# Simple Formulas for Transmission Through Periodic Metal Grids or Plates 

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#### Abstract

A simple closed-form approximate solution is given for the transmission coefficient of a normally incident electromagnetic plane wave through a screen made of periodic metal grids (inductive screen), or made of metal plates (the complementary capacitive screen). Explicit formulas are also presented for cascading screens and dielectric slabs. When compared with the exact solution, our approximate simple formulas show good numerical accuracy.


## I. INTRODUCTION

THIS PAPER studies the transmission of electromagnetic waves through a thin screen made of periodic grids or plates shown in Fig. 1. Such a screen finds application in diverse areas; it may be used as an antenna radome [1]-[5], as a microwave frequency selective surface [6]-[9], as a laser mirror [10]-[16], as a solar filter [17]-[20], and as an artificial dielectric [21], [22]. Under the assumptions that the incident field is a plane wave and that the screen is of infinite size, the present transmission problem can be solved rigorously by the standard mode-matching technique [1]-[3], [19]. Unfortunately, the solution is contained in an infinitely large matrix equation, which must be truncated and inverted numerically with the aid of a computer. Hence, it is desirable to develop a simple closed-form solution to the transmission problem. Not only does the simple solution eliminate the need for a complex computer program, it also gives the explicit functional dependence of various design parameters, and allows one to isolate the "cause" and "effect."

Simple formulas for the transmission/reflection coefficient through the screen in Fig. 1 have been reported in the literature. The earliest one was given by MacFarlane in 1946 [23]. Two of the most popular and useful formulas are given by Chen [3] and Ulrich [15]. Comparatively speaking, Chen's formula is more accurate while Ulrich's formula is simpler. In the present paper, we present a refined version which combines the merits of both Chen's and Ulrich's formulas. Furthermore, by using a scattering matrix approach, we extend the formula to cover the case of cascading several screens, which can be the screens shown in Fig. 1, or dielectric slabs.

As expected, our simple formula has the following limitations. 1) The accuracy of the power transmission coefficient is within about 5 percent for most cases of practical interest. 2) The formula is valid only when the incident direction of the plane wave is normal to the screen and the periodicity of the cells in the screen is less than one wavelength. 3) When cascading, the separation between screens/dielectric slabs cannot be very small.

## II. SINGLE SCREEN WITH ZERO THICKNESS

Let us consider the scattering problem sketched in Fig. 1. The incident field from the lower half-space is given by

$$
\begin{equation*}
\vec{E}^{i}(\vec{r})=\hat{u} e^{-j k z}, \quad \text { for } z<0 \tag{1}
\end{equation*}
$$

[^0]

Fig. 1. A metal screen illuminated by a normally incident plane wave.


Fig. 2. Two types of metal screens (top view). (a) Inductive. (b) Capacitive.
where the time factor $\exp (+j \omega t)$ has been suppressed, and $k=2 \pi / \lambda=\omega / c$ is the wavenumber. The unitary vector $u$ satisfies the relation $u \cdot u^{*}=1$ and $u \cdot z=0$. It specifies the polarization of the incident field, e.g., $u=x$ for a linearly polarized field, and $u=(x \pm j y) / \sqrt{2}$ for a circularly polarized field. The screen is made of metal periodic grids/plates, and it can be either of the following two types.

1) Inductive screen (Fig. 2(a)), which reflects at low frequencies and transmits at high frequencies (high-pass screen).
2) Capacitive screen (Fig. 2(b)), which transmits at low frequencies and reflects at high frequencies (low-pass screen).

Because of the periodic nature of the problem, we may represent the scattered field by a double Fourier series (Floquet space harmonics), namely,

$$
\vec{E}(\vec{r})=\left\{\begin{array}{c}
\hat{u} T e^{-j k z}+\hat{u} \sum_{p} \sum_{q} T_{p q} Q_{p q}(x, y) e^{-j \gamma_{p q} z},  \tag{2}\\
\hat{u} R e^{+j k z}+\hat{u} \sum_{p} \sum_{q} R_{p q} Q_{p q}(x, y) e^{+j \gamma_{p q} z}, \\
\\
\text { for } z<0
\end{array}\right.
$$

The double summations in (2) are over the range $p, q=0$, $\pm 1, \pm 2, \cdots$, except $p=q=0$. They represent the so-called "grating lobes." The transverse variation of the ( $p, q$ ) th grating lobe is

$$
Q_{p q}(x, y)=\exp \left[-j \frac{2 \pi}{a}(p x+q y)\right],
$$

and its propagation constant is

$$
\gamma_{\rho q}=\left[k^{2}-\left(\frac{2 \pi}{a}\right)^{2}\left(p^{2}+q^{2}\right)\right]^{1 / 2}
$$

Throughout this paper, we assume that the spacing $a$ is small so that

$$
\begin{equation*}
(a / \lambda)<1 \tag{3}
\end{equation*}
$$

Hence, $\left\{\gamma_{p q}\right\}$ are all negative imaginary, and the fields of the grating lobes decay exponentially away from the screen. Thus, under condition (3), the quantities of practical interest are the transmission coefficient $T$ and the (voltage) reflection coefficient $R$ of the main beam. Their determinations are discussed below.

Circuit Model: As long as the screen has zero thickness ( $\tau=0$ in Fig. 1), the scattering problem in Fig. 1 can be exactly replaced by an equivalent transmission line problem sketched in Fig. 3. The screen is described by a normalized shunt admittance $2 Y$. From the transmission line theory, it is a simple matter to show that

$$
\begin{align*}
& T=\frac{1}{1+Y}  \tag{4a}\\
& F=\frac{-1}{1+(1 / Y)}=T-1 \tag{4b}
\end{align*}
$$

which applies equally to the inductive (Fig. 2(a)) and capacitive (Fig. 2(b)) screens. From the Babinet principle [24], the coefficients of these two complementary screens are related as follows:

$$
\begin{equation*}
T_{\text {cap }}=-R_{\text {ind }}, \quad R_{\text {cap }}=-T_{\text {ind }} \tag{5}
\end{equation*}
$$

From here on, we concentrate on the inductive screen. Let us write the coefficients in polar form

$$
\begin{equation*}
T_{\mathrm{ind}}=\left|T_{\mathrm{ind}}\right| e^{j \theta_{1}}, \quad R_{\mathrm{ind}}=\left|R_{\mathrm{ind}}\right| e^{j \theta_{2}} \tag{6a}
\end{equation*}
$$

Because of the conservation of energy

$$
\begin{equation*}
|T|^{2}+|R|^{2}=1 \tag{7}
\end{equation*}
$$

and the fact $Y_{\text {ind }}=-j\left|Y_{\text {ind }}\right|$, it may be shown that

$$
\begin{align*}
\cos \theta_{1} & =\left|T_{\text {ind }}\right|, & 0 \leqslant \theta_{1} \leqslant(\pi / 2)  \tag{6b}\\
\theta_{2} & =\theta_{1}+(\pi / 2), & (\pi / 2) \leqslant \theta_{2} \leqslant \pi \tag{6c}
\end{align*}
$$

As a check, for $c=0$ in Fig. 2(a) (a perfect conducting plane), we have from (8) that $\theta_{1}=\pi / 2$ and $\theta_{2}=\pi$, as expected.

Approximate Formulas for Admittance The scattering problem in Fig. 1 can be formulated exactly in terms of an infinite set of linear equations [1]-[3]. After truncating the infinite set of equations at a large finite number (say 50 ), it may be numerically solved with the aid of a computer. We call such a solution the "exact solution." By matching the extensive data that we have generated from the exact solution, the following approximate formula for the admittance, referred to. as LZ, is obtained
$Y_{\mathrm{ind}}=Y_{\mathrm{cap}}^{-1} \approx(-j)\left(\beta-\beta^{-1}\right) \frac{\left[\left(\frac{a}{c}\right)+\frac{1}{2}\left(\frac{a}{\lambda}\right)^{2}\right]}{\ln \csc \left(\frac{\pi}{2} \frac{\delta}{a}\right)}$,


Fig. 3. Transmission line model for the scattering problem in Fig. 1 when $\tau=0$.
where

$$
\begin{equation*}
\beta=\left(1-0.41 \frac{\delta}{a}\right) /(a / \lambda), \quad \delta=(a-c) / 2 \tag{8b}
\end{equation*}
$$

Note that $Y_{\text {ind }}$ depends only on two parameters: $a / \lambda$ and $c / a$, and it is independent of the polarization parameter $u$. The particular functional form in (8) is inspired by the work of Ulrich [15]. It is interesting to note that a total transmission ( $Y_{\text {ind }} \rightarrow 0$ and $T_{\text {ind }}=1$ ) occurs at

$$
\begin{equation*}
\frac{a}{\lambda}=1-0.41\left(\frac{\delta}{a}\right) \tag{9}
\end{equation*}
$$

For most practical screens, $(\delta / a) \leqslant 0.3$. Hence, total transmission occurs when $a$ is slightly less than one wavelength.

Other Formulas in the Literature: Based on Marcuvitz's solution [26] for a one-dimensional periodic grid, Ulrich [15] ${ }^{1}$ presents an approximate formula for $Y_{\text {ind }}$, namely,

$$
\begin{equation*}
Y_{\mathrm{ind}} \approx(-j)\left(\beta_{1}-\beta_{1}^{-1}\right) \frac{1}{\ln \csc \left(\frac{\pi}{2} \frac{\delta}{a}\right)}, \quad \text { (Ulrich) } \tag{10a}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta_{1}=\left(1-0.27 \frac{\delta}{a}\right) /(a / \lambda) \tag{10b}
\end{equation*}
$$

When compared with our formula in (8), we note the factor in the square bracket in (8a) is absent in (10a), and $\beta$ in ( $8 b$ ) is slightly different from $\beta_{1}$ in (10b). Another approximate formula is given by Chen [3], namely,

$$
\begin{align*}
Y_{\text {ind }} \approx & (-j) 2 \sqrt{\left(\frac{\lambda}{a}\right)^{2}-1}\left[\frac{\cos \left(\frac{\pi c}{a}\right)^{2}}{1-\left(\frac{2 c}{a}\right)^{2}}\right]^{2} \\
& -\frac{1}{\sqrt{\left(\frac{\lambda}{a}\right)^{2}-1}}\left[\frac{\left.\sin \left(\frac{\pi c}{a}\right)\right]^{2}}{\left(\frac{\pi c}{a}\right)}\right] \\
& +\left[\sqrt{2\left(\frac{\lambda}{a}\right)^{2}-1}-\sqrt{2\left(\frac{\lambda}{a}\right)^{2}-1}\right] \\
& \left.\cdot\left[\frac{1}{\cos \left(\frac{\pi c}{a}\right)^{2}}\right]^{2}\left[\frac{\left.\sin \left(\frac{\pi c}{a}\right)\right]^{2}}{\left(\frac{\pi c}{a}\right)^{2}}\right]^{(\text {(Chen). }}\right] \tag{11}
\end{align*}
$$

${ }^{1}$ A factor 2 in the denominator is missed in [15, eq. (13)].


Fig. 4. Power transmission coefficient $|T|^{2}$ of an inductive screen illuminated by a normally incident plane wave. The screen aperature dimension is $c=0.7 a$ and its thickness $\tau$ is zero.

In [7], [8], Arnaud, Pelow, and Anderson give a formula similar to Ulrich's, namely, ${ }^{2}$

$$
\begin{equation*}
Y_{\mathrm{ind}} \approx(-j) \frac{1}{2\left(\frac{a}{\lambda}\right) \ln \left[\csc \left(\pi \frac{\delta}{a}\right)\right]}, \quad(\mathrm{APA}) \tag{12}
\end{equation*}
$$

The argument of the cosecant function in (12) differs from that in (10a) by a factor of two.

Numerical Results: Results for the transmission coefficient T as a function of $a / \lambda$ for an inductive screen with zero thickness are presented in Fig. 4. The "exact" solution is the numerical solution based on the analysis of [1]-[3]. Four approximate solutions are calculated from (4a) with $Y$ given in (8a), (10a), (11), or (12). For (c/a) $\geqslant 0.7$, both LZ's and Chen's solutions have good accuracy. For (c/a) $<0.7$ (small aperture size), all simple formulas are no longer reliable.

## III. SINGLE INDUCTIVE SCREEN WITH FINITE THICKNESS

All of the formulas in Section II apply to an inductive or capacitive screen with zero thickness ( $\tau=0$ in Fig. 1). When the thickness is not zero, the aperture section of an inductive screen may be considered as a square waveguide. The dominant mode in the waveguide is the transverse electric ( $\mathrm{TE}_{10}$ ) mode whose propagation constant is

$$
\Gamma= \begin{cases}+\sqrt{k^{2}-(\pi / c)^{2}}, & \text { if } c>0.5 \lambda  \tag{13}\\ -j \sqrt{(\pi / c)^{2}-k^{2}}, & \text { if } c<0.5 \lambda\end{cases}
$$

Based on one-mode approximation for the aperture field, Chen [3] found an approximate formula for the transmission coefficient of a thick inductive screen ${ }^{3}$ namely,

$$
\begin{align*}
T_{\mathrm{thick}} \approx & e^{i k \tau}\left[\frac{1}{1+Y_{\mathrm{ind}}-Z \tan (\Gamma \tau / 2)}\right. \\
& \left.-\frac{1}{1+Y_{\mathrm{ind}}-Z \cot (\Gamma \tau / 2)}\right] \tag{14a}
\end{align*}
$$

2 Equations (1) and (2) of [7] contain misprints. Our (12) above is obtained from (2) and (12) of [8].

3 The exponential factor in (14) is missed in [3, eq. (7)].


Fig. 5. Power transmission coefficient $|T|^{2}$ of an inductive screen as a function of screen thickness $\tau$ and $a=0.7 \lambda$.


Fig. 6. Power transmission coefficient $|T|^{2}$ of an inductive screen as a function of $a / \lambda$.
where

$$
\begin{equation*}
Z=i\left(\frac{\pi^{2}}{8}\right)\left(\frac{a}{c}\right)^{2}(\Gamma / k) \tag{14b}
\end{equation*}
$$

For numerical calculations, $Y_{\text {ind }}$ given in (8) and (11) are used in (14). The results are presented in Figs. 5 and 6. We note that (14) is fairly accurate. Two remarks about the thickness effect can be made. 1) When the $\mathrm{TE}_{10}$ mode in the square waveguide section of the screen is below cutoff ( $c<0.5 \lambda$ ), (14a) is approximately equivalent to

$$
\begin{equation*}
T_{\text {thick }} \approx T_{\text {thin }} e^{j(k-\Gamma) \tau} \tag{15}
\end{equation*}
$$

or

$$
\begin{equation*}
\left|T_{\text {thick }} / T_{\mathrm{thin}}\right|^{2} \approx-54.6 \frac{\tau}{\lambda} \sqrt{\left(\frac{\lambda}{2 c}\right)^{2}-1} \mathrm{~dB} \tag{16}
\end{equation*}
$$

TABLE I
RATIO OF $\left|T_{\text {thick }} / T_{\text {thin }}\right|^{2}$

| $a / \lambda$ | $c / \lambda$ | $\tau / \lambda$ | Exact | From (14) | From (16) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.3 | 0.21 | 0.06 | -7.5 dB | -7.5 dB | -7.1 dB |
| 0.3 | 0.24 | 0.06 | -6.0 | -6.0 | -6.0 |
| 0.7 | 0.49 | 0.14 | -1.28 | -1.25 | -1.6 |

The numerical data in Table I demonstrate the accuracy of (16). It is observed from (15) and (16) that, when $c<0.5 \lambda$, the thickness effect introduces the $\mathrm{TE}_{10}$ mode attenuation in the transmission coefficient. 2) When $c>0.5 \lambda$, the total transmission ( $T=1$ ) of a thick screen occurs at a lower frequency than that of a thin screen. As an example, for $c / a=$ 0.7 , the total transmission of a thin screen occurs at $a / \lambda=$ 0.94 , which may be calculated from (9) or observed from the numerical curve in Fig. 6. For the same screen with a finite thickness $\tau=0.2 a$, the total transmission occurs at $a / \lambda=0.85$.

As a final remark, the formulas in (14)-(16) are applicable to an inductive screen, but not to a capacitive screen. In fact, when the screen is thick, relations in (4b), (5), and (6) are no longer valid.

## IV. CASCADING SCREENS AND DIELECTRIC SLABS

In practical applications, we often cascade metal screens and dielectric slabs in order to obtain the desired transmission characteristics and/or mechanical strength. In this section, we provide a formula for calculating the transmission through such a cascade structure.

As sketched in Fig. 7, let us assume that there are $N$ (possibly different) sheets in cascading. For a typical $n$th sheet, its reference plane is located at $z=d_{1}+d_{2}+\cdots+d_{n-1}$. With respect to this reference plane, the sheet is symmetrical and its (transmission, reflection) coefficients are denoted by ( $T_{n}, R_{n}$ ). If the sheet is a metal screen (inductive or capacitive), we may calculate its coefficient by the simple formulas in Sections II and III. If the sheet is a dielectric slab (Fig. 8), its coefficients are given by the well-known expressions,

$$
\begin{align*}
& T_{n}=\frac{\left(1-r^{2}\right)}{1-r^{2} \exp \left(-j 2 k^{\prime} \tau\right)} e^{j\left(k-k^{\prime}\right) \tau}  \tag{17a}\\
& R_{n}=\frac{r\left[1-\exp \left(-j 2 k^{\prime} \tau\right)\right]}{1-r^{2} \exp \left(-j 2 k^{\prime} \tau\right)} e^{j k \tau}, \tag{17~b}
\end{align*}
$$

where $k^{\prime}=k \sqrt{\epsilon}, \epsilon$ is the relative dielectric constant of the slab, and

$$
\begin{equation*}
r=\frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}} \tag{18}
\end{equation*}
$$

When the slab is lossy, $\epsilon$ has a negative imaginary part. The square root $\sqrt{\epsilon}$ should also have a negative imaginary part.

The interaction among the $N$ sheets can be accounted for by using the scattering matrices [26]. To be exact, the matrices are of infinite order. In the present paper, we use the so-called "one-mode interaction." It means that only the main beam, not the grating lobes (the fields represented by the double summation in (2)), is used in calculating the interaction. This approximation is valid when the intersheet distances ( $d_{1}$, $\left.d_{2}, \cdots, d_{N-1}\right)$ are large in terms of wavelength. As shown later by numerical example, good accuracy of the one-mode interaction is maintained for a surprisingly small $d_{n}$.

Using the one-mode interaction, our final results of $(R, T)$


Fig. 7. Cascading of $N$ sheets of metal screens/dielectric slabs.


Fig. 8. Transmission through a dielectric slab.
for the cascading structure in Fig. 7 are

$$
\begin{align*}
& T=A-(B C / D)  \tag{19a}\\
& R=-(C / D) \tag{19b}
\end{align*}
$$

The coefficients ( $A, B, C, D$ ) are calculated in the following steps. First, for each sheet, we determine a $2 \times 2$ scattering matrix $\overline{\bar{S}}_{n}$, where

$$
\begin{align*}
& \overline{\bar{S}}_{n}=\left[\begin{array}{ll}
T_{n}\left(1-\frac{R_{n}^{2}}{T_{n}^{2}}\right) & \frac{R_{n}}{T_{n}} e^{j 2 k l_{n}} \\
-\frac{R_{n}}{T_{n}} e^{-j 2 k l_{n}} & \frac{1}{T_{n}}
\end{array}\right]  \tag{20a}\\
& l_{n}=d_{1}+d_{2}+\cdots+d_{n-1}, \quad n=1,2, \cdots N . \tag{20b}
\end{align*}
$$

Then

$$
\left[\begin{array}{ll}
A & B  \tag{21}\\
C & D
\end{array}\right]=\overline{\bar{S}}_{N} \overline{\bar{S}}_{N-1} \cdots \overline{\bar{S}}_{3} \overline{\bar{S}}_{2} \overline{\bar{S}}_{1}
$$

For the special case $N=2,(19)$ is simplified to become

$$
\begin{align*}
T & =\frac{T_{1} T_{2}}{1-R_{1} R_{2} \exp \left(-j 2 k d_{1}\right)}  \tag{22a}\\
R & =R_{1}+\frac{T_{1}^{2} R_{2}}{1-R_{1} R_{2} \exp \left(-j 2 k d_{1}\right)} e^{-j 2 k d_{1}} \tag{22b}
\end{align*}
$$

We emphasize that in applying the formulas in (19)-(22), $R_{n}$ is the reflection coefficient of the screen $n$ in reference to the plane $z=d_{n}$ (Fig. 7), namely, the ratio of the reflected and incident electric fields at $z=d_{n}$. One should not use reflection coefficients which refer to other planes. Another remark concerning the present cascading formula is in order.


Fig. 9. Power transmission coefficient $|T|^{2}$ of a double inductive screen with spacing $d_{1}=0.5 a$.

Within the approximation of the one-mode interaction, the final result in (19) is independent of the relative horizontal position of the screens. In other words, as long as the spacings $d_{1}, d_{2}, \cdots$, are maintained in Fig. 7, (19) remains valid when screens are slid or rotated in their respective horizontal planes. In practical applications, sliding or rotating may be used for the suppression of higher order space harmonics and/or the cross polarization.

Double Screen: Figs. 9 and 10 show the transmission coefficients of a double inductive screen with interscreen spacing $d_{1}=0.5 a$, and $0.2 a$. We note that the present "one-mode" formula in (22) is accurate only when $d_{1}>0.5 a$.

Thin Double Screen Approximated by Thick Single Screen: When the spacing $d_{1}$ of a double screen is small, the present "one-mode" formula in (22) is no longer accurate, as seen from Fig. 10. However, such a double screen can be wellapproximated by a single screen with its thickness $\tau$ equal to $d_{1}$. This approximation holds well for $a<\lambda$ (Fig. 10). In conclusion, in cascading two identical screens, (22) applies when spacing $d_{1}$ is large, and the thick screen approximation applies when $d_{1}$ is small.

Single Screen on Dielectric Slab: The "one-mode" formula in (22) remains reasonably accurate for spacing $d_{1}=0.1 a$ (Fig. 11). However, it fails to predict the rapid oscillation near $a \approx \lambda$ when $d_{1}$ is reduced to zero.

Cascading Complementary Screens: The two screens (both of zero thickness) in Fig. 2 are complementary, i.e., when one properly lies on top of the other, they form an infinite screen with no perforation. Their transmission and reflection coefficients are related in the manner described in (4). Now, let us consider the two cascading complementary screens. The transmission coefficient of the composite screen is obtainable from (22a) and (5), namely,

$$
\begin{equation*}
T=\frac{-T_{\text {ind }} R_{\text {ind }}}{1+T_{\text {ind }} R_{\text {ind }} \exp \left(-j 2 k d_{1}\right)} \tag{23}
\end{equation*}
$$



Fig. 10. Power transmission coefficient $|T|^{2}$ of a double inductive screen with spacing $d_{1}=0.2 a$ and a thick signel screen with $\tau=$ $0.2 a$.


Fig. 11. Power transmission coefficient $|T|^{2}$ of an inductive screen and a dielectric slab with spacing $d_{1}=0.1 a$.
wavelength $\lambda_{0}$

$$
\begin{equation*}
\left|Y_{\text {ind }}\right|=1, \quad \text { when } \lambda=\lambda_{0} \tag{24a}
\end{equation*}
$$

Substitution of (24a) into (4a) gives

$$
\begin{equation*}
T_{\mathrm{ind}}=0.707 e^{j 45^{\circ}}, \quad R_{\mathrm{ind}}=0.707 e^{j 135^{\circ}} \tag{24b}
\end{equation*}
$$

Let us choose the spacing between the screens such that

$$
\begin{equation*}
d_{1}=\frac{1}{2} n \lambda_{0}, \quad n=1,2,3, \cdots \tag{25}
\end{equation*}
$$

Under the conditions in (24) and (25), we calculate $T$ from


Fig. 12. Power transmission coefficient $|T|^{2}$ of a double screen made of an inductive screen and its complementary capacitive screen.
(23) with the results

$$
\begin{equation*}
T=1, \quad \text { when } \lambda=\lambda_{0} \tag{26}
\end{equation*}
$$

Thus, the composite screen has a sharp transmission peak at $\lambda=\lambda_{0}$. Based on the exact numerical solution, we have determined the resonance wavelength $\lambda_{0}$ as a function of $a$ and $c$, and the result may be presented in a simple formula, i.e.,

$$
\begin{equation*}
\frac{a}{\lambda_{0}} \approx 1.502-1.266 \frac{c}{a} . \tag{27}
\end{equation*}
$$

For example, when $c / a=0.7$, (27) predicts that the resonance defined in (24) occurs at $a=0.616 \lambda_{0}$, which agrees extremely well with the exact solution in Fig. 12. Making use of (27) in (25), we obtain the interscreen spacing necessary for resonance:

$$
\begin{equation*}
\frac{d_{1}}{a}=\frac{n}{3.004-2.532(c / a)} \quad n=1,2,3, \cdots \tag{28}
\end{equation*}
$$

In summary, when (27) is satisfied, a single inductive or a single capacitive screen has a transmission coefficient $|T|=$ 0.707 ( 3 dB transmission loss). When we cascade two complementary screens whose geometries satisfy (27) and (28), the transmission coefficient is $T=1$ (total transmission). In Fig. 12, we plot $T$ of a composite screen with $c / a=0.7$, and $d_{1}$ given by (28) with $n=1,2$, and 3 . As predicted by (27), total transmission occurs at $a / \lambda=0.616$. For $n=1$ (smallest spacing $d_{1}$ ), the resonance curve is relatively broad. For $n=3$, the curve is sharper, and has three other peaks with $|T|^{2}$ equal to $-8.5,-1.8$, and -3.8 dB .

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