TABLE I
MAINBEAM DIRECTIVITY LOSS (IN DECIBELS)

ν	Δu	u_j	$M = 2$	3	4	5
Sinc Pattern						
1/2	0.024	0.073	0.1	1.1	4.4	
1	0.049	0.073		1.2	4.7	10.4
Chebyshev Pattern						
1/2	0.012	0.099	0.004	−0.2	0.1	
1	0.024	0.100		−0.2	0.05	2.0

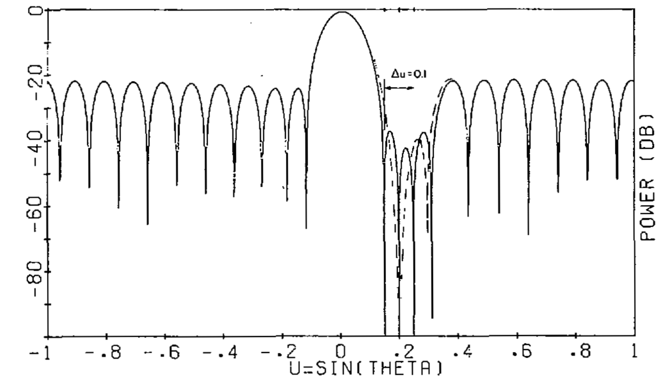

Fig. 6. Higher order nulling versus multiple nulling. Shown is 20-dB Chebyshev pattern with third-order null imposed at $u = 0.20$ (dashed line) and same pattern with three nulls imposed at $u = 0.15, 0.20, 0.25$ (full line). Former pattern contains 1.6 dB more power within Δu than latter. Sidelobe-peak cancellation is 10 dB and 17 dB, respectively. $N = 21, d = \lambda/2$.

written as the sum of the original pattern and a set of M weighted cancellation beams [3]. In this case the nth cancellation beam is given by the nth derivative, with respect to u, of a sinc beam at $u = u_0$.

We have also evaluated this approach, but omit the analysis here, since this method turns out to be less effective for wide-band sidelobe suppression than the multiple nulling method. This conclusion is obvious when the maximum magnitude of the residual pattern is used as a measure, but it holds even in the sense of integrated power over the nulling sector. Qualitatively, this can be understood in view of the fact that an Mth-order derivative null is equivalent to a cluster of M infinitesimally closely spaced single nulls at the center of the nulling sector. This scheme therefore cannot be expected to provide pattern control as efficiently as when the nulls are spaced evenly over the sector.

An illustrative comparison of a pattern with a third-order null ($p = dp/du = d^2p/du^2 = 0$) and a pattern with three closely spaced single nulls is shown in Fig. 6. Clearly, the multiple nulling technique provides superior sidelobe cancellation over the desired nulling sector ($\Delta u = 0.1, u_j = 0.2$).

V. SUMMARY AND CONCLUSION

The problem of wide-band nulling has been analyzed in an attempt to relate the sidelobe cancellation over a given bandwidth to the number of null constraints imposed on the antenna pattern. Two constraint methods, multiple nulling and derivative nulling, were considered and the former was found to be the more effective method.

The mathematical complexity of the problem has been reduced considerably through appropriate approximations. To first order the sidelobe cancellation is shown to be independent of the actual pattern type and determined by only two parameters: the number of null constraints M and the number of sidelobes ν to be cancelled. Since ν directly translates into a desired nulling bandwidth $\Delta f/f_0$ or a nulling sector Δu, either one of the latter may be used as alternative variable.

An exact analytic solution to the problem seems difficult to derive and therefore a numerical solution is offered. From this set of curves, the number of pattern nulls required to suppress a jammer over a given bandwidth can be conveniently estimated. This number is indicative of how many degrees of freedom a fully adaptive antenna system must allocate to attain a specified wide-band nulling performance.

ACKNOWLEDGMENT

The author wishes to thank Dr. R. Mailloux for suggesting the problem and Dr. R. Shore for the computer programming.

REFERENCES

[1] J. Mayhan, "Nulling limitations for a multiple beam antenna," *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 769–779, Nov. 1976.
[2] S. Applebaum, "Adaptive antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 585–598, Sept. 1976.
[3] H. Steyskal, "Synthesis of antenna patterns with nulls," *IEEE Trans. Antennas Propagat.*, vol. AP-30, Mar. 1982.
[4] K. Takao and K. Komiyama, "An adaptive antenna for rejection of wideband interference," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-16, pp. 452–459, July 1980.
[5] L. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*. Englewood Cliffs, NJ: Prentice-Hall, 1973.
[6] C. Drane, "Dolph-Chebyshev excitation coefficient approximation," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 781–782, Nov. 1964.

Simple Method for Pattern Nulling by Phase Perturbation

HANS STEYSKAL, MEMBER, IEEE

Abstract—**A method of sidelobe nulling, which involves perturbing the array illumination phase only, is presented. The general nonlinear problem is linearized by assuming the perturbations to be small, and an analytic solution is derived. Illustrative examples of sinc and Chebyshev patterns with imposed nulls are given.**

INTRODUCTION

Methods for forming nulls in the radiation pattern of an antenna in order to suppress directional interference presently receive much attention. A problem of particular interest is pattern nulling by perturbation of the aperture illumination phase, since in a phased array the required controls are already incorporated. Although singular cases are certainly conceivable where phase control alone is insufficient, for nulls in the sidelobe region of normal antenna patterns, it seems to be quite effective. The phys-

Manuscript received March 16, 1982; revised June 17, 1982.
The author is with the Electromagnetic Sciences Division, Rome Air Development Center, Hanscom AFB, MA 01731.

ical reason is of course that at a low pattern level an imposed null represents a relatively small pattern change.

In this communication we present a deterministic method for the synthesis of the desired pattern nulls using phase perturbation only. By assuming the perturbations to be small, the general nonlinear problem is linearized and an analytic solution is derived. To ensure the validity of the initial assumption, the small perturbation condition is explicitly imposed on the solution. The approach therefore leads to good nulling performance even though it is approximate.

Publications on this problem appear to be limited to [1] and [2], which focus on the adaptive minimization of a generalized performance index. The present method differs from [1] mainly in that it provides a direct rather than an iterative solution. Also, due to the difference in approximation criteria, the resultant cancellation beams reflect the general characteristics of the original pattern.

METHOD OF SOLUTION

Consider the pattern

$$f_0(u) = \sum_{-N}^{N} a_n e^{-inkdu} \tag{1}$$

of a linear uniform array of $2N + 1$ isotropic elements. Here $u = \sin\theta$, where θ denotes the angle from broadside, d is the element spacing, and a_n is the complex excitation of the nth element. We limit our attention to the usual case where $f_0(u)$ is a real function so that a_0 is real and $a_{-n} = a_n^*$, with the asterisk denoting complex conjugate. Our problem is to find $2N + 1$ excitation phase perturbations ϕ_n, such that the perturbed pattern $f(u)$ has nulls at the prescribed directions $u = u_m$, $m = 1, \cdots, M$.

Performing a Taylor expansion of the phase factor of the perturbed excitation coefficients $a_n \exp(i\phi_n)$ and retaining the first two terms, we obtain

$$\begin{aligned} f(u) &\cong \sum_n a_n e^{-inkdu} + i \sum_n a_n \phi_n e^{-inkdu} \\ &= f_0(u) + f_c(u). \end{aligned} \tag{2}$$

The term $f_c(u)$ represents a cancellation pattern which can be used to achieve the M desired nulls. These M conditions constitute $2M$ real equations for the $2N + 1$ unknown ϕ_n and since usually $2M < 2N + 1$, the solution is not yet completely determined. Therefore, we also impose the condition that the perturbations be small (in a mean square sense), since our approach is based on this very assumption. This leads to the final system of equations

$$\begin{cases} f_0(u_m) + f_c(u_m) = 0, \qquad m = 1, \cdots, M & \text{(3a)} \\ \sum_n \phi_n^2 = \text{minimum}. & \text{(3b)} \end{cases}$$

Defining a $(2N + 1)$-dimensional phase perturbation vector $\bar{\phi} = (\phi_{-N}, \cdots, \phi_N)$ and constraint vectors $\bar{c}_m = (a_{-N} \exp(iNkdu_m), \cdots, a_N \exp(-iNkdu_m))$, it is shown in the Appendix that the solution of (3) can be written in the form

$$\bar{\phi} = \sum_{m=1}^{M} \lambda_m \operatorname{Im} \bar{c}_m \tag{4}$$

where $\operatorname{Im} \bar{c}$ denotes the imaginary component of the complex vector $\bar{c}$ and the M unknown coefficients λ_m are determined from the M linear equations (3a). Note that this usually constitutes a very small system of equations, which is easily inverted. Further, it is found that $\phi_{-n} = -\phi_n$ and when f_0 is an even (odd) function, then f_c is a real odd (even) function.

For the case of a single null imposed on an even pattern, the cancellation pattern is found to be, using (2) and (4),

$$f_c(u) = (1/2)\lambda_1 \left[\sum_{-N}^{N} a_n^2 e^{-inkd(u-u_1)} - \sum_{-N}^{N} a_n^2 e^{-inkd(u+u_1)} \right]. \tag{5}$$

The pattern is seen to consist of a pair of beams, pointing at $u = u_1$ and $u = -u_1$, respectively, and each beam corresponds to an aperture amplitude distribution which is the initial amplitude squared. An initial sinc pattern will thus lead to sinc-type cancellation beams, and, interestingly, an initial Chebyshev pattern will lead to Chebyshev-type cancellation beams. The latter relation is only of an approximate qualitative nature and follows from the fact that the cancellation beam is the convolution of the initial pattern with itself. For multiple nulls the cancellation pattern clearly is a properly weighted sum of such beam pairs.

The property of a beam-pair cancellation pattern has been shown earlier in [1]. However, due to the error criterion chosen, the beams obtained there are necessarily sinc-beams, whereas ours preserve the characteristic of the initial pattern.

RESULTS

The method for pattern nulling presented above is based on an approximation. To validate the approach, therefore, a few illustrative examples of perturbed patterns have been calculated, using perturbed excitation coefficients $a_n \exp(i\phi_n)$ instead of the coefficients a_n in (1).

A sinc pattern and a 30-dB Chebyshev pattern, each with one prescribed null, are shown in Figs. 1(a) and 2(a). Although the nulls are not perfect, due to the approximations inherent in the method, the initial sidelobe levels have been reduced by 48 and 60 dB, respectively. Note the unavoidable sidelobe increase at $u = -u_1$ since the cancellation patterns f_c are odd functions. These patterns are shown in Figs. 1(b) and 2(b), which verify the expected characteristics: a sinc-type beam pair in the former case, a Chebyshev-type beam pair in the latter case. The phase perturbations corresponding to these two cases are listed in Table I. For the Chebyshev pattern the perturbations are considerably smaller than for the sinc pattern, due to the lower initial sidelobe level.

With closely spaced multiple nulls the pattern can be suppressed over an extended angular sector, which for a phase scanned array is tantamount to an increased nulling bandwidth. This is shown in Fig. 3, where with three nulls a 36-dB sidelobe cancellation is achieved over a sector $\Delta u = 0.012$, corresponding to about 5 percent bandwidth. A similar example with five prescribed nulls is shown in Fig. 4. Although this obviously strains our approximate method, still 22-dB cancellation is obtained over $\Delta u = 0.024$, corresponding to about 10 percent nulling bandwidth. For comparison, Fig. 5 shows the nulled pattern when both the illumination amplitude and phase are perturbed [3], which gives 85-dB cancellation. A unified perspective on different methods for pattern nulling may be found in [4], which also contains additional examples.

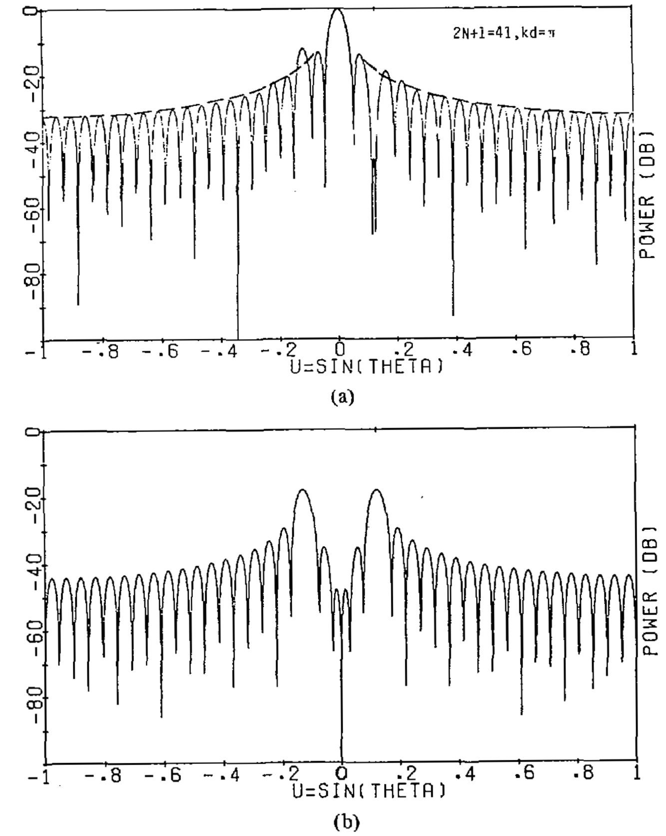

Fig. 1. (a) Pattern with one null imposed at u = 0.123. Dashed line shows initial sinc-pattern envelope. Sidelobe cancellation is 48 dB. (b) Cancellation pattern.

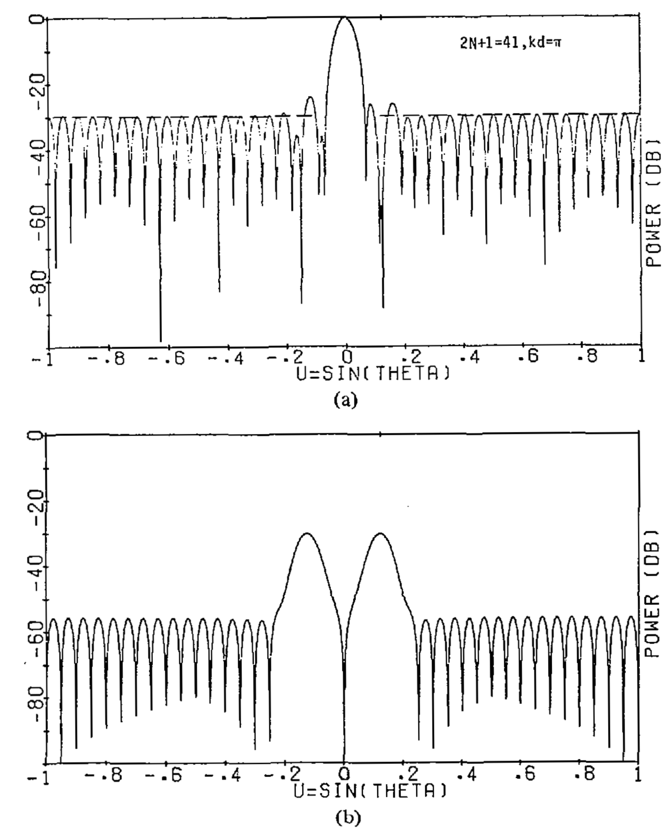

Fig. 2. (a) Pattern with one null imposed at u = 0.123. Dashed line shows initial 30-dB Chebyshev pattern envelope. Sidelobe cancellation is 60 dB. (b) Cancellation pattern.

TABLE I

COMPUTED PHASE PERTURBATIONS FOR FIGS. 1, 2

PATTERN:	SINC	30DB CHEBYSHEV
ELEMENT NO.	ϕ_n (°)	ϕ_n (°)
0 (Array Center)	0	0
1	5	2
2	10	3
3	13	4
4	14	5
5	14	4
6	10	3
7	6	2
8	1	0
9	-5	-1
10	-10	-2
11	-12	-3
12	-14	-3
13	-14	-2
14	-11	-2
15	-7	-1
16	-2	0
17	4	0
18	9	1
19	12	1
20	14	2

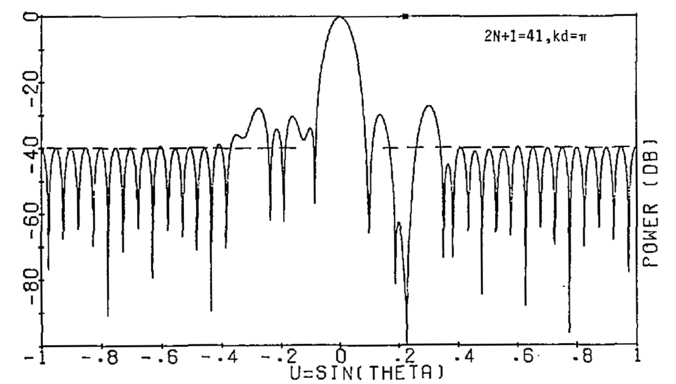

Fig. 3. Pattern with three nulls imposed at u = 0.214, 0.22, and 0.226, respectively. Dashed line shows initial 40-dB Chebyshev pattern envelope. Sidelobe cancellation is 36 dB.

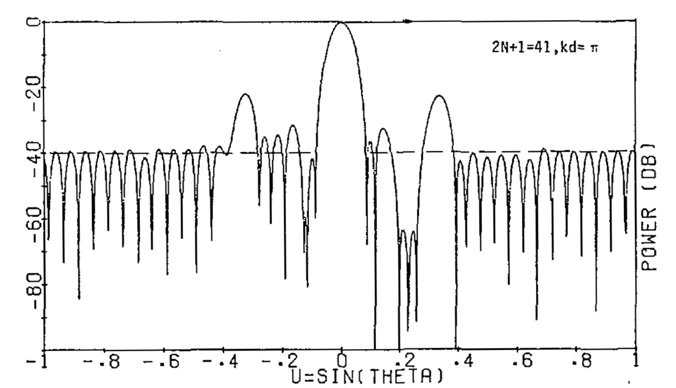

Fig. 4. Pattern with five nulls imposed at u = 0.214, 0.22, 0.226, 0.232, and 0.238, respectively. Dashed line shows initial 40-dB Chebyshev pattern envelope. Sidelobe cancellation is 22 dB.

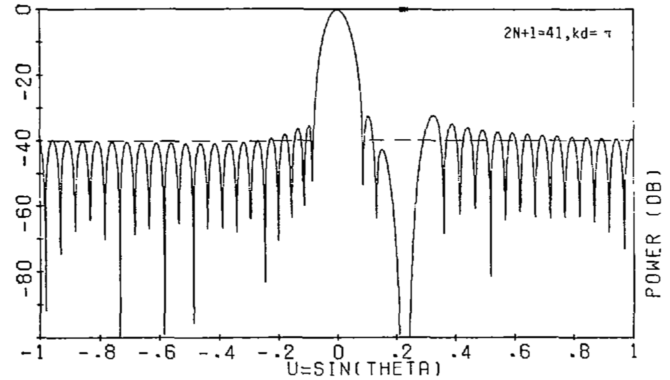

Fig. 5. Same case as in Fig. 4 when in addition to excitation phase also amplitude is perturbed. Sidelobe cancellation is 85 dB.

CONCLUSION

We conclude that this simple approximate approach to the nonlinear problem of phase-only pattern nulling works satisfactorily so long as the nulls are imposed in the sidelobe region and the number of nulls $M \ll$ the number of elements $(2N + 1)$. A general limitation of phase-only nulling methods appears to be that they are incapable of nulling two jammers, which are disposed perfectly symmetrically about the main beam. This, however, is a rather academic situation and, for a modest number of interference sources, therefore, these methods should prove useful. A matter of practical interest is the smallness of the phase perturbations, which possibly demands a larger number of phase shifter bits than that required for the beam scanning function alone.

APPENDIX

Equation (3) can be written, using the vectors introduced in the text, as

$$\begin{cases} (\bar{\phi}, \bar{c}_m{}^*) = if_0(u_m) \qquad m = 1, \cdots, M \\ \|\bar{\phi}\|^2 = \text{minimum} \end{cases} \tag{6}$$

where as usual the inner product and the norm are defined by $(\bar{x}, \bar{y}) = \Sigma x_n y_n{}^*$ and $\|\bar{x}\| = (\bar{x}, \bar{x})^{1/2}$. Separating (6) into real and imaginary parts leads to the system of real equations

$$\begin{cases} (\bar{\phi}, \text{Re}\,\bar{c}_m) = 0, & \text{(7a)} \\ (\bar{\phi}, \text{Im}\,\bar{c}_m) = f_0(u_m), \qquad m = 1, \cdots, M. & \text{(7b)} \\ \|\bar{\phi}\|^2 = \text{minimum} & \text{(7c)} \end{cases}$$

Ignoring for a moment (7a), the solution to (7b) and (7c) is given by (see [5, p. 811])

$$\bar{\phi} = \sum_{m=1}^{M} \lambda_m \,\text{Im}\, c_m \tag{8}$$

where the constants λ_m are determined by the M equations (7b). Further, it is easily shown that

$$(\text{Re}\,\bar{c}_m, \text{Im}\,\bar{c}_n) = 0, \qquad \forall\, m, n. \tag{9}$$

The vector defined by (8) thus satisfies also (7a), and consequently it constitutes the desired solution.

ACKNOWLEDGMENT

I thank Dr. R. Shore for several enlightening discussions, and Ms. Y. Gersht for the computer programming.

REFERENCES

[1] C. Baird and G. Rassweiler, "Adaptive sidelobe nulling using digitally controlled phase-shifters," *IEEE Trans. Antennas Propagat.*, vol. AP-24, Sept. 1976.
[2] E. Mendelovicz and E. Oestreich, "Phase-only adaptive nulling with discrete values," in *IEEE/AP-S Int. Symp. Digest*, 1979.
[3] H. Steyskal, "Synthesis of antenna patterns with nulls," *IEEE Trans. Antennas Propagat.*, vol. AP-30, Mar. 1982.
[4] R. Shore and H. Steyskal, "Nulling in linear array patterns with minimization of weight perturbations," Rep. RADC-TR-82-32, Feb. 1982.
[5] R. Bellman, *Introduction to Matrix Analysis*, 2nd ed. New York: McGraw-Hill, 1970.

Scattering by a Rotating Circular Cylinder with Finite Conductivity

DANIEL DE ZUTTER

Abstract—**The effect of motion on the signal scattered by a rotating circular cylinder with finite conductivity is investigated. The problem is solved by means of the "instantaneous rest-frame" hypothesis. The analysis shows that a surface current must be taken into account to calculate the jump in the tangential magnetic field at the cylindrical surface. This holds even in the case of finite conductivity. For a perfectly conducting cylinder, the influence of the motion on the fields is negligible. This is shown by considering the limit of high but finite σ.**

I. INTRODUCTION

The field scattered by a moving body, illuminated by an incident wave, is influenced by the motion of the scatterer. This phenomenon can be exploited to gain information on the properties and the state of motion of the target. In the past, much attention has been given to targets in translational motion with constant velocity. In this case the special theory of relativity can be used to find the scattered fields and the associated Doppler spectra [1], [2]. The simplest approach to the problem of rotating targets is to apply the quasi-stationary method. There, the fields in the presence of the rotating body are calculated by assuming that at any given time, they have the value corresponding to the instantaneous position of the body [3]-[5]. According to this method, the fields scattered by a rotating homogeneous circular cylinder or sphere should not be influenced by the rotation.

A more accurate analysis shows that the rotation produces an additional field of order Ω/ω, where Ω is the angular velocity and ω the angular frequency of the incident wave [3], [6]. This analysis uses the instantaneous rest-frame hypothesis [3], according to which a scattering problem involving an accelerated body can be solved by using the local boundary conditions and constitutive equations of the special relativity. The constant velocity appearing in these equations must now be replaced by the time and/or space-dependent velocity at the various points of the accelerated body. The validity of this hypothesis is generally accepted for sufficiently low accelerations [3], [7], [8]. In this communication, the theory will be used to investigate scattering by a rotating, homogeneous circular cylinder with finite conductivity. A solution of this problem already exists for nonconduc-

Manuscript received May 14, 1982; revised August 9, 1982.
The author is with the Electromagnetism and Acoustics Laboratory, University of Ghent, Belgium.

0018-926X/83/0100-0166$1.00 © 1983 IEEE