SIMPLE RELATIONSHIP BETWEEN POWER INPUT AND MIXING TIME IN TURBULENT AGITATED VESSEL

SETSURO HIRAOKA AND RYUZO ITO
Department of Industrial Chemistry, Nagoya Institute of Technology, Nagoya 466

Introduction

An agitated vessel has often been used as a mixing device, and its ability as a mixer is usually estimated by mixing or blending time. Mixing time for various type impellers has been studied by many investigators and correlated with vessel dimensions and operating conditions based on dimensional analysis, or directly with other characteristics, as power input, impeller jet flow rate, turbulent properties, etc. Van de Vusse[10] proposed the correlation of mixing time with impeller jet flow rate, and Kamiwano *et al.*[4] derived the dependencies of mixing time both on impeller jet flow rate and eddy diffusion, i.e., $N\theta$ being correlated with N_{qd} and N_P.

$$\frac{1}{N\theta}=k\left\{\left(\frac{d}{D}\right)^3 N_{qd}+0.21\left(\frac{d}{D}\right)\sqrt{\frac{N_P}{N_{qd}}}\right\}\{1-e^{-13(d/D)}\} \tag{1}$$

On the other hand, the authors have proposed a relationship between power input and impeller jet flow rate, so it is expected that mixing time can be correlated with power input only.

This paper deals with the derivation of correlation variables to mixing time based on the model analysis of circulation flow, and proposes the correlation of mixing time with friction factor, i.e., power input.

1. Model Analysis

Considering a system of fluid particle on a stream line circulation flow, the mass balance of component A in this particle is expressed by the use of substantial derivative as; (see **Fig. 1**)

$$\frac{DC_A}{Dt}=\varepsilon_{\mathrm{turb}}\nabla^2 C_A \tag{2}$$

where the eddy diffusivity is assumed to be constant. As it has been reported by Nagase *et al.*[5] that the dimensionless velocity profiles of circulation flow based on circulation flow rate and vessel diameter are almost identical for some impellers, Eq. (2) is rewritten in dimensionless form by using circulation time, V/Q_c, and vessel diameter, D, as the characteristc variables.

$$\frac{DC_A^*}{D(tQ_c/V)}=\left(\frac{\varepsilon_{\mathrm{turb}}V}{Q_cD^2}\right)\nabla^{*2}C_A^* \tag{3}$$

where the superscript, *, means dimensionless variable. Equation (3) gives two independent dimensionless variables, (tQ_c/V) and $(\varepsilon_{\mathrm{turb}}V/Q_cD^2)$, controlling the mixing process in the fluid particle. Under a fixed condition to the end of turbulent mixing process, mixing time, θ, can be expressed as;

$$\left(\frac{\theta Q_c}{V}\right)=f_1\left(\frac{\varepsilon_{\mathrm{turb}}V}{Q_cD^2}\right) \tag{4-a}$$

or

$$\left(\frac{\theta\varepsilon_{\mathrm{turb}}}{D^2}\right)=f_2\left(\frac{\varepsilon_{\mathrm{turb}}V}{Q_cD^2}\right) \tag{4-b}$$

Two dimensionless variables in Eq. (4-b) are modified with such measurable quantities as friction factor, rotational speed, impeller jet flow rate and impeller dimensions. The eddy diffusivity is assumed to be constant proportional to the apparent friction velocity at the impeller tip specifying the flow pattern in turbulent agitated vessel and to the vessel diameter relating to the scale of large eddy, i.e.,

$$\varepsilon_{\mathrm{turb}}\propto(\beta u_d^*)D=\sqrt{\frac{f}{2}}\left(\beta^2\frac{\pi}{2}ND^2\right) \tag{5}$$

where the relation between u_d^* and $\sqrt{f/2}$, previously proposed[2], is used to derive the last equality, and the friction factor can be calculated from power input[1].

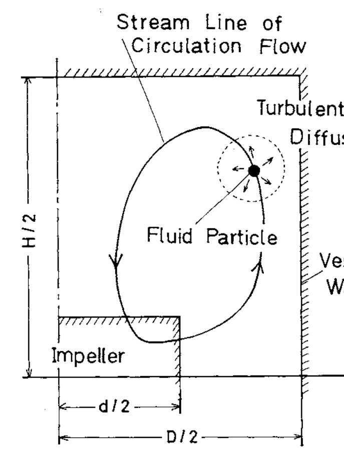

Fig. 1 Model of circulation flow and diffusion process

Received July 19, 1976. Correspondence concerning this article should be addressed to S. Hiraoka. R. Ito is at Dept. of Chem. Eng., Osaka Univ., Toyonaka 560.

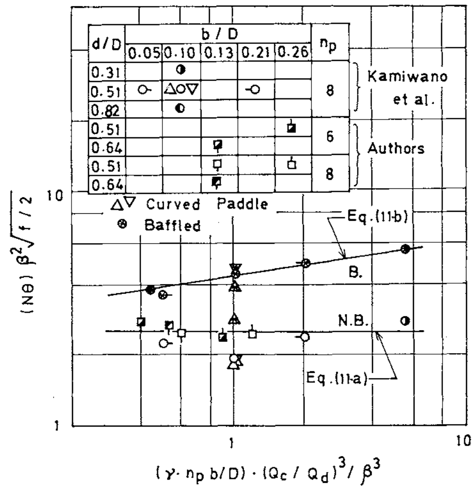

Fig. 2 Correlation of mixing time with friction factor (Impeller at vessel center)

$$\frac{f}{2}=\frac{\tau_w}{\rho v_\theta^2}=N_P\frac{4(d/D)^2(d/H)}{\pi^4\beta^2(1+\alpha)} \tag{6}$$

On the other hand, impeller jet flow rate is expressed as[3];

$$Q_d=1.04\left(\gamma\cdot\frac{n_p b}{D}\right)^{1/3}\left(\frac{V}{D}\right)u_d^* \tag{7}$$

Therefore,

$$\left(\frac{\theta\varepsilon_{\text{turb}}}{D^2}\right)\propto(N\theta)\beta^2\sqrt{\frac{f}{2}} \tag{8-a}$$

$$\left(\frac{\varepsilon_{\text{turb}}V}{Q_c D^2}\right)\propto\frac{\beta}{(\gamma\cdot n_p b/D)^{1/3}}\left(\frac{Q_d}{Q_c}\right) \tag{8-b}$$

Eq. (8-b) depends only on the impeller dimensions, because the ratio of circulation flow rate to impeller jet flow rate has been given by Yamamoto[11] as follows;

$$\frac{Q_c}{Q_d}=1+0.16\left\{\left(\frac{D}{d}\right)^2-1\right\} \tag{9}$$

As a result, Eq. (4-b) is rewritten as follows;

$$(N\theta)\beta^2\sqrt{\frac{f}{2}}=f_3\left\{\left(\gamma\cdot\frac{n_p b}{D}\right)\left(\frac{Q_c}{Q_d}\right)^3\Big/\beta^3\right\} \tag{10}$$

2. Correlation of Experimental Data

The mixing time measured by electroconductivity method of Kamiwano *et al.*[4] are replotted on the new variables in **Fig. 2**, in which some data by the authors are also plotted. In these experiments paddle impellers were set at the vessel center.

For the non-baffled agitated vessel, it appears in Fig. 2 that the ordinate has a constant value for all impeller dimensions. Whereas for the baffled agitated vessel, the ordinate is higher than that for the non-baffled vessel and depends slightly on the abscissa. As a result, the following correlation equations are obtained.

$$(N\theta)\beta^2\sqrt{\frac{f}{2}}\doteqdot 2.5 \qquad \text{(non-baffled)} \tag{11-a}$$

$$(N\theta)\beta^2\sqrt{\frac{f}{2}}=4.3\left\{\left(\gamma\cdot\frac{n_p b}{D}\right)\left(\frac{Q_c}{Q_d}\right)^3\Big/\beta^3\right\}^{0.15} \qquad \text{(baffled)} \tag{11-b}$$

Equations (11-a) and (11-b) are transformed into the usual form as follows.

For the non-baffled vessel, the friction factor is correlated with the modified Reynolds number in turbulent range.

$$\frac{f}{2}=0.121Re_G^{-1/3}=0.121\left\{\frac{\pi}{4}\left(\frac{\beta D}{d}\right)\ln\frac{D}{d}\right\}^{-1/3}Re_d^{-1/3} \tag{12}$$

Combining Eqs. (6), (11-a) and (12), and using the power expression of (d/D) for the range, $0.3<(d/D)<0.8$, Eq. (11-a) is approximately represented by;

$$N\theta=11.5\left(\frac{D}{d}\right)^{1.23}\left(\frac{H}{d}\right)^{0.5}\sqrt{\frac{1+\alpha}{N_P}}=4.8\left(\frac{D}{d}\right)^{0.82}Re_d^{1/6} \tag{13}$$

For the baffled vessel, power input is approximately expressed as;

$$N_{P\max}\doteqdot 6.6\left(\frac{n_p b}{d}\right) \tag{14}$$

This equation satisfies well the maximum power input data by Nagata *et al.*[6,7]. The same procedure above transforms Eq. (11-b) into;

$$N\theta=16.8\left(\frac{n_p b}{d}\right)^{0.15}\left(\frac{D}{d}\right)^{1.40}\left(\frac{H}{d}\right)^{0.5}\sqrt{\frac{1+\alpha}{N_{P\max}}}=7.2\left(\frac{n_p b}{d}\right)^{-0.35}\left(\frac{D}{d}\right)^{1.40}\left(\frac{H}{d}\right)^{0.5} \tag{15}$$

3. Discussion

For the non-baffled vessel, Eq. (13) shows that $N\theta$ is proportional to the one-sixth power of impeller Reynolds number, and this result coincides with data by Yamamoto[11].

For the baffled vessel, Norwood and Metzner[8] had shown the following correlation equation for the range of $Re_d>10^5$.

$$\frac{(N\theta)(d/D)^2}{(N^2d/g)^{1/6}(H/D)^{1/2}}\doteqdot 5 \tag{16-a}$$

This is rearranged into;

$$N\theta\doteqdot 5\left(\frac{N^2d}{g}\right)^{1/6}\left(\frac{D}{d}\right)^{3/2}\left(\frac{H}{d}\right)^{1/2} \tag{16-b}$$

Comparison of Eqs. (15) and (16-b) shows about the same dependence of $N\theta$ on the vessel dimensions, but different on the rotational speed.

Van de Vusse[10] has also derived the following correlation from his data for some types of impellers for $Re_d\doteqdot 10^5$.

$$\left(\frac{N\theta d^2p}{V}\right)\left(\frac{\rho d^2N^2}{gH\Delta\rho}\right)^{0.25}\doteqdot 9 \tag{17-a}$$

which is transformed into;

$$N\theta=7.1\left(\frac{N^2d}{g}\right)^{-0.25}\left(\frac{\Delta\rho}{\rho}\right)^{0.25}\left(\frac{d}{p}\right)\left(\frac{D}{d}\right)^2\left(\frac{H}{d}\right)^{0.75} \tag{17-b}$$

Equation (17-b) shows a higher dependence of $N\theta$ on the vessel dimensions compared with that in Eq. (15). This is probably due to the predominant circulation flow of the propeller-type impeller. The same tendency is found in the results by Prochazka and Landau[9].

When the impeller is set near the bottom, it has been reported by Kamiwano *et al.*[4] that the mixing time is slightly shorter than that predicted by Eq. (15).

Concluding Remarks

From the model analysis of circulation flow, two independent variables for mixing time are derived and used to correlate the available experimental data. The simple correlation equation (11-a) obtained for the non-baffled vessel is agreement with the data by Yamamoto. And the correlation equation (11-b) for the baffled vessel shows almost the same dependency of $N\theta$ on the vessel dimensions as that given by Norwood and Metzner, but different on the rotational speed.

Nomenclature

b	= impeller width	[cm]
C_A	= concentration of component A	[mol/cm^3]
D	= vessel diameter	[cm]
d	= impeller diameter	[cm]
f	= friction factor	[—]
g	= gravitational acceleration	[cm/sec^2]
H	= vessel height	[cm]
N	= rotational speed	[sec^{-1}]
n_p	= number of blade	[—]
P	= power input	[g·cm^2/sec^3]
p	= pitch	[cm]
Q_c	= circulation flow rate	[cm^3/sec]
Q_d	= impeller jet flow rate	[cm^3/sec]
t	= time	[sec]
u_d^*	= apparent friction velocity at impeller tip ($=(D/d)\sqrt{\tau_w/\rho}$)	[cm/sec]
V	= vessel volume	[cm^3]
v_θ	= characteristic velocity ($=(\pi/2)Nd\beta$)	[cm/sec]
α	= ratio of torque at bottom wall to that at side	[—]
β	= variable defined by $2\ln(D/d)/(D/d-d/D)$	[—]
γ	= variable defined by $\{(d/D)^5\ln(D/d)/\beta^5\}^{1/3}$	[—]
ε_{turb}	= eddy diffusivity of mass	[cm^2/sec]
θ	= mixing time	[sec]
ρ	= fluid density	[g/cm^3]
τ_w	= average shear stress at wall	[g/cm·sec^2]
N_P	= $P/\rho N^3d^5$	
N_{qd}	= Q_d/Nd^3	
Re_d	= $d^2N\rho/\mu$	

Literature Cited

1) Hiraoka, S., I. Yamada, N. Doi, H. Takeda, A. Kawai, Y. Usui and R. Ito: *Bull. Nagoya Inst. Tech.*, **26**, 239 (1974).
2) Hiraoka, S., R. Ito, I. Yamada, K. Sawada, M. Ishiguro and S. Kawamura: *J. Chem. Eng. Japan*, **8**, 156 (1975).
3) Hiraoka, S. and R. Ito: *ibid*, **8**, 323 (1975).
4) Kamiwano, M., K. Yamamoto and S. Nagata: *Kagaku Kōgaku*, **31**, 365 (1967).
5) Nagase, Y., T. Iwamoto, S. Fujita and T. Yoshida: *ibid.*, **38**, 519 (1974).
6) Nagata, S., K. Yamamoto, T. Yokoyama and S. Shiga: *Memo. Fac. Eng. Kyoto Univ.*, **19**, 274 (1957).
7) Nagata, S.: "Mixing principle and applications" Kodansha-a Halsted Press Book (1975).
8) Norwood, K. W. and A. B. Metzner: *AIChE J.*, **6**, 432 (1960).
9) Prochazka, J. and J. Landau: *Collect. Czech. Chem. Commun.*, **26**, 2961 (1961).
10) Van de Vusse, J. G.: *Chem. Eng. Sci.*, **4**, 178, 209 (1955).
11) Yamamoto, K.: "Saikin no Kagaku Kogaku", Maruzen (1964).