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SIMPLE-SEMI-CONDITIONAL VERSIONS OF MATRIX GRAMMARS WITH A REDUCED REGULATING MECHANISM

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Abstract. This paper discusses some conditional versions of matrix grammars. It establishes several characterizations of the family of the recursively enumerable languages based on these grammars. In fact, making use of the Geffert Normal forms, the present paper demonstrates these characterizations based on matrix grammars with conditions of a limited length, a reduced number of nonterminals, and a reduced number and size of matrices.

Keywords: Descriptional complexity, matrix grammars, simple-semi-conditional grammars

1 INTRODUCTION

Regulated grammars are significantly more powerful than ordinary context-free grammars, and this increase of the generative power represents their indisputable advantage. However, this advantage is achieved by an additional regulating mechanism. It is thus more than natural to reduce this regulating mechanism without any decrease of the generative power. The present paper discusses this reduction in terms of matrix grammars, which belong to the very basic types of regulated grammars. More specifically, it introduces simple-semi-conditional versions of these grammars and reduces their regulating mechanism. In these versions, a production may have an attached word, called *a context condition*, and its application requires that the attached condition occurs in the rewritten sentential form or, on the contrary, does not occur there.

Unfortunately, with the conditions of length one, these grammars do not increase their power at all as follows from Theorems 6.3.1 and 6.3.2 in [1]. As these grammars define only a proper subfamily of the family of recursively enumerable languages (see Theorem 2.12 on page 129 in [8]), the matrix grammars with context conditions of length one cannot define this family either, so they are hardly of any interest.

However, this paper considers simple-semi-conditional versions of matrix grammars with conditions longer than one and demonstrates that they increase their generative power. Indeed, the resulting conditional matrix grammars characterize the entire family of recursively enumerable languages. This paper presents several characterizations of this family by simple-semi-conditional versions of matrix grammars that have only one condition attached to their productions or matrices. As a matter of fact, these characterizations are achieved based on reduced versions of these grammars. This reduction consists in simultaneously bounding

1. number and length of conditions
2. number of nonterminals
3. number and size of matrices.

More specifically, Section 3 introduces matrix simple-semi-conditional grammars in whose every production has no more than one attached condition. It demonstrates that any recursively enumerable language can be described by a matrix simple-semi-conditional grammar with a single matrix containing six rules having conditions of length three while all the other rules are context-free in the grammar. Section 4 introduces simple-semi-conditional matrix grammars in which conditions are attached to matrices rather than productions. It proves that seven-nonterminal simple-semi-conditional matrix grammars define the family of recursively enumerable languages with two matrices having context conditions of length three. Section 5 compares the achieved results and proposes some open-problem areas.

2 DEFINITIONS

We assume that the reader is familiar with the language theory (see [5]).

Let V be an alphabet. The cardinality of V is denoted by $\#_V$. V^* represents the free monoid generated by V under the operation of concatenation. The unit of V is denoted by ε. Set $V^+ = V^* - \{\varepsilon\}$; algebraically, V^+ is thus the free semigroup generated by V under the operation of concatenation. For a word, $w \in V^*$, $|w|$, $\mathrm{alph}(w)$, and $\mathrm{reversal}(w)$ denote the length of w, the set of letters occurring in w, and the reversal of w, respectively. For every symbol $X \in V$, $\#_X w$ denotes the number of occurrences of X in w. For a language, $L \subseteq V^*$, we set $\mathrm{alph}(L) = \{a : a \in \mathrm{alph}(w)$ for some $w \in L\}$, and $\mathrm{reversal}(L) = \{\mathrm{reversal}(w) : w \in L\}$.

A *context-free grammar* is a quadruple, $G = (V, T, P, S)$, where V is an alphabet, $T \subset V$, and $S \in V - T$. P is a finite set of productions of the form $A \to x$, where $A \in V - T$ and $x \in V^*$. If $A \to x \in P$ and $u, v \in V^*$, then $uAv \Rightarrow_G uxv$ in G. Let $\Rightarrow_G^*$ denote the transitive-reflexive closure of $\Rightarrow_G$. The *language of* G, $L(G)$, is defined as $L(G) = \{y\colon S \Rightarrow_G^* y, y \in T^*\}$.

Next, we recall the definition of matrix grammars. (In the theory of regulated grammars, there also exist these grammars with appearance checking; these versions, however, are not discussed in this paper.)

A *matrix grammar* (see [1]) is a quadruple, $G = (V, T, M, S)$, where V is an alphabet, $T \subseteq V$, and $S \in V - T$. M is a finite set of sequences of the form $(A_1 \to x_1, \ldots, A_n \to x_n)$, where $A_i \in V - T$ and $x_i \in V^*$ for some $n \geq 1$; $(A_1 \to x_1, \ldots, A_n \to x_n)$ is called a *matrix*, and its members are called *productions*. If $(A_1 \to x_1, \ldots, A_n \to x_n) \in M, z_1, \ldots, z_{n+1} \in V^*$ for some $n \geq 1$, $z_j = u_j A_j v_j$ and $z_{j+1} = u_j x_j v_j$ for some $u_j, v_j \in V^*$, $1 \leq j \leq n$, then $z_1 \Rightarrow_G z_{n+1}[(A_1 \to x_1, \ldots, A_n \to x_n)]$ in G or, simply, $z_1 \Rightarrow_G z_{n+1}$. Let $\Rightarrow_G^*$ denote the reflexive-transitive closure of $\Rightarrow_G$. The *language of* G, $L(G)$, is defined as $L(G) = \{y\colon S \Rightarrow_G^* y, y \in T^*\}$. A matrix of the form $(A \to x_1, \ldots, A \to x_n)$ with $n \geq 2$ is called a *multi-production matrix*; a matrix of the form $(A \to x)$ is a *one-production matrix*. Observe that the application of any one-production matrix $(A \to x)$ is made in an ordinary context-free way; for simplicity, instead of $(A \to x)$, we hereafter write $A \to x$.

Let **M** and **RE** denote the families of matrix and recursively enumerable languages, respectively. Recall that $\mathbf{M} \subset \mathbf{RE}$ (see Theorem 2.12 on page 129 in [8]).

3 MATRIX SIMPLE-SEMI-CONDITIONAL GRAMMARS

3.1 Definitions

A *matrix simple-semi-conditional grammar* (*mssc*-grammar for short) represents a combination of simple-semi-conditional grammars (see [6]) and matrix grammars (see [1]).

Formally, a *mssc*-grammar is a quadruple $G = (V, T, M, S)$ where V, T and S have the same meaning as in a matrix grammar and M is a finite set of sequences of the form

$$((A_1 \to x_1, Q_1, R_1), \ldots, (A_n \to x_n, Q_n, R_n)),$$

where $n \geq 1, A_i \in V - T, x_i \in V^*, Q_i, R_i \in V^+ \cup \{0\}$ so that $Q_i = 0$ or $R_i = 0$ for $1 \leq i \leq n$. The Qs and Rs above are called the *permitting* and *forbidding* conditions, respectively; 0 is a special symbol, $0 \notin V$, meaning that a condition is missing. The length of the longest condition represents the *degree of* G; if all conditions are 0, then G's degree is zero. The sequences in M are called *ssc*-matrices, and they are divided into one-production *ssc*-matrices and multi-production *ssc*-matrices by analogy with ordinary matrices. For brevity we simplify $((A \to X, 0, 0))$ to $A \to x$ hereafter. If $m\colon ((A_1 \to x_1, Q_1, R_1), \ldots, (A_n \to x_n, Q_n, R_n)) \in M, z_1, \ldots, z_{n+1} \in V^*$ for some $n \geq 1, z_j = u_j A_j v_j, z_{j+1} = u_j x_j v_j$ for some $u_j, v_j \in V^*, Q_j \in u_j A_j v_j$

or $Q_j = 0, R_j \notin u_j A_j v_j$ or $R_j = 0$, for $1 \leq j \leq n$, then $z_1 \Rightarrow_G z_{n+1}[((A_1 \rightarrow x_1, Q_1, R_1), (A_2, \rightarrow x_2, Q_2, R_2), \ldots, (A_n \rightarrow x_n, Q_n, R_n))]$ or, simply, $z_1 \Rightarrow_G z_{n+1}$; to express $z_1 \Rightarrow_G z_{n+1}$ as the n consecutive applications of the productions in matrix m, write:

$$\begin{array}{llll} z_1 & {}_{1-m}\!\Rightarrow_G & z_2 & [m\colon (A_1 \rightarrow x_1, Q_1, R_1)] \\ & {}_{2-m}\!\Rightarrow_G & z_3 & [m\colon (A_2 \rightarrow x_2, Q_2, R_2)] \\ & \vdots & & \\ & {}_{n-m}\!\Rightarrow_G & z_{n+1} & [m\colon (A_n \rightarrow x_n, Q_n, R_n)]. \end{array}$$

Let $\Rightarrow_*$ denote the transitive and reflexive closure of $\Rightarrow$. The *language of* G, $L(G)$, is defined as $L(G) = \{y\colon S \Rightarrow_G^*, y \in T^*\}$.

3.2 Matrix Simple-Semi-Conditional Grammars of Degree 3

Theorem 1. For every recursively enumerable language L, there exists a mssc-grammar G' of degree 3 satisfying the following conditions:

1. $L = L(G')$.
2. G' contains only one multi-production matrix with no more than six productions; all the other matrices are one-production matrices without any condition.
3. G' contains no more than seven nonterminals.

Proof. Let $L \in$ **RE**. Without any loss of generality, we assume that L is generated by a grammar G of the form $G = (V, T, P \cup \{ABC \rightarrow \varepsilon\}, S)$ such that P contains only context-free productions and $V - T = \{S, A, B, C\}$ (see [4]). Next, we define the *mssc*-grammar $G' = (V', T, P', S)$, where $V' = V \cup \{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ (assume that $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\} \cap W = \emptyset$), and P' is constructed in the following way:

1. if $H \rightarrow \alpha \in P$, $H \in V - T$, $\alpha \in V^*$, then add $(H \rightarrow \alpha, 0, 0)$ to P';
2. add the following matrix to P': $m\colon \{(A \rightarrow \widetilde{A}, 0, \widetilde{A}), (B \rightarrow \widetilde{B}, 0, \widetilde{B}), (C \rightarrow \widetilde{C}, 0, \widetilde{C}), (\widetilde{A} \rightarrow \varepsilon, \widetilde{A}\widetilde{B}\widetilde{C}, 0), (\widetilde{B} \rightarrow \varepsilon, 0, 0), (\widetilde{C} \rightarrow \varepsilon, 0, 0)\}$.

   Next, we prove that $L(G') = L(G)$.

Basic idea: Productions of matrix m simulates the application of $ABC \rightarrow \varepsilon$ in G' as follows. First, one occurrence of A, B and C are rewritten with $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$, respectively. Then, we check that the marked letters form a subword $\widetilde{A}\widetilde{B}\widetilde{C}$ by the fourth production of m. If so, G' erases these three consecutive symbols; otherwise, G' cannot complete this matrix.

Formal proof: To establish $L(G) = L(G')$ formally, we first prove the following claim.

Claim 1. $S \Rightarrow_{G'}^* x'$ implies $\#_{\widetilde{X}} x' \leq 1$ for each $\widetilde{X} \in \{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$, where $x' \in (V')^*$.

Proof. By inspection of productions in P', the only production that can generate $\widetilde{X}$ is of the form $(X \to \widetilde{X}, 0, \widetilde{X})$. This production can be applied only when no $\widetilde{X}$ occurs in the rewritten sentential form. Thus, it is impossible to derive x' from S such that $\#_{\widetilde{X}} x' \geq 2$. $\square$

Let g be a finite substitution from $(V')^*$ to V^* defined as follows:

1. for all $X \in V$: $g(X) = \{X\}$;
2. $g(\widetilde{A}) = \{A\}$,
   $g(\widetilde{B}) = \{B, AB\}$,
   $g(\widetilde{C}) = \{C, ABC\}$.

Claim 2. $S \Rightarrow_G^* x$ if and only if $S \Rightarrow_{G'}^* x'$ for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Proof. The claim is proved by induction on the length of derivations.

Only if: We prove that

$$S \Rightarrow_G^m x \quad \text{implies} \quad S \Rightarrow_{G'}^* x,$$

where $m \geq 0$, $x \in V^*$. This is established by induction on m.

Basis: Let $m = 0$. That is $S \Rightarrow_G^0 S$. Clearly, $S \Rightarrow_{G'}^0 S$.

Induction Hypothesis: Suppose that the claim holds for all derivations of length m or less, for some $m \geq 0$.

Induction Step: Let us consider $S \Rightarrow_G^{m+1} x$, $x \in V^*$. Since $m+1 \geq 1$, there is some $y \in V^+$ and $p \in P \cup \{ABC \to \varepsilon\}$ such that $S \Rightarrow_G^m y \Rightarrow_G x$ $[p]$. By the induction hypothesis, there is a derivation $S \Rightarrow_{G'}^* y$.

There are two cases that cover all possible forms of p:

(i) $p = H \to y_2 \in P$, $H \in V - T$, $y_2 \in V^*$. Then $y = y_1 H y_3$ and $x = y_1 y_2 y_3$, $y_1, y_3 \in V^*$. Because $(H \to y_2, 0, 0) \in P'$, we have $S \Rightarrow_{G'}^* y_1 H y_3 \Rightarrow_{G'} y_1 y_2 y_3$ $[(H \to y_2, 0, 0)]$ and $y_1 y_2 y_3 = x$.

(ii) $p = ABC \to \varepsilon$. Then $y = y_1 ABC y_3$ and $x = y_1 y_3$, $y_1, y_3 \in V^*$. In this case, there is the following derivation which uses matrix m:

$$\begin{array}{llll}
S & \Rightarrow_{G'}^* & y_1 ABC y_3 & \\
 & {}_{1-m}\Rightarrow_{G'} & y_1 \widetilde{A} BC y_3 & [m\colon (A \to \widetilde{A}, 0, \widetilde{A})] \\
 & {}_{2-m}\Rightarrow_{G'} & y_1 \widetilde{A}\widetilde{B} C y_3 & [m\colon (B \to \widetilde{B}, 0, \widetilde{B})] \\
 & {}_{3-m}\Rightarrow_{G'} & y_1 \widetilde{A}\widetilde{B}\widetilde{C} y_3 & [m\colon (C \to \widetilde{C}, 0, \widetilde{C})] \\
 & {}_{4-m}\Rightarrow_{G'} & y_1 \widetilde{B}\widetilde{C} y_3 & [m\colon (\widetilde{A} \to \varepsilon, \widetilde{A}\widetilde{B}\widetilde{C}, 0)] \\
 & {}_{5-m}\Rightarrow_{G'} & y_1 \widetilde{C} y_3 & [m\colon (\widetilde{B} \to \varepsilon, 0, 0)] \\
 & {}_{6-m}\Rightarrow_{G'} & y_1 y_3 & [m\colon (\widetilde{C} \to \varepsilon, 0, 0)].
\end{array}$$

If: By induction on n, we prove that

$$S \Rightarrow_{G'}^{n} x' \quad \text{implies} \quad S \Rightarrow_{G}^{*} x$$

for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Basis: Let $n = 0$. That is, $S \Rightarrow_{G'}^{0} S$. It is obvious that $S \Rightarrow_{G}^{0} S$ and $S \in g(S)$.

Induction Hypothesis: Assume that the claim holds for all derivations of length n or less for some $n \geq 0$.

Induction Step: Consider a derivation $S \Rightarrow_{G'}^{n+1} x'$, $x' \in (V')^*$. Since $n+1 \geq 1$, there is some $y' \in (V')^+$ and $p' \in P'$ such that $S \Rightarrow_{G'}^{n} y' \Rightarrow_{G'} x'$ $[p']$ and by the induction hypothesis there is also a derivation $S \Rightarrow_{G}^{*} y$ such that $y \in g(y')$.

By inspection of P' the following cases (i) through (xi) covers all possible forms of p':

(i) $p' = (H \to y_2, 0, 0) \in P'$, $H \in V - T$, $y_2 \in V^*$. Then $y' = y_1' H y_3'$, $x' = y_1' y_2 y_3'$, $y_1', y_3' \in (V')^*$ and y has the form $y = y_1 Z y_3$, where $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and $Z \in g(H)$. Because for all $X \in V - T$ such that $g(X) = \{X\}$, the only Z is H and thus $y = y_1 H y_3$. By the definition of P' (see (1)), there exists a production $p = H \to y_2$ in P and we can construct the derivation $S \Rightarrow_{G}^{*} y_1 H y_3 \Rightarrow_{G} y_1 y_2 y_3$ $[p]$ such that $y_1 y_2 y_3 = x$, $x \in g(x')$.

(ii) $p' = m\colon (A \to \widetilde{A}, 0, \widetilde{A})$. Then $y' = y_1' A y_3'$, $x' = y_1' \widetilde{A} y_3'$, $y_1', y_3' \in (V')^*$ and $y = y_1 Z y_3$, where $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and $Z \in g(A)$. Because $g(A) = \{A\}$ the only Z is A, so we can express $y = y_1 A y_3$. Having the derivation $S \Rightarrow_{G}^{*} y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $A \in g(\widetilde{A})$.

(iii) $p' = m\colon (B \to \widetilde{B}, 0, \widetilde{B})$. By analogy with (ii), $y' = y_1' B y_3'$, $x' = y_1' \widetilde{B} y_3'$, $y = y_1 B y_3$, where $y_1', y_3' \in (V')^*$, $y_1 \in g(y_1')$, $y_3 \in g(y_3')$; thus $y \in g(x')$ because $B \in g(\widetilde{B})$.

(iv) $p' = m\colon (C \to \widetilde{C}, 0, \widetilde{C})$. By analogy with (ii), $y' = y_1' C y_3'$, $x' = y_1' \widetilde{C} y_3'$, $y = y_1 C y_3$, where $y_1', y_3' \in (V')^*$, $y_1 \in g(y_1')$, $y_3 \in g(y_3')$; thus $y \in g(x')$ because $C \in g(\widetilde{C})$.

(v) $p' = m\colon (\widetilde{A} \to \varepsilon, \widetilde{A}\widetilde{B}\widetilde{C}, 0)$. By the permitting condition of this production string $\widetilde{A}\widetilde{B}\widetilde{C}$ surely occurs in y'. By Claim 1 no more than one $\widetilde{A}$ occurs in y'. Therefore, y' must be of form $y' = y_1' \widetilde{A}\widetilde{B}\widetilde{C} y_3'$, where $y_1', y_3' \in (V')^*$ and $\widetilde{A} \notin \mathrm{sub}(y_1' y_3')$. Then $x' = y_1' \widetilde{B}\widetilde{C} y_3'$ and y is of the form $y = y_1 Z y_3$, where $y_1 \in g(y_1'), y_3 \in g(y_3')$ and $Z \in g(\widetilde{B}\widetilde{C})$. Because $g(\widetilde{B}\widetilde{C}) = \{BC, ABC, BABC, ABABC\}$ we obtain $y = y_1 ABC y_3$. By the induction hypothesis we have a derivation $S \Rightarrow_{G}^{*} y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $AB \in g(\widetilde{B})$ and $C \in g(\widetilde{C})$.

(vi) $p' = m\colon (\widetilde{B} \to \varepsilon, 0, 0)$. By the definition of *mssc*-grammar and Claim 1, the only sentential form in which we can use this production is that G' obtains from the previous sentential form. That means $y' = y_1' \widetilde{B}\widetilde{C} y_3'$, where $y_1', y_3 \in (V')^*$ and $\widetilde{B} \notin \mathrm{sub}(y_1' y_3')$. Then $x' = y_1' \widetilde{C} y_3'$ and y is of the form $y = y_1 Z y_3$, where $y_1 \in g(y_1'), y_3 \in g(y_3')$ and $Z \in g(\widetilde{C})$. Because $g(\widetilde{C}) = \{C, ABC\}$, we obtain

$y = y_1ABCy_3$. By the induction hypothesis, we have a derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $ABC \in g(\widetilde{C})$.

(vii) $p' = m\colon(\widetilde{C} \to \varepsilon, 0, 0)$. Then, $y' = y_1'\widetilde{C}y_3'$ and $x' = y_1'y_3'$, where $y_1', y_3' \in (V')^*$. Express $y = y_1Zy_3$ so that $y_1 \in g(y_1'), y_3 \in g(y_3')$ and $Z \in g(\widetilde{C})$, where $g(\widetilde{C}) = \{C, ABC\}$. Let $Z = ABC$. Then, $y = y_1ABCy_3$ and there exists the derivation $S \Rightarrow_G^* y_1ABCy_3 \Rightarrow_G y_1y_3[ABC \to \varepsilon]$, where $y_1y_3 = x, x \in g(x')$.

We have completed the proof and established Claim 2 by the principle of induction. □

Observe that $L(G) = L(G')$ follows from Claim 2. Indeed, according to the definition of g, we have $g(a) = \{a\}$ for all $a \in T$. Thus, from Claim 2, we have for any $x \in T^*$:

$$S \Rightarrow_G^* x \quad \text{if and only if} \quad S \Rightarrow_{G'}^* x.$$

Consequently $L(G) = L(G')$, so the first part of the theorem holds.

The rest of the theorem follows from the construction of G'. □

3.3 Matrix Simple-Semi-Conditional Grammars of Degree 2

Theorem 2. For every recursively enumerable language L, there exists a mssc-grammar G' of degree two satisfying the following conditions:

1. $L = L(G')$.
2. G' contains only two multi-production matrices with no more than four productions in them; the other matrices are one-production matrices without any condition.
3. G' contains no more than six nonterminals.

Proof. Let L be a recursively enumerable language. From [4], we can assume that L is generated by a grammar G of the form

$$G = (V, T, P \cup \{AB \to \varepsilon, CC \to \varepsilon\}, S)$$

such that P contains only context-free productions and

$$V - T = \{S, A, B, C\}.$$

We construct an *mssc*-grammar G' as follows:

$$\begin{aligned} &G' = (V', T, P', S), \quad \text{where} \\ &V' = V \cup W, \\ &W = \{\widetilde{X}, \widetilde{Y}\},\ V \cap W = \emptyset. \end{aligned}$$

The set of productions P' is defined in the following way:

1. if $H \to \alpha \in P$, $H \in V - T$, $\alpha \in V^*$, then add $(H \to \alpha, 0, 0)$ to P';
2. add the following matrices to P':

$$m_1\colon \{ \quad \begin{array}{l}(A \to \widetilde{X}, 0, \widetilde{X}),\\ (B \to \widetilde{Y}, 0, \widetilde{Y}),\\ (\widetilde{X} \to \varepsilon, \widetilde{X}\widetilde{Y}, 0),\\ (\widetilde{Y} \to \varepsilon, 0, 0)\}\end{array} \qquad m_2\colon \{ \quad \begin{array}{l}(C \to \widetilde{X}, 0, \widetilde{X}),\\ (C \to \widetilde{Y}, 0, \widetilde{Y}),\\ (\widetilde{X} \to \varepsilon, \widetilde{X}\widetilde{Y}, 0),\\ (\widetilde{Y} \to \varepsilon, 0, 0)\}.\end{array}$$

Next we prove that $L(G') = L(G)$.

Basic idea: Notice that G' contains only two matrices, m_1 and m_2, with three conditional productions and one context-free production. These matrices simulate the application of $AB \to \varepsilon$ and $CC \to \varepsilon$ as follows. Consider m_1. First, one occurrence of A and one occurrence of B are rewritten with $\widetilde{X}$ and $\widetilde{Y}$, respectively. Then, m_1 checks whether the marked letters form a substring $\widetilde{X}\widetilde{Y}$. If so, G' erases these consecutive symbols; otherwise, G' cannot complete this matrix. $CC \to \varepsilon$ is simulated in a similar way by using the other matrix.

Formal proof: To establish $L(G) = L(G')$ formally, we first prove the following claim.

Claim 3. $S \Rightarrow^*_{G'} x'$ implies $\#_{\widetilde{Q}} x' \leq 1$ for each $\widetilde{Q} \in \{\widetilde{X}, \widetilde{Y}\}$, where $x' \in (V')^*$.

Proof. By inspection of productions in P', the only production that can generate $\widetilde{Q}$ is of the form $(Q \to \widetilde{Q}, 0, \widetilde{Q})$. This production can be applied only when no $\widetilde{Q}$ occurs in the rewritten sentential form. Thus, it is impossible to derive x' from S such that $\#_{\widetilde{Q}} x' \geq 2$. □

Let g be a finite substitution from $(V')^*$ to V^* defined as follows:

1. for all $X \in V$: $g(X) = \{X\}$;
2. $g(\widetilde{X}) = \{A, C\}$,
   $g(\widetilde{Y}) = \{B, AB, C, CC\}$.

Claim 4. $S \Rightarrow^*_G x$ if and only if $S \Rightarrow^*_{G'} x'$ for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Proof. This claim is proved by induction on the length of derivations.

Only if: We prove that

$$S \Rightarrow^m_G x \quad \text{implies} \quad S \Rightarrow^*_{G'} x,$$

where $m \geq 0$, $x \in V^*$. This is established by induction on m.

Basis: Let $m = 0$. That is $S \Rightarrow^0_G S$. Clearly, $S \Rightarrow^0_{G'} S$.

Induction Hypothesis: Suppose that the claim holds for all derivations of length m or less for some $m \geq 0$.

Induction Step: Let us consider a derivation $S \Rightarrow_G^{m+1} x,\ x \in V^*$. Since $m+1 \geq 1$, there is some $y \in V^+$ and $p \in P \cup \{AB \to \varepsilon, CC \to \varepsilon\}$ such that $S \Rightarrow_G^m y \Rightarrow_G x\ [p]$. By the induction hypothesis there is a derivation $S \Rightarrow_{G'}^* y$.

There are three cases that cover all possible forms of the production p:

(i) $p = H \to y_2 \in P,\ H \in V - T,\ y_2 \in V^*$. Then $y = y_1Hy_3$ and $x = y_1y_2y_3$, $y_1, y_3 \in V^*$. Because we have $(H \to y_2, 0, 0) \in P'$, $S \Rightarrow_{G'}^* y_1Hy_3 \Rightarrow_{G'} y_1y_2y_3\ [(H \to y_2, 0, 0)]$ and $y_1y_2y_3 = x$.

(ii) $p = AB \to \varepsilon$. Then $y = y_1ABy_3$ and $x = y_1y_3,\ y_1, y_3 \in V^*$. In this case there is the following derivation which uses matrix m:

$$
\begin{array}{rll}
S & \Rightarrow_{G'}^* & y_1ABy_3 \\
& {}_{1-m_1}\Rightarrow_{G'} & y_1\widetilde{X}By_3 \quad [m_1\colon (A \to \widetilde{X}, 0, \widetilde{X})] \\
& {}_{2-m_1}\Rightarrow_{G'} & y_1\widetilde{X}\widetilde{Y}y_3 \quad [m_1\colon (B \to \widetilde{Y}, 0, \widetilde{Y})] \\
& {}_{3-m_1}\Rightarrow_{G'} & y_1\widetilde{Y}y_3 \quad [m_1\colon (\widetilde{X} \to \varepsilon, \widetilde{X}\widetilde{Y}, 0)] \\
& {}_{4-m_1}\Rightarrow_{G'} & y_1y_3 \quad [m_1\colon (\widetilde{Y} \to \varepsilon, 0, 0)]
\end{array}
$$

(iii) $p = CC \to \varepsilon$. Then $y = y_1CCy_3$ and $x = y_1y_3,\ y_1, y_3 \in V^*$. In this case there is the following derivation which uses matrix m:

$$
\begin{array}{rll}
S & \Rightarrow_{G'}^* & y_1CCy_3 \\
& {}_{1-m_2}\Rightarrow_{G'} & y_1\widetilde{X}Cy_3 \quad [m_2\colon (C \to \widetilde{X}, 0, \widetilde{X})] \\
& {}_{2-m_2}\Rightarrow_{G'} & y_1\widetilde{X}\widetilde{Y}y_3 \quad [m_2\colon (C \to \widetilde{Y}, 0, \widetilde{Y})] \\
& {}_{3-m_2}\Rightarrow_{G'} & y_1\widetilde{Y}y_3 \quad [m_2\colon (\widetilde{X} \to \varepsilon, \widetilde{X}\widetilde{Y}, 0)] \\
& {}_{4-m_2}\Rightarrow_{G'} & y_1y_3 \quad [m_2\colon (\widetilde{Y} \to \varepsilon, 0, 0)]
\end{array}
$$

If: By induction on $n \geq 0$, we prove that

$$S \Rightarrow_{G'}^n x' \quad \text{implies} \quad S \Rightarrow_G^* x$$

for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Basis: Let $n = 0$. That is, $S \Rightarrow_{G'}^0 S$. It is obvious that $S \Rightarrow_G^0 S$ and $S \in g(S)$.

Induction Hypothesis: Assume that the claim holds for all derivations of length n or less, for some $n \geq 0$.

Induction Step: Consider a derivation $S \Rightarrow_{G'}^{n+1} x'$, $x' \in (V')^*$. Since $n+1 \geq 1$, there is some $y' \in (V')^+$ and $p' \in P'$ such that $S \Rightarrow_{G'}^n y' \Rightarrow_{G'} x'\ [p']$ and by the induction hypothesis there is also a derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$.

By inspection of P' the following cases (i) through (xi) cover all possible forms of p':

(i) $p' = (H \to y_2, 0, 0) \in P'$, $H \in V - T$, $y_2 \in V^*$. Then $y' = y'_1 H y'_3$, $x' = y'_1 y_2 y'_3$, $y'_1, y'_3 \in (V')^*$ and y has the form $y = y_1 Z y_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(H)$. Because for all $X \in V - T$ such that $g(X) = \{X\}$, the only Z is H and thus $y = y_1 H y_3$. By the definition of P' (see (1)) there exists a production $p = H \to y_2$ in P and we can construct the derivation $S \Rightarrow_G^* y_1 H y_3 \Rightarrow_G y_1 y_2 y_3$ $[p]$ such that $y_1 y_2 y_3 = x$, $x \in g(x')$.

(ii) $p' = m_1 \colon (A \to \widetilde{X}, 0, \widetilde{X})$. Then $y' = y'_1 A y'_3$, $x' = y'_1 \widetilde{X} y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1 Z y_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(A)$. Because $g(A) = \{A\}$ the only Z is A, so we can express $y = y_1 A y_3$. Having the derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$ it is easy to see that also $y \in g(x')$ because $A \in g(\widetilde{X})$.

(iii) $p' = m_1 \colon (B \to \widetilde{Y}, 0, \widetilde{Y})$. By analogy with (ii), $y' = y'_1 B y'_3$, $x' = y'_1 \widetilde{Y} y'_3$, $y = y_1 B y_3$, where $y'_1, y'_3 \in (V')^*$, $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and thus $y \in g(x')$ because $B \in g(\widetilde{Y})$.

(iv) $p' = m_1 \colon (\widetilde{X} \to \varepsilon, \widetilde{X}\widetilde{Y}, 0)$. By the permitting condition of this production string $\widetilde{X}\widetilde{Y}$ surely occurs in y'. By Claim 3 no more than one $\widetilde{X}$ occurs in y'. Therefore, y' must be of form $y' = y'_1 \widetilde{X}\widetilde{Y} y'_3$, where $y'_1, y'_3 \in (V')^*$ and $\widetilde{X} \notin \mathrm{sub}(y'_1 y'_3)$. Then $x' = y'_1 \widetilde{B} y'_3$ and y is of the form $y = y_1 Z y_3$, where $y_1 \in g(y'_1), y_3 \in g(y'_3)$ and $Z \in g(\widetilde{Y})$. Because $g(\widetilde{Y}) = \{B, AB, C, CC\}$ we obtain $y = y_1 AB y_3$. By the induction hypothesis we have a derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $AB \in g(\widetilde{Y})$.

(v) $p' = m_1 \colon (\widetilde{Y} \to \varepsilon, 0, 0)$. Then, $y' = y'_1 \widetilde{Y} y'_3$ and $x' = y'_1 y'_3$, where $y'_1, y'_3 \in (V')^*$. Express $y = y_1 Z y_3$ so that $y_1 \in g(y'_1), y_3 \in g(y'_3)$ and $Z \in g(\widetilde{Y})$, where $g(\widetilde{Y}) = \{B, AB, C, CC\}$. Let $Z = AB$. Then, $y = y_1 AB y_3$ and there exists the derivation $S \Rightarrow_G^* y_1 AB y_3 \Rightarrow_G y_1 y_3 [AB \to \varepsilon]$, where $y_1 y_3 = x, x \in g(x')$.

(vi) $p' = m_2 \colon (C \to \widetilde{X}, 0, \widetilde{X})$. By analogy with (ii), $y' = y'_1 C y'_3$, $x' = y'_1 \widetilde{X} y'_3$, $y = y_1 C y_3$, where $y'_1, y'_3 \in (V')^*$, $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and thus $y \in g(x')$ because $C \in g(\widetilde{X})$.

(vii) $p' = m_2 \colon (C \to \widetilde{Y}, 0, \widetilde{Y})$. By analogy with (ii), $y' = y'_1 C y'_3$, $x' = y'_1 \widetilde{Y} y'_3$, $y = y_1 C y_3$, where $y'_1, y'_3 \in (V')^*$, $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and thus $y \in g(x')$ because $C \in g(\widetilde{Y})$.

(viii) $p' = m_2 \colon (\widetilde{X} \to \varepsilon, \widetilde{X}\widetilde{Y}, 0)$. By the permitting condition of this production string $\widetilde{X}\widetilde{Y}$ surely occurs in y'. By Claim 3 no more than one $\widetilde{X}$ occurs in y'. Therefore, y' must be of form $y' = y'_1 \widetilde{X}\widetilde{Y} y'_3$, where $y'_1, y'_3 \in (V')^*$ and $\widetilde{X} \notin \mathrm{sub}(y'_1 y'_3)$. Then $x' = y'_1 \widetilde{Y} y'_3$ and y is of the form $y = y_1 Z y_3$, where $y_1 \in g(y'_1), y_3 \in g(y'_3)$ and $Z \in g(\widetilde{Y})$. Because $g(\widetilde{Y}) = \{B, AB, C, CC\}$ we obtain $y = y_1 CC y_3$. By the induction hypothesis we have a derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $CC \in g(\widetilde{Y})$.

(ix) $p' = m_2 \colon (\widetilde{Y} \to \varepsilon, 0, 0)$. Then, $y' = y'_1 \widetilde{Y} y'_3$ and $x' = y'_1 y'_3$, where $y'_1, y'_3 \in (V')^*$. Express $y = y_1 Z y_3$ so that $y_1 \in g(y'_1), y_3 \in g(y'_3)$ and $Z \in g(\widetilde{Y})$, where

$g(\widetilde{Y}) = \{B, AB, C, CC\}$. Let $Z = CC$. Then, $y = y_1CCy_3$ and there exists the derivation $S \Rightarrow_G^* y_1CCy_3 \Rightarrow_G y_1y_3[CC \rightarrow \varepsilon]$, where $y_1y_3 = x, x \in g(x')$.

We have completed the proof and established Claim 4 by the principle of induction. □

Observe that $L(G) = L(G')$ follows from Claim 4. Indeed, according to the definition of g, we have $g(a) = \{a\}$ for all $a \in T$. Thus, from Claim 4, we have for each $x \in T^*$:

$$S \Rightarrow_G^* x \quad \text{if and only if} \quad S \Rightarrow_{G'}^* x.$$

Consequently, $L(G) = L(G')$. The rest of this theorem follows from the construction of G'. □

4 SIMPLE-SEMI-CONDITIONAL MATRIX GRAMMARS

4.1 Definitions

A *simple-semi-conditional matrix grammar* (*sscm*-grammar for short) is another combination of matrix grammars (see [1]) and simple-semi-conditional grammars (see [6]).

A *sscm*-grammar is a quadruple $G = (V, T, P, S)$, where V, T, and S are defined as in Section 2. P is a finite set of *matrices with context conditions* of the form

$$(((A_1 \rightarrow x_1), \ldots, (A_n \rightarrow x_n)), Q, R)$$

where $n \geq 1$, $A_i \rightarrow x$ is a context-free production and $Q, R \in V^* \cup \{0\}$, (0 means that condition is missing, $0 \notin V$). According to the matrix of the above form, G makes a derivation step $u \Rightarrow_G v$, where $u, v \in V^*$ if $Q \in \text{alph}(y_1Ay_2), R \notin \text{alph}(y_1Ay_2)$, and u directly derives v according to $A_1 \rightarrow x_1, \ldots, A_n \rightarrow x_n$ in an ordinary matrix-grammar way (see Section 3). The language of G, $L(G)$, is defined as usual. The length of the longest condition in G represents the degree of G.

A matrix of the form $((A \rightarrow x), 0, 0)$ is simplified to $A \rightarrow x$ hereafter.

4.2 Simple-Semi-Conditional Matrix Grammars of Degree 3

Theorem 3. Every recursively enumerable language, L, can be defined by sscm-grammar G' satisfying the following conditions:

1. $L = L(G')$
2. G' contains no more than two matrices with context conditions
3. G' contains no more than seven nonterminals.

Proof. Let L be a recursively enumerable language. From [4], we can assume that L is generated by a grammar G of the form

$$G = (V, T, P \cup \{ABC \rightarrow \varepsilon\}, S)$$

such that P contains only context-free productions and

$$V - T = \{S, A, B, C\}.$$

We construct an *sscm*-grammar G' as follows:

$$\begin{aligned} &G' = (V', T, P', S) \\ &V' = V \cup W \\ &W = \{\widetilde{A}, \widetilde{B}, \widetilde{C}\} \\ &V \cap W = \emptyset. \end{aligned}$$

The set of productions, P', is defined in the following way:

1. if $H \to \alpha \in P$, $H \in V - T$, $\alpha \in V^*$, then add $((H \to \alpha), 0, 0)$ to P';
2. add the following matrices to P':

$$\begin{aligned} &m_1\colon (\{(A \to \widetilde{A}), (B \to \widetilde{B}), (C \to \widetilde{C})\}, 0, \widetilde{A}), \\ &m_2\colon (\{(\widetilde{A} \to \varepsilon), (\widetilde{B} \to \varepsilon), (\widetilde{C} \to \varepsilon)\}, \widetilde{A}\widetilde{B}\widetilde{C}, 0). \end{aligned}$$

Next we prove that $L(G') = L(G)$.

Basic idea: Matrices m_1 and m_2 simulate the application of $ABC \to \varepsilon$ in G' as follows. First, A, B and C are rewritten with $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$, respectively. Then, G' checks whether the marked letters form a substring $\widetilde{A}\widetilde{B}\widetilde{C}$. If so, G' erases these consecutive symbols by m_2; otherwise, G' cannot complete this matrix.

Formal proof: To establish $L(G) = L(G')$ formally, we first prove the following claim.

Claim 5. $S \Rightarrow^*_{G'} x'$ implies $\#_{\widetilde{x}} x' \leq 1$ for all $\widetilde{x} \in \{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$, where $x' \in (V')^*$.

Proof. By inspection of productions in P', the only way of generating $\widetilde{x}$ is by using m_1. This matrix can be applied only when no $\widetilde{A}$ occurs in the rewritten sentential form. Because the only way of rewriting $\widetilde{x}$s is by using m_2, it is impossible to derive x' from S such that $\#_{\widetilde{x}} x' \geq 2$. □

Let g be a finite substitution from $(V')^*$ to V^* defined as follows:

1. for all $X \in V$: $g(X) = \{X\}$;
2. $g(\widetilde{A}) = \{A\}$,
   $g(\widetilde{B}) = \{B\}$,
   $g(\widetilde{C}) = \{C\}$.

Claim 6. $S \Rightarrow^*_G x$ if and only if $S \Rightarrow^*_{G'} x'$ for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Proof. The claim is proved by induction on the length of derivations.

Only if: We prove that

$$S \Rightarrow_G^m x \quad \text{implies} \quad S \Rightarrow_{G'}^* x,$$

where $m \geq 0$, $x \in V^*$. This is established by induction on m.

Basis: Let $m = 0$. That is $S \Rightarrow_G^0 S$. Clearly, $S \Rightarrow_{G'}^0 S$.

Induction Hypothesis: Suppose that the claim holds for all derivations of length m or less, for some $m \geq 0$.

Induction Step: Let us consider a derivation $S \Rightarrow_G^{m+1} x,\ x \in V^*$. Since $m + 1 \geq 1$, there is some $y \in V^+$ and $p \in P \cup \{ABC \to \varepsilon\}$ such that $S \Rightarrow_G^m y \Rightarrow_G x\ [p]$. By the induction hypothesis, there is a derivation $S \Rightarrow_{G'}^* y$.

There are two cases that cover all possible forms of production p:

(i) $p = H \to y_2 \in P,\ H \in V - T,\ y_2 \in V^*$. Then $y = y_1 H y_3$ and $x = y_1 y_2 y_3$, $y_1, y_3 \in V^*$. Because we have $(H \to y_2, 0, 0) \in P'$, $S \Rightarrow_{G'}^* y_1 H y_3 \Rightarrow_{G'} y_1 y_2 y_3\ [(H \to y_2, 0, 0)]$ and $y_1 y_2 y_3 = x$.

(ii) $p = ABC \to \varepsilon$. Then $y = y_1 ABC y_3$ and $x = y_1 y_3,\ y_1, y_3 \in V^*$. In this case, there is the following derivation that uses matrix m:

$$\begin{aligned} S &\Rightarrow_{G'}^* y_1 ABC y_3 \\ &\Rightarrow_{G'} y_1 \widetilde{A}\widetilde{B}\widetilde{C} y_3 \quad [m_1\colon (\{(A \to \widetilde{A}), (B \to \widetilde{B}), (C \to \widetilde{C})\}, 0, \widetilde{A})] \\ &\Rightarrow_{G'} y_1 y_3 \qquad\quad [m_2\colon (\{(\widetilde{A} \to \varepsilon), (\widetilde{B} \to \varepsilon), (\widetilde{C} \to \varepsilon)\}, \widetilde{A}\widetilde{B}\widetilde{C}, 0)]. \end{aligned}$$

If: By induction on $n \geq 0$, we prove that

$$S \Rightarrow_{G'}^n x' \quad \text{implies} \quad S \Rightarrow_G^* x$$

for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Basis: Let $n = 0$. That is, $S \Rightarrow_{G'}^0 S$. It is obvious that $S \Rightarrow_G^0 S$ and $S \in g(S)$.

Induction Hypothesis: Assume that the claim holds for all derivations of length n or less, for some $n \geq 0$.

Induction Step: Consider a derivation $S \Rightarrow_{G'}^{n+1} x'$, $x' \in (V')^*$. Since $n+1 \geq 1$, there is some $y' \in (V')^+$ and $p' \in P'$ such that $S \Rightarrow_{G'}^n y' \Rightarrow_{G'} x'\ [p']$ and, by the induction hypothesis, there is also a derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$.

By inspection of P', the following cases (i) through (v) cover all possible forms of p':

(i) $p' = (H \to y_2, 0, 0) \in P',\ H \in V - T,\ y_2 \in V^*$. Then $y' = y_1' H y_3'$, $x' = y_1' y_2 y_3'$, $y_1', y_3' \in (V')^*$ and y has the form $y = y_1 Z y_3$, where $y_1 \in g(y_1'),\ y_3 \in g(y_3')$ and $Z \in g(H)$. Because for all $X \in V - T$ such that $g(X) = \{X\}$, the only Z is H and thus $y = y_1 H y_3$. By the definition of P' (see (1)), there exists a production

$p = H \rightarrow y_2$ in P, and we can construct the derivation $S \Rightarrow_G^* y_1 H y_3 \Rightarrow_G y_1 y_2 y_3$ $[p]$ such that $y_1 y_2 y_3 = x$, $x \in g(x')$.

(ii) $p' = m_1$: $(\{(A \rightarrow \widetilde{A}), (B \rightarrow \widetilde{B}), (C \rightarrow \widetilde{C})\}, 0, \widetilde{A})$. Next, we examine each production contained in this matrix.

(a) $p' = (A \rightarrow \widetilde{A})$ Then $y' = y_1' A y_3'$, $x' = y_1' \widetilde{A} y_3'$, $y_1', y_3' \in (V')^*$ and $y = y_1 Z y_3$, where $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and $Z \in g(A)$. Because $g(A) = \{A\}$ the only Z is A, so we can express $y = y_1 A y_3$. Having the derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$ it is easy to see that also $y \in g(x')$ because $A \in g(\widetilde{A})$.

(b) $p' = (B \rightarrow \widetilde{B})$. By analogy with (a), $y' = y_1' B y_3'$, $x' = y_1' \widetilde{B} y_3'$, $y = y_1 B y_3$, where $y_1', y_3' \in (V')^*$, $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and thus $y \in g(x')$ because $B \in g(\widetilde{B})$.

(c) $p' = (C \rightarrow \widetilde{C})$. By analogy with (a), $y' = y_1' C y_3'$, $x' = y_1' \widetilde{C} y_3'$, $y = y_1 C y_3$, where $y_1', y_3' \in (V')^*$, $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and thus $y \in g(x')$ because $C \in g(\widetilde{C})$.

(iii) $p' = m_4$: $(\{(\widetilde{A} \rightarrow \varepsilon), (\widetilde{B} \rightarrow \varepsilon), (\widetilde{C} \rightarrow \varepsilon)\}, \widetilde{A}\widetilde{B}\widetilde{C}, 0)$. By the permitting condition of this production string $\widetilde{A}\widetilde{B}\widetilde{C}$ surely occurs in y'. By Claim 5 no more than one $\widetilde{A}, \widetilde{B}$ and $\widetilde{C}$ occurs in y'. Therefore, y' must be of form $y' = y_1' \widetilde{A}\widetilde{B}\widetilde{C} y_3'$, where $y_1', y_3' \in (V')^*$ and $\widetilde{A}, \widetilde{B}, \widetilde{C} \notin \mathrm{sub}(y_1' y_3')$. Then $x' = y_1' y_3'$ and y is of the form $y = y_1 y_3$, where $y_1 \in g(y_1')$ and $y_3 \in g(y_3')$. By the induction hypothesis we have a derivation $S \Rightarrow_G^* y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $A \in g(\widetilde{A}), B \in g(\widetilde{B})$ and $C \in g(\widetilde{C})$.

We have completed the proof and established Claim 6 by the principle of induction. □

Observe that $L(G) = L(G')$ follows from Claim 6. Indeed, according to the definition of g, we have $g(a) = \{a\}$ for all $a \in T$. Thus, from Claim 6, we have for each $x \in T^*$:

$$S \Rightarrow_G^* x \quad \text{if and only if} \quad S \Rightarrow_{G'}^* x.$$

Consequently, $L(G) = L(G')$. The rest of this theorem follows directly from the construction of G'. □

5 SUMMARY

This paper proves that family **RE** is characterized by these grammars:

1. seven-nonterminal *mssc*-grammar of degree three with only 1 multi-production matrix;
2. seven-nonterminal *mssc*-grammar of degree two with only 2 multi-production matrices;

3. seven-nonterminal *sscm*-grammar of degree three with only 2 multi-production matrices.

In all these cases, we thus obtain the characterization of **RE** based on reduced ssc-versions of matrix grammars. These results are of some interest because ordinary matrix grammars do not characterize **RE** even if they are not reduced at all as already stated in Section 1.

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