decision latency. As a comparison, also displayed in Fig. 2 is the performance of the optimal coherent detection of the minimum-shift keying (MSK) scheme, a binary full response $(L = 1)$ CPM with $h = 0.5$.

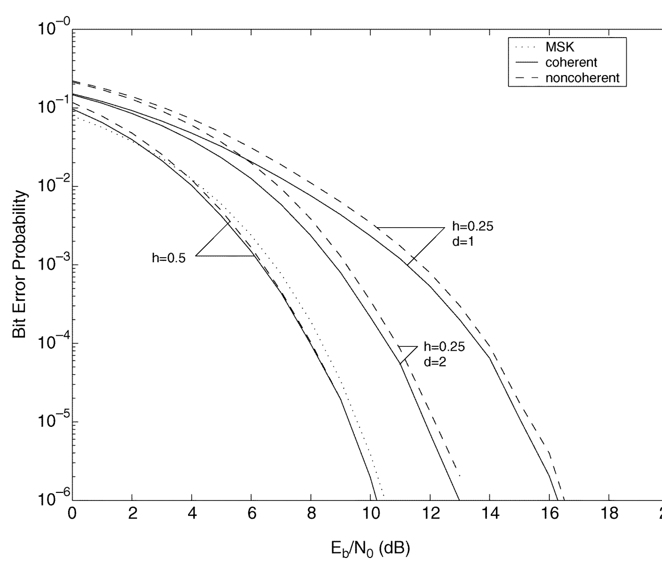

Fig. 2. Performance of the simplified SD receiver for quaternary 2RC scheme with $h = 0.25$ and 0.5.

The performance of the simplified DSD receiver using four trellis states for the quaternary 2RC scheme with $h = 0.25$, 0.32, and 0.625 is shown in Fig. 3. The Viterbi algorithms with

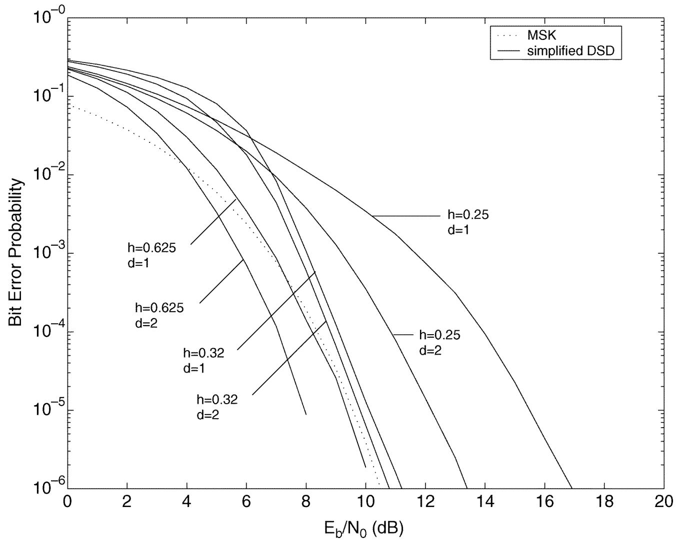

Fig. 3. Performance of the simplified DSD receiver using four trellis states for quaternary 2RC scheme with $h = 0.25$, 0.32, and 0.625.

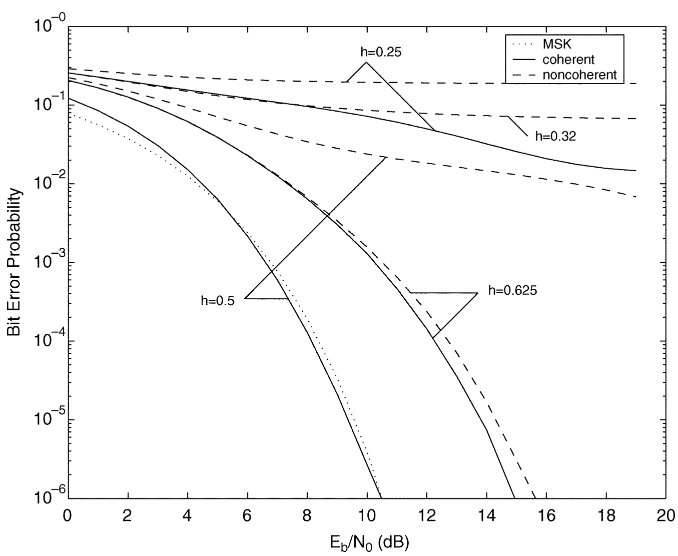

Fig. 4. Performance of the simplified MF receiver for quaternary 2RC scheme with $h = 0.25$, 0.32, 0.5, and 0.625.

different decision delays of one symbol $(d = 1)$ and two symbols $(d = 2)$ are tested, respectively. Compared with the results shown in Fig. 2, we see that the performance of the simplified DSD receiver for $h = 0.25$ is similar to that of the simplified noncoherent SD receiver (both use four trellis states), which indicates that the proposed DSD is indeed an alternative to MLSE noncoherent SD.

Fig. 4 shows the performance of the simplified MF receiver for the quaternary 2RC scheme. For $h = 0.5$, the performance is very close to that of the simplified SD receiver for coherent receiving, but degraded significantly for noncoherent receiving. For $h = 0.625$, some performance degradations are observed for both coherent and noncoherent receivers. For $h = 0.25$ and 0.32, the simplified MF receiver seems not suitable.

It is of interest to know from the above simulation that when suitable receivers are used, the performances for some quaternary 2RC schemes with larger modulation indexes, such as $h = 0.5$ and 0.625, are even better than that of MSK. These results indicate that it is feasible to use multilevel CPM with the reduced-complexity receivers to achieve higher data rates.

V. CONCLUSIONS

It has been shown that the switched linear modulation model for CPM signal approximation combined with new phase state symbol definitions can be exploited to further reduce the complexity of different CPM receivers. For a rational modulation index M-ary CPM signal, the simplified SD receiver always offers the best performance among the proposed three simple receivers, and is also less complicated than the simplified DSD receiver if the required number of trellis states is less than M. For an irrational modulation index M-ary CPM signal, the simplified DSD receiver offers better performance than the noncoherent simplified MF receiver, and is also a low-complexity alternative to the noncoherent SD receiver for some rational modulation index M-ary CPM signals with more than M required trellis states. The simplified MF receiver has the lowest complexity, but it offers the worst performance and is only useful for the CPM signal with a relatively larger modulation index.

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