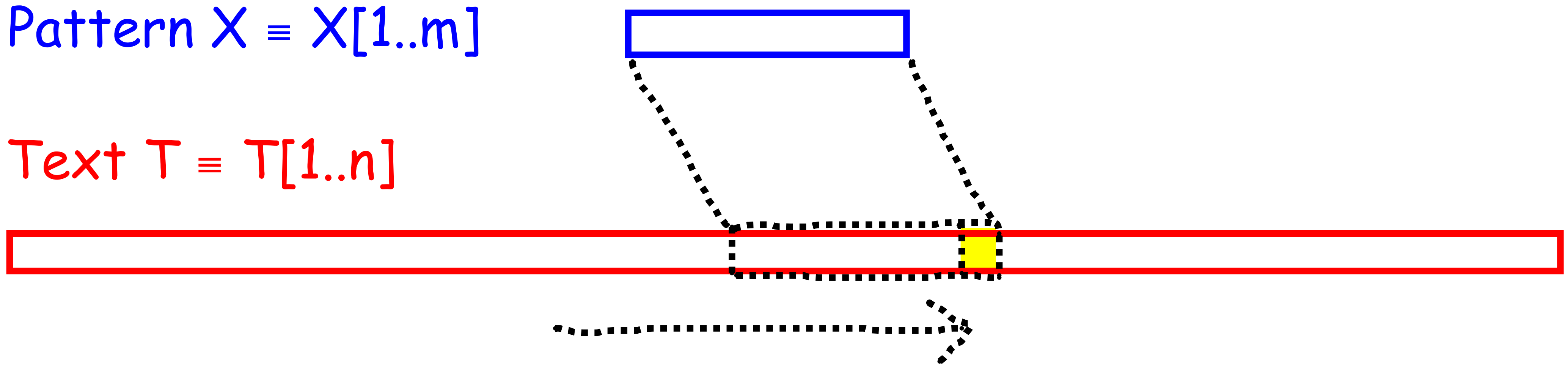
# Simple Real-Time Constant-Space String Matching

Dany Breslauer, Roberto Grossi and Filippo Mignosi

# Real-time string matching

Pattern X ≡ X[1..m]

Text T ≡ T[1..n]



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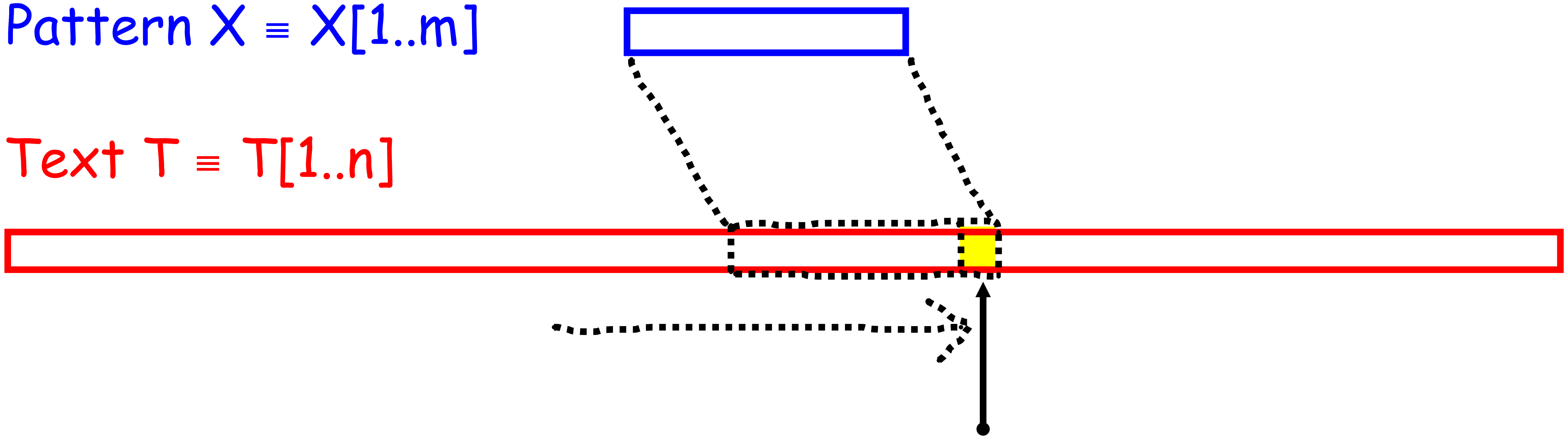

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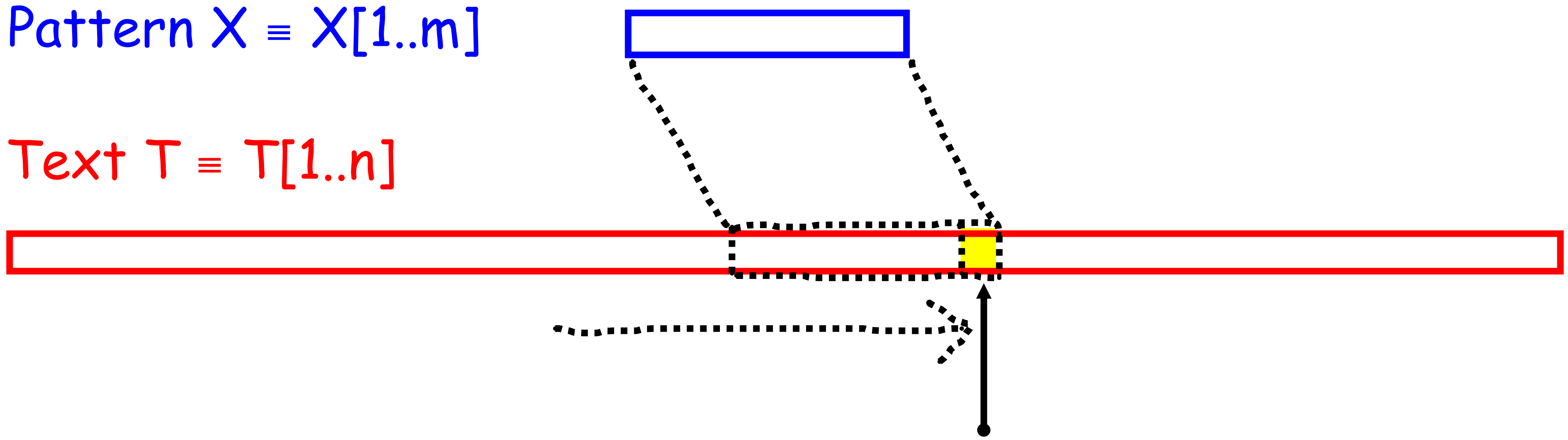


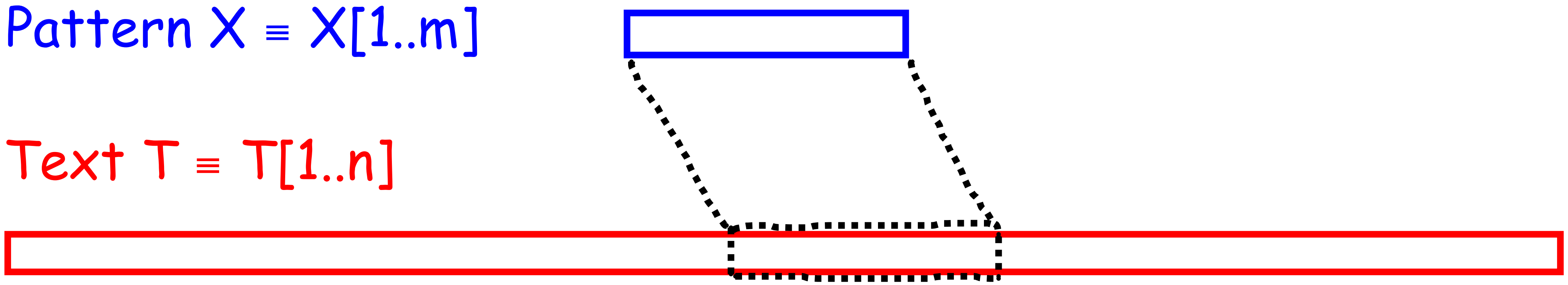
O(1) worst-case time to answer after reading the text symbol

Different from real-time streaming s.m., where X and T cannot be entirely stored!

# Constant-space string matching

Pattern X ≡ X[1..m]

Text T ≡ T[1..n]



(a.k.a. in-place)

O(1) working space apart from that required by X and T

O(log n) bits

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- Take a simple version of the constant-space Crochemore-Perrin (CP) algorithm

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- Make CP also real-time by running **two** instances simultaneously

# Some related work

- Galil '81: real-time string matching
- Galil, Seiferas '83: constant space
- Karp, Rabin '87: randomized constant space real-time
- Crochemore, Perrin '91: constant space
- Gasieniec, Plandowski, Rytter '95: constant space
- Gasienec, Kolpakov '04: real-time + sublinear space (extends GPR'95)
- • • more papers [Crochemore, Rytter '91,'95] [Crochemore '92] [...]
- Porat, Porat '09: randomized streaming, O(log m) space, no real-time
- Breslauer, Galil '10: randomized real-time streaming, O(log m) space

# Our result

- Real-time constant-space string matching
  - O(1) words in addition to those for read-only X and T
  - O(1) worst-case time to answer after each text symbol

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deterministic

- Real-time constant-space string matching
  O(1) words in addition to those for read-only X and T
  O(1) worst-case time to answer after each text symbol

Not to be confused with

Montecarlo

- Real-time streaming string matching
  O(log m) memory words (X and T cannot be kept)
  O(1) worst-case time to answer after each text symbol

## We propose a simple way to combine the two features

- Take a simple version of the constant-space Crochemore-Perrin (CP) algorithm

- Make CP also real-time by running two instances simultaneously

## Simple version of the Crochemore-Perrin (CP) algorithm

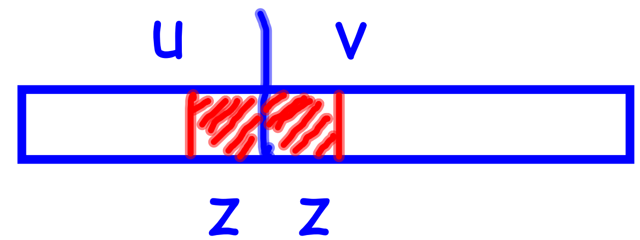
Consider a non-empty prefix-suffix factorization X = u v

The local period is the shortest z such that

z is suffix of u or vice versa

and

z is a prefix of v or vice versa

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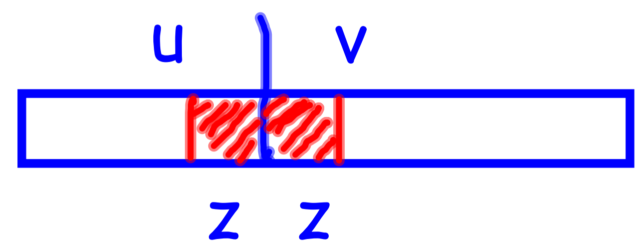


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Example: X = abaaaba

X = u v

| | | |
|---|---|---|
| a baaaba | ab aaaba | aba aaba |
| ba ba | aaab aaab | a a |
| z | | |

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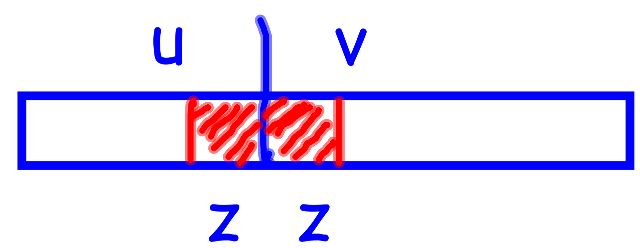


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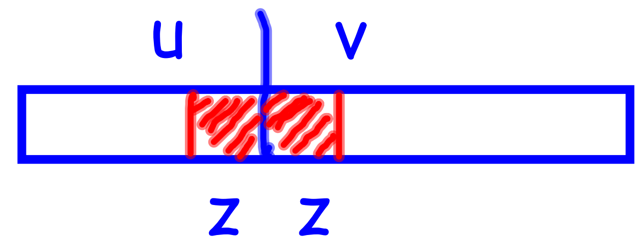


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$\mu(u,v) \equiv$ length of the local period

Critical factorization if $\mu(u,v) = \pi(X)$ [len. of the period of $X$]

Example:

| | | |
|---|---|---|
| X = u v | | |
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| | | |
|---|---|---|
| | X = u v | |
| a baaaba | ab aaaba | aba aaba |
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| | z | |

Example:

```
 a  baaaba
ba  ba
```

```
X = u   v
   ab   aaaba
 aaab   aaab
        z
```

```
aba  aaba
  a  a
```

critical!

the period is "abaa"

Example:

| | | |
|---|---|---|
| | | X = u v |
| a baaaba | ab aaaba | aba aaba |
| ba ba | aaab aaab | a a |
| | | z |

Example:

a baaaba

ba ba

ab aaaba

aaab aaab

aba aaba

a a

Critical Factorization Theorem (Cesari and Vincent):

Among $\pi(X) - 1$ consecutive factorizations:

at least one is a critical factorization

Example:

```
 a  baaaba        ab  aaaba        aba  aaba
ba  ba          aaab  aaab           a  a
```

Critical Factorization Theorem (Cesari and Vincent):

Among $\pi(X) - 1$ consecutive factorizations:

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There always exists a critical factorization $X = u\,v$ such that $|u| < \pi(X)$

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How to handle mismatches?

# We propose a simple way to combine the two features

- Take a simple version of the constant-space Crochemore-Perrin (CP) algorithm

- Make CP also real-time by running two instances simultaneously

## Basic Real-Time Algorithm

Interleave O(1) comparisons from the forward scan with O(1) comparisons from the back fill

X = ab  aaaba  critical factorization

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z

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≠

abaabaaabaa

mismatch

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Interleave O(1) comparisons from the forward scan
with O(1) comparisons from the back fill

X = ab  aaaba  critical factorization

$$\overbrace{\text{abaa}}^{z}\text{aba} \qquad\qquad \xleftrightarrow{|z|+1}\ \text{abaaaba}$$

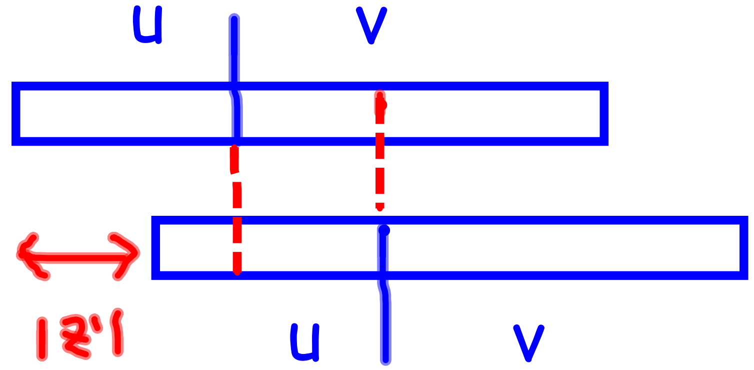
$$\text{abaa}\overset{\neq}{\text{b}}\text{aaabaa} \qquad\qquad \text{abaabaaabaa}$$

shift by |z|+1 positions

(and charge the O(|z|+1) cost to the symbols in z in real time)

By contradiction, suppose there is a valid shift that is shorter...

... recall that $|u| < \pi(X)$, the length of the period



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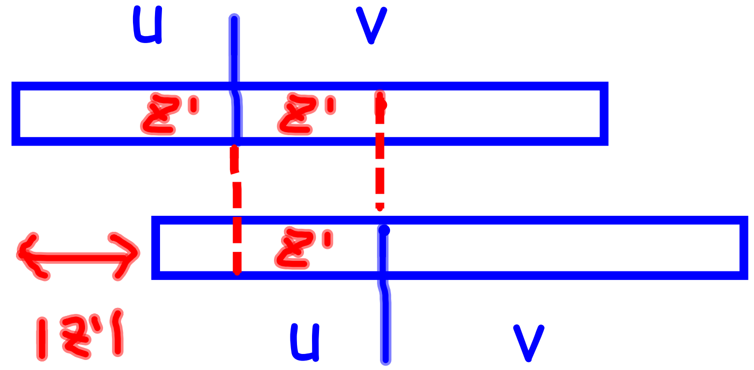


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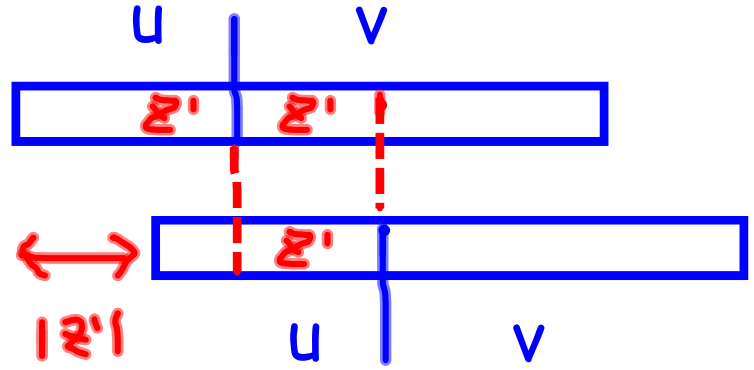


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Contradiction: a local period at u v that is shorter than $\pi(X)$!!

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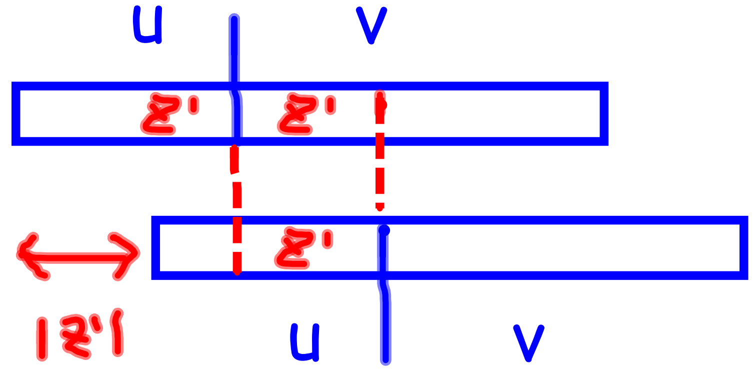


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Contradiction: a local period at u v that is shorter than $\pi(X)$!!

It follows from the Crochemore-Perrin result [other case $|z'| \geqslant \pi(X)$ not displayed: periodicity rules out occurrences]

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Output an occurrence when the forward scan terminates (and interrupt the back fill if needed)

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- if $z \neq v \Rightarrow$ shift by $|z|+1$ positions and reset $z$ = empty
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Total cost is $O(1)$ worst-case per symbol: the algorithm is real-time

Q: What if $|u| > |v|$?

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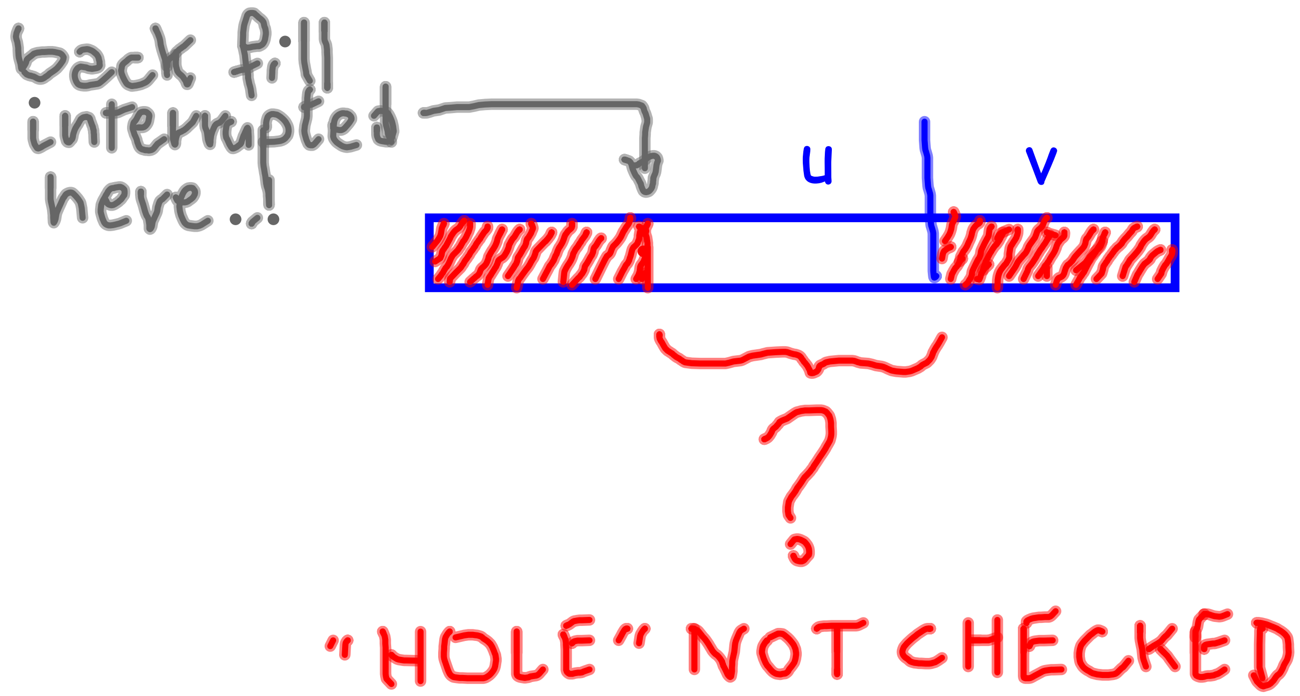

## Real-Time Variation of CP

Consider a 3-way non-empty factorizaton $X = u\ v\ w$ such that

$X = (uv)\ w$ is a critical factorization with $|uv| \le |w|$

OR

$X = (uv)\ w$ is a critical factorization, and
$X' = u\ (vv')$ is a critical factorization for a prefix $X'$ of $X$
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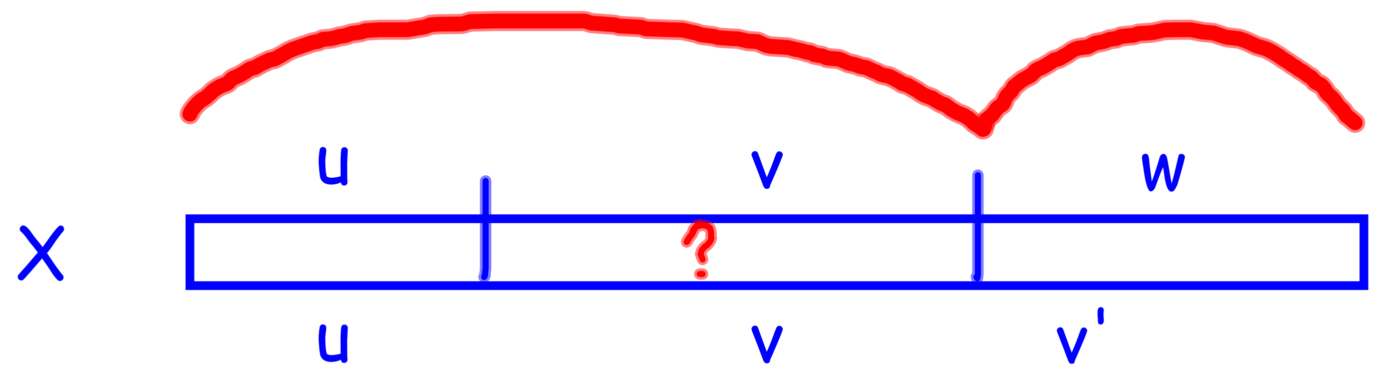


Recall we may leave a "hole" to the left of $w$:
this hole has to be covered by $X'$...

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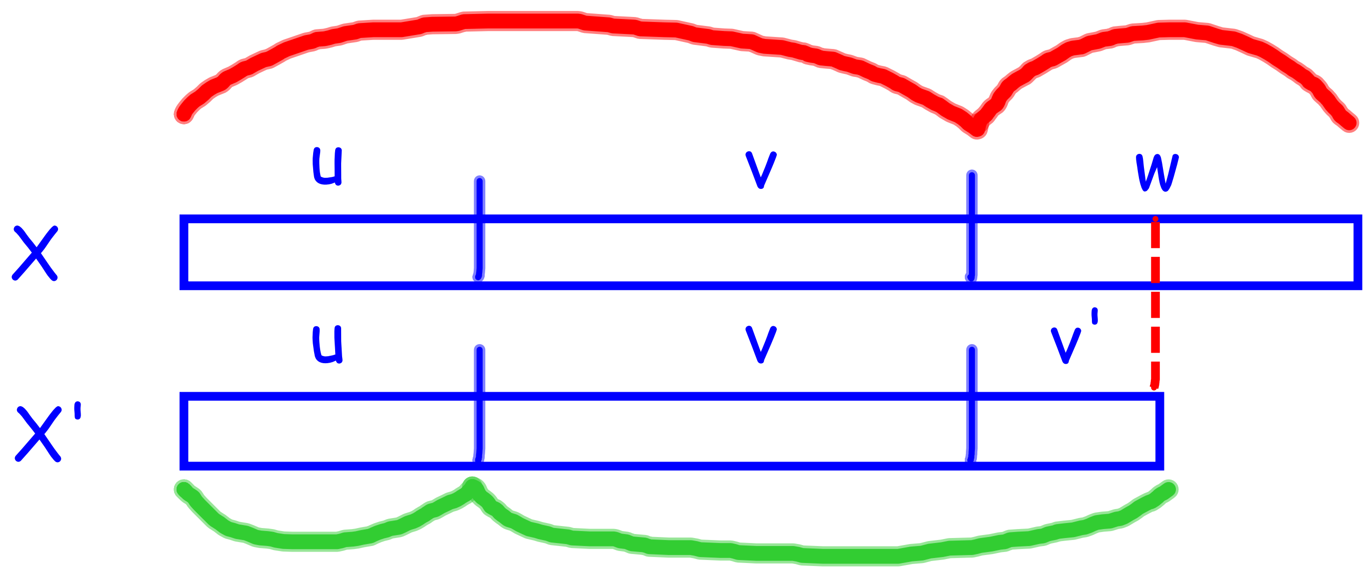


Note that $X'$ is entirely matched since $|u| \leq |vv'|$

## Real-Time Variation of the CP Algorithm

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Simple pseudocode

## Pattern preprocessing

GOAL:

Find the desired 3-way non-empty factorizaton $X = u\ v\ w$
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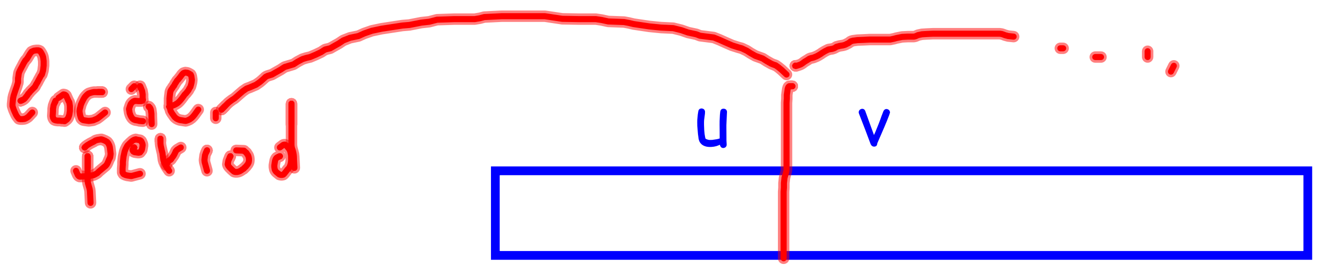
Some more definitions...

A factorization $u\ v$ is left-external if $|u| \leq \mu(u,v)$ for non-empty $u, v$




Define $L(X) = \{\ u\ v : X = u\ v$ is left-external $\}$

L(X) non-empty because of the Critical Factorization Theorem

# Pattern preprocessing

Let $X = u_1\ w$ be the first critical factorization in $L(X)$

HINT: use CP preprocessing on the prefixes of X

Lemma: $u\ v \in L(X) \Rightarrow$ prefix $X' = u'\ v'$ s.t. $\mu(u',v') = \mu(u,v)$

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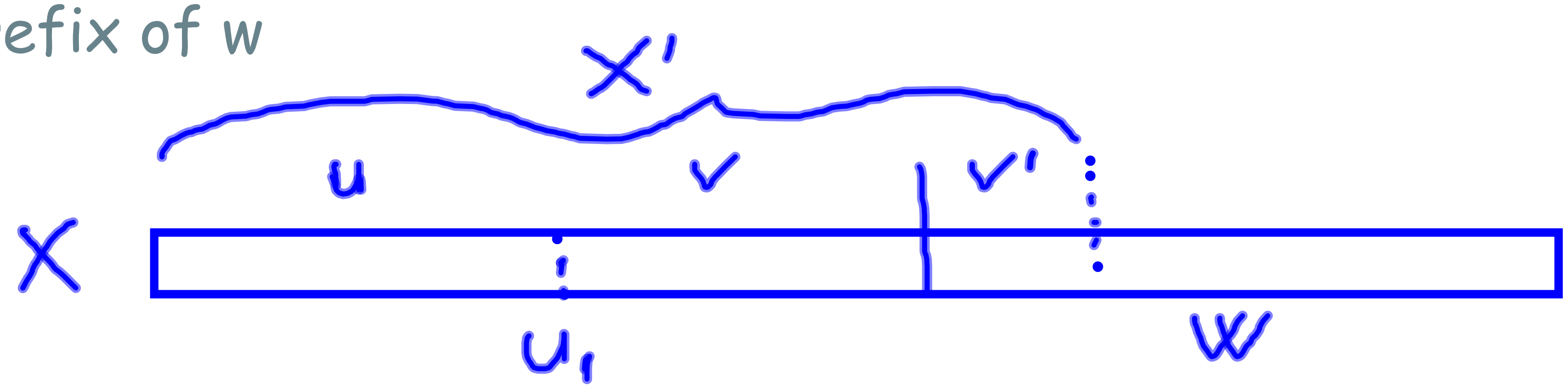

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It is $|u| \leq \mu(u, v) \leq \mu(u, vv') = \mu(u, vw) \leq |vv'|$

Q.E.D.

Questions?