Simple model of current-induced spin torque in domain walls

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The effective spin pressure induced by an electric current on a domain wall in a ferromagnet is determined using a simple classical model, which allows us to extend previous theories to arbitrary domain-wall widths. In particular, the role of spatially nonuniform components of the torques are analyzed in detail. We find that in the steady state, the main effect of the current is a distortion of the wall, which should enhance depinning. We also discuss the nonadiabatic part of the torque and find that this term, responsible for the pressure on the wall, depends on the nature of spin-flip scattering events.

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Traditionally, spin electronics has been dealing with resistance changes induced by different magnetic configurations, like those encountered in "giant magnetoresistance" and "tunneling magnetoresistance" devices. This has led to important developments in information technology applied in computer read heads and the more recent magnetic memories (or MRAMs) for high-density information storage. For the latter, one has to be able to read and write the magnetic information. Writing is presently achieved by local application of a magnetic field using current lines crossing on each individual cell. This procedure is the main source for cross-talk between magnetic elements because of field leakages, which can influence neighboring memories. Thus, it would be most useful to switch the magnetic configurations directly with a current flowing in the spin valve. The relevant effect is called spin torque and it has been predicted by Slonczewski[1] and Berger[2] and observed in the late 1990's.[3] It is now well known that magnetization can be reversibly switched in ferromagnetic-normal metal-ferromagnetic trilayers by a large current crossing the interfaces. Besides, it appears that this effect can also produce microwave-frequency oscillations of the thin layer,[4] possibly allowing coherent microwave generation by an assembly of nanopillars.[5]

An alternative way to control magnetic configurations is to move domain walls (DWs) with a current as first proposed by Berger[6] and demonstrated a long time ago.[7] Recently, theoretical studies[8] and numerical calculations[9] have shown that a large part of the current-induced torque does not push the DWs and the pressure only originates from spin-flip events relaxing the conduction electrons' spins. We propose here a simple picture of the relevant physics involved in the generation of the different components of the torque in domain walls. The basis for it lies in understanding the evolution of the spin of a conduction electron as it crosses a domain wall and its reaction on the local moment. The relevant interaction is that between localized (the local magnetization) and delocalized (conduction) electrons, which can be expressed with the s-d Hamiltonian: $H_{s\text{-}d}=-J_{ex}\vec{s}\cdot\vec{S}$ where J_{ex} is the exchange interaction, $\langle\vec{S}\rangle/S=-\vec{M}/M_s$ refers to localized spins, and $\vec{s}$ refers to the conduction electrons spins. The exchange interaction splits the conduction electrons in two populations with spins parallel (up) or antiparallel (down) to the local moments. A current generates a plane wave of electrons whose wave functions can be expressed as a spinnor with two components, up and down, traveling with different wave vectors. The question is then to analyze how these two components evolve when the electrons are forced to cross a region where localized moments change direction in space, i.e., a domain wall. The proper way to do this is to write Schrödinger's equation and match wave functions and their derivatives at the borders of the DW. Solutions of the problem are complex for any particular shape of the domain-wall width, but analytical solutions can be easily found for the two limits of an abrupt, or a very long domain wall (see, for example, Ref. 10). There is another way of treating the problem, which is to consider conduction electrons as free particles entering a region in space where a local field changes direction (the DW). Their spin evolution is then obtained by writing the Landau-Lifshitz equation. This is simpler, but one has to overcome conceptual problems linked with the nature of the electrons crossing, which are band particles. These two visions of the problem are actually equivalent in the limit where the amount of reflected wave can be neglected.[10] Hence, this is valid for a DW width much greater than the Fermi wavelength.[11] Several works in that field (addressing more particularly DW resistance) have chosen either the "wave" approach[12–14] or the "particle" one.[15,16] Because in the wave approach one has to use perturbation theory, spin torques have been calculated in the limits of wide, or very narrow domain walls. Here, we propose to use the simplicity of the particle approximation to describe the spin evolution in the domain walls and to extend previous results to intermediate values of the DW width.

CLASSICAL MODEL

If both magnetization and electron magnetic moment ($\vec{\mu}=-g\mu_B\vec{s}$) are considered classical vectors, the conduction spin dynamics obeys the basic precession equation with a damping term corresponding to spin-flip scattering,

$$\frac{d\vec{\mu}}{dt}=\frac{J_{ex}S}{\hbar}\vec{m}\times\vec{\mu}-\frac{1}{\tau_{sf}}(\vec{\mu}-\vec{\mu}_{eq}), \tag{1}$$

where $\vec{m}=\vec{M}/M_s$ is the unitary vector of magnetization. The first part of the right-hand side is similar to the classical Landau-Lifshitz equation with the external field replaced by

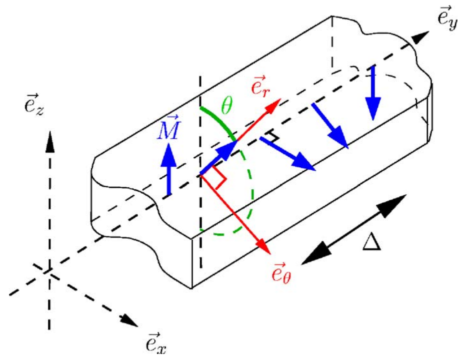

FIG. 1. (Color online) Bloch domain wall: conventions and notations.

the exchange field, which reflects the difference between the *s-d* Hamiltonian considered here and the traditional Zeeman one. The second term corresponds to spin-flip relaxation of the magnetic moment toward the local magnetization, hence $\vec{\mu}_{eq}$ is taken along $\vec{m}$. In usual ferromagnetic materials, the spin-flip relaxation time τ_{sf} is large compared to the Larmor period $T_L=2\pi\tau_{ex}$, so we take for the following: $\tau_{ex}/\tau_{sf}\ll 1$.

We consider here the frame moving with the electron crossing the DW, in which the magnetization varies continuously in time. For simplicity, we consider the case of a Bloch wall, but the following derivation would equally apply to a magnetization rotation around any axis (as long as the system can be considered one dimensional). The moving frame is defined by the three vectors $(\vec{e}_r,\vec{e}_\theta,\vec{e}_y)$ as shown in Fig. 1. The evolution of the electron magnetic moment is thus simply described by

$$\frac{d\vec{\mu}}{dt}=\begin{pmatrix}\dot{\mu}_r-\dot{\theta}\mu_\theta\\ \dot{\mu}_\theta+\dot{\theta}\mu_r\\ \dot{\mu}_y\end{pmatrix}=\frac{SJ_{ex}}{\hbar}\vec{m}\times\vec{\mu}-\frac{1}{\tau_{sf}}\begin{pmatrix}\mu_r-\dfrac{g\mu_B}{2}\\ \mu_\theta\\ \mu_y\end{pmatrix}. \quad (2)$$

Defining $\tau_{ex}=\hbar/SJ_{ex}$ we get

$$\dot{\mu}_r-\dot{\theta}\mu_\theta=\frac{1}{\tau_{sf}}\left(\mu_r-\frac{g\mu_B}{2}\right),$$

$$\dot{\mu}_\theta+\dot{\theta}\mu_r=-\frac{\mu_y}{\tau_{ex}}-\frac{1}{\tau_{sf}}\mu_\theta,$$

$$\dot{\mu}_y=\frac{\mu_\theta}{\tau_{ex}}-\frac{1}{\tau_{sf}}\mu_y. \quad (3)$$

This leads to periodic terms describing conduction-electrons spin precessing with the so-called Larmor period. It is convenient to consider first a long DW with a linear variation of the magnetization angle θ (the second derivative of the angle is neglected $\ddot{\theta}=0$, and the electrons spin remains mainly aligned with the magnetization), which leads to the simplified equations,

$$\ddot{\mu}_\theta+\frac{2}{\tau_{sf}}\dot{\mu}_\theta+\frac{1}{\tau_{ex}^2}\mu_\theta=-\frac{\dot{\theta}}{\tau_{sf}}\frac{g\mu_B}{2},$$

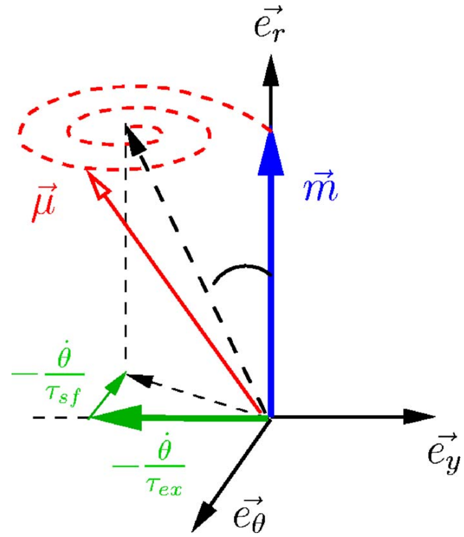

FIG. 2. (Color online) Evolution of the conduction-electron magnetic moment in the domain wall in the frame rotating with the magnetization. The precession of $\vec{\mu}$ is not around the local magnetization but around a tilted effective field.

$$\ddot{\mu}_y+\frac{2}{\tau_{sf}}\dot{\mu}_y+\frac{1}{\tau_{ex}^2}\mu_y=-\frac{\dot{\theta}}{\tau_{ex}}\frac{g\mu_B}{2}. \quad (4)$$

In fact, the magnetic moment $\vec{\mu}$ is precessing around an effective field (Fig. 2) whose direction is given by the sum of the magnetization, its rotation vector, and a small component along e_θ coming from the spin relaxation: $\vec{m}-\dot{\theta}\tau_{ex}\vec{e}_y-\dot{\theta}\tau_{sf}\vec{e}_\theta$. Hence, during DW crossing, the conduction spins lag from the local magnetization, which induces spin mixing and an extra resistance as described in Ref. 16. The average over time of the mistracking between $\vec{\mu}$ and the magnetization is not zero as it lies on the effective field,

$$\langle\vec{\mu}\rangle=\frac{g\mu_B}{2}\begin{pmatrix}1\\ -\dot{\theta}\tau_{sf}\\ -\dot{\theta}\tau_{ex}\end{pmatrix}. \quad (5)$$

This is a crucial point as it explains why, even if we consider a large number of Larmor oscillations of $\vec{\mu}$ in the wall, the resulting effect on the magnetization will not be averaged to zero. The reaction torque on the local magnetization can be expressed, for each electron per unit volume, using

$$\frac{\delta\vec{M}}{\delta t}=\frac{1}{\tau_{ex}}\vec{\mu}\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}. \quad (6)$$

The resulting torque can be decomposed into a constant and a periodic term.[10] For long DWs, the periodic part averages to zero and two constant *spin-torque* terms appear. Considering first the situation with no spin flip (taking τ_{sf} infinite),

$$\left.\frac{\delta\vec{M}}{\delta t}\right|_{st} = \frac{1}{\tau_{ex}}\begin{pmatrix}\frac{g\mu_B}{2}\\ 0\\ \langle\mu_y\rangle\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = -\frac{g\mu_B}{2}\dot{\theta}\vec{e}_\theta, \quad (7)$$

which is not in a proper direction to drive the DW into a steady motion[9] (it only pushes for a time in the ns range taken by the demagnetizing field to stop the motion). The effect of the spin-flip-related term is therefore important as it leads to a torque in the right direction for pushing the DW,

$$\left.\frac{\delta\vec{M}}{\delta t}\right|_{sf} = \frac{1}{\tau_{ex}}\langle\mu_\theta\rangle\vec{e}_\theta\times\vec{m} = \frac{g\mu_B}{2}\dot{\theta}\frac{\tau_{ex}}{\tau_{sf}}\vec{e}_y. \quad (8)$$

This conribution, known as the "beta term," was recently derived by Li and Zhang[8] who pointed out the importance of this nonadiabatic component for current-induced DW motion. However, it appears that this contribution is only obtained when considering the reaction from the conduction electrons through the Landau-Lifshitz equation. In other words, this is true for spin-flip scattering events that do not conserve the total magnetization. Indeed, if one expresses instead that $\frac{\delta\vec{M}}{\delta t}=-\frac{\delta\vec{\mu}}{\delta t}$, another term reenters the equation given by

$$\frac{1}{\tau_{sf}}\langle\mu_y\rangle\vec{e}_y = -\frac{g\mu_B}{2}\dot{\theta}\frac{\tau_{ex}}{\tau_{sf}}\vec{e}_y, \quad (9)$$

which cancels out the one derived previously. This means that the beta term comes from the nonadiabaticity of the whole system consisting of the magnetization plus the conduction spins. Hence, for conservative spin-flip scattering events like electron-magnon scattering, the overall nonadiabatic spin torque should be zero. On the other hand, spin-flip events due to phonons (made possible when spin-orbit scattering is present), or spin-orbit impurities, remove some magnetization from the system (which is taken as an angular momentum) and generate a nonzero spin pressure on the DW. Keeping this in mind but considering there is no easy way to give the relative importance of the two kinds of scattering events, we will in the following, keep the value of Eq. (8) as a maximum for the nonadiabatic torque.

Under an electric-current density $\vec{j}=j\vec{e}_y$, and choosing to introduce the magnetization gradient in the wall, we then get for the two torques,

$$\left.\frac{d\vec{M}}{dt}\right|_{st} = \frac{jP}{e}\frac{g\mu_B}{2}\frac{\partial\vec{m}}{\partial y}, \quad (10)$$

$$\left.\frac{d\vec{M}}{dt}\right|_{sf} = -\frac{jP}{e}\frac{g\mu_B}{2}\frac{\tau_{ex}}{\tau_{sf}}\left(\vec{m}\times\frac{\partial\vec{m}}{\partial y}\right), \quad (11)$$

where the polarization P is added as a prefactor to account for the partial polarization of the charge carriers. So the adiabatic spin torque accounted by various authors[10,17–19] can be explained as the reaction from the precession of the conduction electrons magnetic moments around the effective field. The second term originating from the nonadiabatic spin-flip scattering has an amplitude reduced by a factor τ_{ex}/τ_{sf} typically around 1/30. When writing the micromagnetic equations of motion of the DW, it has been shown that only this (small) second term applies a pressure that pushes the wall.[8,19] This can also be understood in the following manner: from the Landau-Lifshitz equation it can be seen that a precession of magnetization is equivalent to a field whose direction is along the rotation vector. Because the main spin evolution of electrons crossing the wall is a rotation following the DW magnetization, the equivalent field they generate is directed perpendicular to the plane of the DW and hence to the magnetization of the domains. As a result, this does not push the wall in any specific direction but only induces a canting of the wall magnetization. On the other hand, spin-flip terms correspond to an effective field perpendicular to both the local magnetization and the electrons direction of propagation. Considering average quantities during DW crossing, it lies globally along the direction of the magnetization of the domain from where the electrons are coming. Therefore, this is applying a pressure, which tends to push the DW along the direction of the conduction electrons.

Hence, our classical model leads to deformation and pressure expressions, which are consistent with the results of semiclassical theories. The advantage of our simple formalism is twofold. It is now possible to study in detail the spatial evolution of the torque along the width of the DW, and it also makes it possible to explore the torque for wall widths of the order of the precession length: $\Delta\simeq\lambda_L$. Indeed, the only essential approximation of the model is that the reflected part of the plane electron wave impinging on the DW can be neglected. This imposes DW widths much larger than the Fermi wavelength of the electrons,[10,11] which is not a very stringent condition since the latter is around 3 Å in metals.

NUMERICAL CALCULATIONS FOR THIN WALLS

We have numerically calculated the current-induced torques exerted locally on the wall, from the electrons magnetic-moment evolution derived from Eq. (1), but now without the simplifications associated to large domain walls. In the limit of spin-flip scattering that does not conserve magnetization (taking the full beta term), pressure and deformation terms labeled, respectively, Γ_p and Γ_d are given by

$$\Gamma_d(y) = \frac{jP}{e}\left\langle\frac{\mu_y(y,v)}{\tau_{ex}}\right\rangle_v, \quad (12)$$

$$\Gamma_p(y) = -\frac{jP}{e}\left\langle\frac{\mu_\theta(y,v)}{\tau_{ex}}\right\rangle_v. \quad (13)$$

In our simulations, these are also averaged on the different directions the Fermi velocity can take on the Fermi sphere. This induces decoherence of the torques as electrons traveling with different components of their velocity on the direction perpendicular to the DW, v_y, have different Larmor precession lengths. The resulting effect is a damping of the averaged spin precession (hence the oscillating part of both torques) after a few Larmor periods $2\pi\tau_{ex}$. This allows us to extend the work of Waintal and Viret[10] who derived the pe-

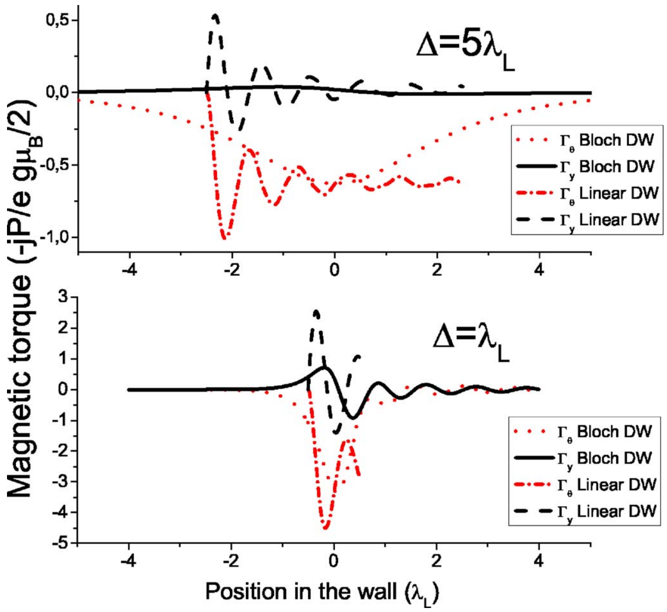

FIG. 3. (Color online) Distortion and pressure torque parameters Γ_d and Γ_p in the domain wall under a current density j with $\Delta=5\lambda_L$ (top curve) and $\Delta=\lambda_L$ (bottom) and $\frac{\lambda_{ex}}{\lambda_{sf}}=1/30$.

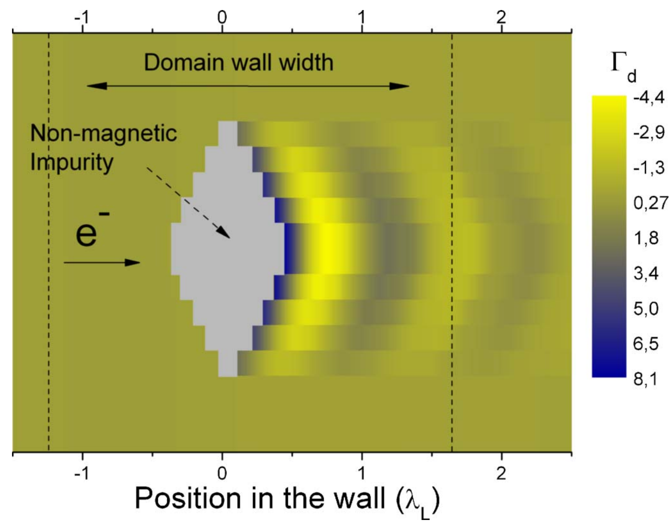

FIG. 4. (Color online) Numerical simulation of the effect of a nonmagnetic impurity on the torque v_p, for a Bloch DW of width $\Delta=2.5\lambda_L$. The figure is a superposition of one-dimensional simulations for the varying width of the nonmagnetic material, considering it does not affect the domain-wall profile.

riodic torque but did not include the spin-flip scattering, which leads to a zero average of Γ_p. Figure 3 presents numerical computations of the torques along different walls: linear and Bloch type, with $\Delta=5\lambda_L$ or $\Delta=\lambda_L$ (with $\lambda_L=\frac{v_F\tau_{ex}}{2\pi}$) and $\lambda_{sf}=50\lambda_{ex}$. An obvious result is that the exact DW shape is important for the torques as also pointed out by Xiao *et al.*[20] For a linear wall (dashed line) where $\dot{\theta}$ is discontinuous, oscillations are enhanced compared to those in a Bloch wall (solid line). Thus, in the pure Bloch walls of bulk materials, the periodic torque is small but in short linear walls, like those expected in constrictions,[21] periodic components are significant. In real systems, where DWs are often pinned on defects or impurities, large torque oscillations can also be expected because of the abrupt perturbations defects have on the local magnetization. This is schematically shown in Fig. 4 for a Bloch wall pinned on a nonmagnetic impurity. Beyond the simplicity of the chosen system, the result of these large torque oscillations should be an efficient depinning of the walls.

Let us now turn to the average torque values on the domain-wall width Δ. To allow for a straightforward comparison with Ref. 14 we introduce the two velocitylike parameters b_j and c_j corresponding to the torques exerted on the magnetization, respectively, in the $\vec{e}_\theta$ and $\vec{e}_y$ directions,

$$b_j=\frac{1}{\Delta}\int_y \frac{1}{\left|\frac{\partial m}{\partial y}\right|}\Gamma_d(y)dy, \quad c_j=-\frac{1}{\Delta}\int_y \frac{1}{\left|\frac{\partial m}{\partial y}\right|}\Gamma_p(y)dy. \tag{14}$$

The dependence of these quantities with Δ is presented in Fig. 5. Asymptotic values for large domain walls ($\Delta \gg \lambda_L$) are those derived previously (not shown). For thin domain walls with width close to the Larmor length λ_L, the oscillatory torque neither averages out to zero nor is it washed out by the average on the Fermi sphere. Interestingly, as Δ is reduced, the c_j term oscillates but globally increases in magnitude. Keeping in mind that the beta term is responsible for a constant positive contribution very likely to be overesti-

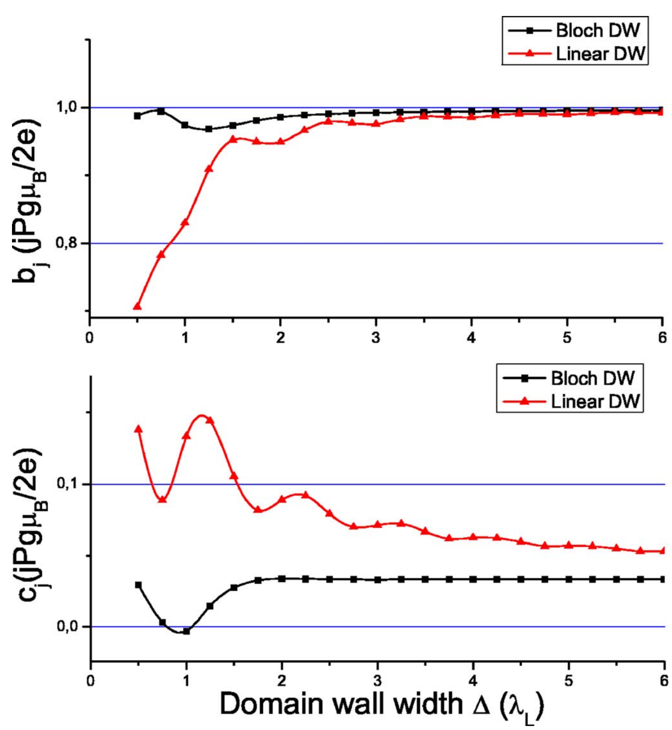

FIG. 5. (Color online) Averaged distortion (b_j) and pressure (c_j) as a function of the domain-wall width Δ for linear and Bloch walls. The component applying pressure (c_j) tends to the beta term in the limit of long walls and increases as thickness decreases towards the Larmor precession lengthscale (especially for linear walls).

mated in the simulations, one can see that the pressure can even be reversed for thin Bloch walls. In any case, in the thin-wall regime, precessing terms clearly dominate. This is understandable because when first entering the wall, all the electrons are precessing in the same way and it takes some time for decoherence to damp the precession. Thus, in appropriate systems, the effective torque could be an order of magnitude larger than that in conventional domain walls. This is particularly relevant for ferromagnetic materials with very large anisotropy, where the predicted domain-wall width is only a few nanometers, which is close to the Larmor precession length λ_L. One could also imagine making walls thin in nanometer-sized constrictions. Interestingly, for thin walls, conduction electrons are injected in the magnetic domain beyond the DW with their spin significantly misaligned. Spin-flip events will eventually realign the spins thus leaving some angular momentum beyond the DW. It is likely that this will result in large magnon emission in the domains. One can conjecture that for a large current, the magnetization beyond the DW could be destabilized. Another way of enhancing the current-induced pressure is by increasing λ_L like in magnetic semiconductors. Interestingly in GaMnAs, DWs have been shown to move under much lower current densities[22] corresponding to an efficiency of 30% for the spin torque (an order of magnitude higher than in NiFe). From the present calculations, the ease of the current-induced domain-wall motion in this system can be understood by the combination of two properties: firstly, the exchange is relatively weak and the anisotropy large, which probably makes for a small ratio of wall thickness to Larmor precession length. Secondly, the very large spin-orbit coupling of the conduction band is likely to induce a considerable amount of pure non-adiabatic (i.e., nonmagnetization conserving) spin-flip scattering events. Hence, the beta term, which governs the pressure on the wall, is expected to be much larger than in 3d metals.

In summary, the model we develop here allows us to study the evolution of the conduction electrons spins as they cross a domain wall. For long walls, the average torque is mainly due to a constant term coming from the global spin rotation while following the magnetization direction. This tends to distort the DW without pushing it. The only contribution applying a pressure is that due to nonadiabatic spin-flip scattering events, which amounts to a few percent of the total torque in 3d metals. For thinner walls, the contribution from the periodic torque, which depends on the exact shape of the DW, becomes important.

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