Simple methods of measuring the net photorefractive phase shift and coupling constant

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We report measurements of the photorefractive phase shift and coupling constant of several photorefractive materials. We solve the problem of beam coupling and diffraction in a material with a dynamically written grating for arbitrary input beams. These solutions are used to determine the beam coupling as a function of the photorefractive phase ϕ and coupling constant g when one beam is either sinusoidally phase modulated or ramped in phase. Experimental results are obtained for $LiNbO_3$, $BaTiO_3$, and for paraelectric potassium lithium tantalate niobate as a function of applied electric field.

The photorefractive grating phase ϕ and the coupling constant g are the two material parameters that describe the interaction of two or more coherent beams in a photorefractive material. Yet the exact determination of these parameters has received little attention to date. Most published reports describe approximate methods of phase determination.[1-4] The treatment by Vahey,[5] although exact, is flawed since he uses an intensity-dependent equation for the index grating. Here we report an exact solution of the coupled equations describing the evolution of two arbitrary beams incident at the Bragg angle on a dynamically written photorefractive grating. The incident beams need not possess the same phase nor the same intensity as the beams that wrote the grating. In our analysis, all the beams are of the same frequency.

Two coherent beams are symmetrically incident on a photorefractive crystal. A dynamically written refractive-index grating is the result. The writing beams are then replaced with two beams with arbitrary intensity and phase and of the same frequency incident at the Bragg angles. We calculate the instantaneous beam coupling experienced by these new beams off the dynamically written grating, which, for the short initial period, is considered fixed. We ignore the time-dependent formation of dynamic gratings written by the new beams.

By using the material of Ref. 6, we can calculate the two-beam coupling of two incident copropagating beams with amplitudes $A(z)$ and $B(z)$ in a photorefractive material (Fig. 1). We use α as the optical absorption and β as the half-angle of beam intersection inside the material. The index of refraction is $n(z) = n_0 + 1/2[\Delta n(z)\exp(i\phi + iKz) + \text{c.c.}]$. Here ϕ is the photorefractive phase between the optical-intensity grating and the induced-index grating. $K = 2k \sin \beta$ is the grating wave vector. Since the index grating is formed dynamically by the writing beams, we have $\Delta n(z) = n_1 A(z)B^*(z)/I(z)$, where $I(z)$ is the total intensity and n_1 is the peak-to-peak amplitude of the index grating when $A(z) = B(z)$. L is the effective thickness of the crystal, with $L = d/\cos \beta$. By using the above, the index grating in the material is readily shown to be $\Delta n = n_1(I_1I_2)^{1/2}[I_1 \exp(-\Gamma z/2) + I_2 \exp(\Gamma z/2)]^{i \cot \phi - 1}$. We have defined $g = \pi n_1/\lambda$ as the material coupling constant, $\Gamma = 2g \sin \phi$, and $I_1 = I_1(0)$ and $I_2 = I_2(0)$.

We calculate the beam coupling of a new set of Bragg-matched beams $T(0) = P_1^{1/2} \exp(i\psi_1)$ and $V(0) = P_2^{1/2} \exp(i\psi_2)$ off this index grating. Analogous to Ref. 6, we write the coupled-mode equations

$$T'(z)\cos \beta = ig\sqrt{I_1I_2} \exp[+i(\phi + \theta)][I_1 \exp(-\Gamma z/2) + I_2 \exp(+\Gamma z/2)]^{+i \cot \phi - 1}V(z) - \frac{\alpha}{2} T(z), \tag{1a}$$

$$V'(z)\cos \beta = ig\sqrt{I_1I_2} \exp[-i(\phi + \theta)][I_1 \exp(-\Gamma z/2) + I_2 \exp(+\Gamma z/2)]^{-i \cot \phi - 1}T(z) - \frac{\alpha}{2} V(z), \tag{1b}$$

where $\theta = \zeta_1(0) - \zeta_2(0) - \psi_1(0) + \psi_2(0)$ is the phase difference between the intensity pattern of the beams that wrote the grating and that of $T(0)$ and $V(0)$. I_1 and I_2 are the intensities of the writing beams. Equations (1a) and (1b) ignore the new dynamic grating that is written by beams $T(z)$ and $V(z)$. This neglect is justified only for a short time, which, however, is amply long (≥ 1 s) to obtain the necessary data.

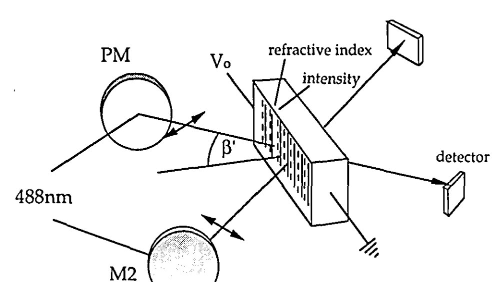

Fig. 1. Experimental setup for determining the photorefractive coupling constant and phase of dynamic gratings. The piezoelectric mirror PM modulates the phase of one beam either sinusoidally or as a ramp.

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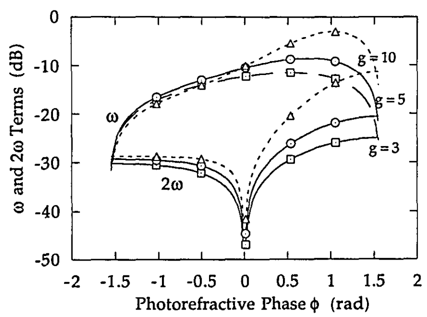

Fig. 2. Intensity $I_1(L)$ of Fig. 1 at ω and 2ω relative to the dc power (=0 dB) when one of two interfering beams is phase modulated at ω as a function of the photorefractive phase ϕ. The results are plotted for coupling constants $gL = 3, 5, 10$.

Equations (1a) and (1b) are satisfied by

$$T(z)\exp(\alpha L/2) = C_1[I_2 + I_1 \exp(-\Gamma z)]^{+i\eta-1/2} + C_2[I_2 \exp(+\Gamma z) + I_1]^{+i\eta-1/2}, \quad (2a)$$

$$V(z)\exp(\alpha L/2) = C_3[I_2 + I_1 \exp(-\Gamma z)]^{-i\eta-1/2} + C_4[I_2 \exp(+\Gamma z) + I_1]^{-i\eta-1/2}, \quad (2b)$$

where $\eta = \cot \phi/2$ and C_j are constants that are determined by the coupled equations and the boundary conditions.

We solve the coefficients C_j for two special cases. The first is the case where one or two of the reading beams are phase shifted relative to recording beams but their intensities are unchanged. This can be accomplished in practice by merely inserting a phase shift in the path of one of the recording beams. We use the boundary conditions $T(0) - I_1^{1/2}(I_1 + I_2)^{i\eta}$ and $V(0) = I_2^{1/2}(I_1 + I_2)^{-i\eta}$. We determine for this case

$$C_1 = \sqrt{I_1/(I_1 + I_2)}\, I_2[1 - \exp(i\theta)], \quad (3a)$$

$$C_2 = \sqrt{I_1/(I_1 + I_2)}\, [I_1 + I_2 \exp(i\theta)], \quad (3b)$$

$$C_3 = \sqrt{I_2/(I_1 + I_2)}\, [I_2 + \exp(i\theta) I_1], \quad (3c)$$

$$C_4 = \sqrt{I_2/(I_1 + I_2)}\, I_1[1 - \exp(i\theta)]. \quad (3d)$$

The output amplitudes $T(L)$ and $V(L)$ are thus determined. When $I_1 = I_2$, the transmitted intensities reduce to

$$P_1(z)/I_1 = 1 - \tanh(\Gamma z/2)\cos\theta - \frac{\sin(g \cos\phi\, z)}{\cosh(\Gamma z/2)} \sin\theta\,, \quad (4a)$$

$$P_1(z)/I_1 = 1 + \tanh(\Gamma z/2)\cos\theta + \frac{\sin(g \cos\phi\, z)}{\cosh(\Gamma z/2)} \sin\theta\,. \quad (4b)$$

The solutions for the case $I_1 \ll I_2$ also follow readily from Eqs. (2) and (3) and agree with previously published results.[1] The results for the cases $\phi = 0$ and $\pi/2$ assume a simple form for arbitrary I_1 and I_2.[7]

The other case considered is diffraction of a single incident beam off the grating. Here, we take $V(0) = 0$ and $T(0)$ as before. In this case, we obtain

$$C_1 = \sqrt{I_1/(I_1 + I_2)}\, I_2\,, \quad (5a)$$

$$C_2 = \sqrt{I_1/(I_1 + I_2)}\, I_1\,, \quad (5b)$$

$$C_3 = \sqrt{I_2/(I_1 + I_2)}\, I_1\,, \quad (5c)$$

$$C_3 = -\, C_4\,. \quad (5d)$$

Again, for the condition $I_1 = I_2$, the transmitted and diffracted intensities reduce to a simple form

$$P_1(z) = \frac{I_1}{2}[1 + \cos(g \cos\phi\, z)/\cosh(g \sin\phi\, z)], \quad (6a)$$

$$P_2(z) = \frac{I_1}{2}[1 - \cos(g \cos\phi\, z)/\cosh(g \sin\phi\, z)]. \quad (6b)$$

By measuring the beam coupling [Ref. 6, Eq. (4.5)] and the diffraction [Ref. 6, Eq. (6)], we can determine g and ϕ in the material.

An alternative method of measuring g and ϕ follows when one of the interfering beams is phase shifted by θ using a piezoelectrically driven mirror. The experimental configuration is shown in Fig. 1. When the mirror is driven sinusoidally we have $\theta(t) = \theta_0 \sin(\omega t)$. If $\omega \gg 1/\tau$, where τ is the writing time of the grating, $\theta_0 \ll \pi/2$, and $I_1 = I_2$, then the powers of the transmitted beams at dc, ω, 2ω are related

Table 1. Values of the Coupling Constant g and the Photorefractive Phase ϕ for the Crystal Samples Tested[a]

	Coupling Constant g (1/cm)			Photorefractive Phase ϕ (rad)		
Material	1	2	3	1	2	3
$LiNbO_3$	12.75	13.67	13.3	0.40	0.42	0.41
$BaTiO_3$	2.25	2.33	2.30	1.5	1.46	1.47
KLTN[b]	2.41	2.45	2.43	0.0	0.00	0.00
KLTN[c]	7.18	7.0	7.1	1.28	1.00	1.1

[a]Column 1 is data from the beam-coupling/diffraction method; column 2 is from the harmonics method; column 3 is the best fit to the data.
[b]KLTN is paraelectric, so its photorefractive response in the absence of an applied field is due to formation of zero-phase ZEFPR gratings.
[c]When a 2200-V/cm field is applied, the electro-optic gratings dominate, and the phase is far from zero.

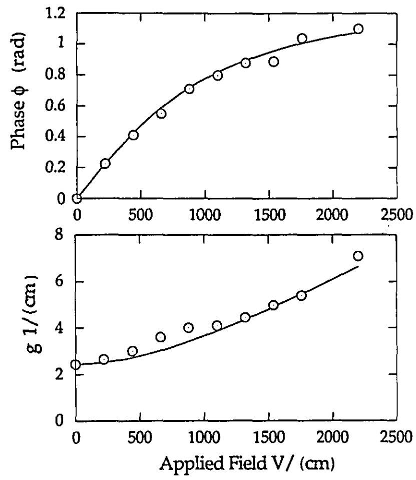

Fig. 3. (a) Photorefractive phase ϕ of gratings written in KLTN versus applied electric field. Solid curve describes the theoretical interaction between a ZEFPR grating and an electro-optic grating. (b) Coupling constant g of KLTN versus applied field. Again, the data are in accord with the theory.

by [Eqs. (4)]

$$\left|\frac{P_{dc}}{P_{2\omega}}\right| = \frac{4}{\theta_0{}^2}[\coth(g \sin \phi\, z) \pm 1], \tag{7a}$$

$$\left|\frac{P_{\omega}}{P_{2\omega}}\right| = \frac{4}{\theta_0}\frac{\sin(g \cos \phi\, z)}{\sinh(g \sin \phi\, z)}, \tag{7b}$$

where the powers are taken to be peak-to-peak values (Fig. 2). The plus in Eq. (7a) refers to the beam that is amplified, and the minus to the beam that is attenuated. If, instead of sinusoidal modulation, the piezoelectric mirror is ramped linearly in phase, then the phase positions during the ramp corresponding to the minimum and maximum of the transmitted intensities can be used to determine g and ϕ. In particular, the maximum of $P_2(z)$ from Eq. (4b) occurs at $\theta_{max} = \tan^{-1}[\sin(g \cos \phi\, z)/\sinh(g \sin \phi\, z)]$. Thus Eqs. (7a) and (7b) are a second method of obtaining g and ϕ. Determining the value of θ_{max} is a third method that does not yield g and ϕ separately. Instead, θ_{max} gives a value for g consistent with any given ϕ. Thus it can corroborate the results obtained by weighted averaging of data from the previous two methods. The harmonics method is expected to be more accurate than the beam-coupling/diffraction method, particularly for phases near $\phi = \pi/2$. The experimental results appear to bear out this prediction. We used these methods to determine g and ϕ experimentally for an iron-doped lithium niobate crystal, an iron-doped barium titanate crystal, and a paraelectric potassium lithium tantalate niobate (KLTN) crystal doped with copper. The KLTN was grown and prepared by us. The laser beams were at 488 nm, with polarization in the plane of their intersection. The c axis of the samples was perpendicular to the bisector of the light beams in the plane of polarization. We used $K = 1.7 \times 10^7$/m and $\theta_0 = 0.0613$ rad at 10.6 kHz for the sinusoidal phase modulation. The phase was not modulated during writing of the grating. The results for the beam-coupling/diffraction method and the harmonics method are shown in Table 1 for the samples tested. Paraelectric KLTN is forbidden from having a photorefractive response without an applied field, yet it does display the recently reported zero-electric-field photorefractive (ZEFPR) effect of paraelectric crystals.[8] As predicted, the ZEFPR gratings show $\phi = 0$ to within the accuracy of the experiment.

In addition, we determined the photorefractive parameters of the KLTN versus applied electric field. The photorefractive response of paraelectric KLTN is described by the quadratic electro-optic effect in conjunction with the ZEFPR effect. The ZEFPR gratings are always $\pi/2$ out of phase with the electro-optically induced index grating. We write $\Delta n(z) = E_{SC}[(\gamma E_0)^2 + \gamma_{Zf}{}^2]^{1/2} \sin(Kz + \phi_E + \alpha)$. Here E_0 is the applied uniform electric field, E_{SC} is the photorefractive space-charge field, γ is given by[9] $\gamma = -n_0{}^3 g(\epsilon\epsilon_0)^2$, g is the relevant quadratic electro-optic coefficient, and $\epsilon\epsilon_0$ is the dielectric constant. γ_{Zf} is experimentally determined by the zero applied-field diffraction, and ϕ_E is the phase between the intensity and electro-optic gratings.[6] $\alpha = \tan^{-1}(\gamma E_0/\gamma_{Zf})$, so that the photorefractive phase $\phi = \phi_E + \alpha$. The coupling constant is given by $g = \pi[(\gamma E_0)^2 + \gamma_{Zf}{}^2]^{1/2} E_{SC,0}/\lambda$. Here $E_{SC,0}$ is the space-charge field for unity modulation depth. The applied-field dependence of the space-charge field follows from Ref. 6.

The sample that we used had a composition of $K_{0.99}L_{0.01}T_{0.71}N_{0.29}$. The acceptor concentration was determined to be $N_A = 1.5 \times 10^{24}$ m^{-3}. We determined that $\epsilon = 12,000$ at 24 °C (the temperature of the experiment). The crystal was paraelectric above its phase transition at 15 °C. Figures 3(a) and 3(b) show good agreement with the theory described above.

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