Simplified Coherent Detection of CPM

S.J. Simmons

Abstract— This proposal avoids the complex matched-filter bank required for optimal detection. Instead, two simple lowpass filters are employed, followed by a sampler and a Viterbi algorithm that accounts for ISI. For representative modulations it is shown that near-optimal performance can be achieved with just two samples per baud.

I. INTRODUCTION

Coherent detectors proposed for CPM modulations with memory typically consist of a bank of (often many) matched filters followed by the Viterbi decoding algorithm. Simplified detector structures have been proposed earlier [1] - [4] that give modest reductions in the number of filters and states required. In [5], the filters are implemented digitally, but still at high computational cost. Matched-filter simplifications in [3] and [6] are based on the observation that CPM signals are virtually imited to a small frequency band, and the number of dimensions per baud is limited by the dimensionality theorem [7]. An approximate signal basis set having small cardinality can then be found such that there is little error in the representation. The receiver proposed in this letter takes this idea of low effective dimensionality, but adopts a more familiar basis set, namely, the time translates of the $sinc()$ function

$$\left\{ \frac{\sin\left(2\pi[t - kT_s]/T_s\right)}{(2\pi[t - kT_s]/T_s)} \right\}.$$

The cardinality of this basis set is simply T/T_s, but only a single filter is required to implement it, where the received signals are first filtered then sampled at rate $1/T_s$. The samples are passed to a Viterbi algorithm operating on a trellis that has been modified to account for the effect of the filtering. Further reduction in the number of states below the number of modulator states, as initially proposed in [8], is not attempted. The reason is that such state-reduction schemes, including [1], [3], are generally highly susceptible to adjacent channel interference, as demonstrated in [10].

II. PROPOSED RECEIVER

The received carrier-frequency signal is of the form

$$r_C(t) = \sqrt{2E_s/T}\cos(\omega_C t + \phi(t) + \theta_0) + n_C(t) \quad (1)$$

where initial phase θ_0 is assumed known, E_s is the transmitted energy per symbol, $\phi(t)$ is the information-bearing phase modulated by M-ary symbols at rate $1/T$, and $n_C(t)$ is AWGN having single-sided power spectral density N_o watts/Hz . The received signal will be dealt with in its equivalent lowpass form,

The author is with the Department of Electrical and Computer Engineering, Queen's University, Kingston, Ontario.
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$r(t) = s(t) + n(t)$. The proposed receiver structure shown in Fig. 1 begins with normal IF filtering followed by quadrature demodulation with local $\sqrt{2}\cos(\omega_c t)$ and $-\sqrt{2}\sin(\omega_c t)$ carriers to get to baseband.
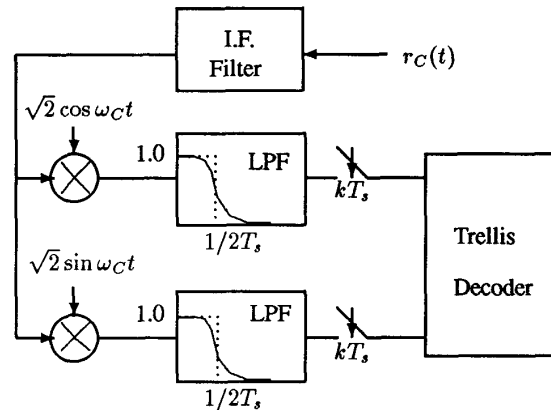

Fig. 1. Proposed CPM receiver structure (ideal brickwall filters dotted, practical filter solid line).

A unity-height low-pass filter of -3 dB bandwidth $B = 1/2T_s$ Hertz follows in each arm of the receiver, and its output is sampled at rate $1/T_s$, generating $S = T/T_s$ samples per baud. Since the original CPM signal $s(t)$ is not bandlimited, this filtering will modify the signal shapes, in effect, causing intersymbol interference.

With ideal brickwall filters, there is no aliasing, and the complex lowpass signal after the ideal filtering, $\check{r}(t) = \check{s}(t) + \check{n}(t)$, can be represented in terms of its complex samples $\check{r}_k = \check{r}(kT_s)$:

$$\check{r}(t) = \sum_k \check{r}_k \frac{\sin \pi[t - kT_s]/T_s}{\pi[t - kT_s]/T_s}. \quad (2)$$

Now consider reception of one of a number of possible transmitted signals $s^i(t)$, of length N symbols. A maximum likelihood detector *working on the filtered signal set* would select the i'th filtered signal $\check{s}^i(t)$ that minimized (since the noise is still white)

$$\Lambda^i = \int_0^{(N+\epsilon)T} [\check{r}(t) - \check{s}^i(t)]^2 \, dt, \quad (3)$$

where ϵ accounts for the increase in effective signal duration due to filtering. We can convert this to the sampled domain to get:

$$\Lambda^i = T_s \sum_k^{S(N+\epsilon)} [\check{r}(kT_s) - \check{s}^i(kT_s)]^2 \quad (4)$$

Minimization of Λ^i therefore involves minimizing the discrete time sum

$$\Delta^i = \sum_{k=1} [\check{r}_k - \check{s}_k^i]^2 \quad (5)$$

An optimal sequence estimator could therefore search a trellis which mapped symbol sequences to their filtered signal samples $\tilde{s}_k$, by assigning metrics to paths in the trellis according to equation (5). As $B \to \infty$, $\tilde{s}(t) \to s(t)$, and $\tilde{r}(t) \to r(t)$, in the squared error sense, and this sampled data sequence estimator must necessarily make the same decisions as an optimal sequence estimator working with the unfiltered signals $r(t)$ and $s(t)$. It therefore approaches the optimal matched-filter receiver as the number of samples per baud, S, becomes large. The normalized squared distance between sample sequences is directly related to the squared distance between the two filtered signals $\tilde{s}^i(t)$ and $\tilde{s}^j(t)$:

$$\delta_{ij}^2 \triangleq \sum_k [\tilde{s}_k^i - \tilde{s}_k^j]^2 = \frac{1}{T_s} \int [\tilde{s}^i(t) - \tilde{s}^j(t)]^2 \, dt = \frac{1}{T_s} \tilde{d}_{ij}^2 \quad (6)$$

The number of sample computations per baud can be minimized by choosing a narrow filter to keep S small, but if the filtering is too severe, the signal distances will be excessively degraded.

To keep the receiver filtering simple, 8-pole (or 4-pole) Butterworth filters are considered for which simple VLSI implementations are readily available. The non-ideal filters will cause some aliasing in the signal samples. Substitution of non-ideal-filtered samples $\tilde{s}_k^i$ and $\tilde{r}_k$ into equations (4) and (5) will therefore be only an approximation to the optimum criterion of equation (3). The non-ideal filter will also, if not Nyquist, cause correlation and increased variance in the noise samples. With 8-pole filters, all these effects are negligible. In practice, the receiver ignores the small noise correlations; simulation results show that the loss in performance is insignificant.

Table I shows degradation in minimum distance for a number of representative CPFSK and CPM modulations, where E_b is the transmitted energy per bit.

TABLE I

NORMALIZED SAMPLED MINIMUM SQUARED DISTANCE, $(\delta_{min}^2 T_s)/2E_b$, AND dB LOSS (BRACKETED) FOR OVERSAMPLED VITERBI RECEIVER.

Modulation	$S = 1$	$S = 2$	$S = 4$
$M = 2, h = 1/2$ FSK	~ 2.0	2.00 (-0.0)	2.00 (-0.0)
$M = 4, h = 2/5$ FSK	~ 1.5	3.03 (~ 0)	3.04 (-0.0)
$M = 4, h = 1/3$ 2RC	~ 1.7	2.18 (~ 0)	2.19 (-0.0)
$M = 4, h = 1/3$ 3RC	~ 1.5	1.67 (-0.0)	1.67 (-0.0)

Note that in all cases only two samples per baud are needed to achieve virtually full d_{min}^2 in the filtered trellis. At $S \geq 4$, there is virtually zero loss, but at $S = 1$ the filtering is too severe (the $S = 1$ numbers are only approximate due to filter memory truncation for computational purposes). It should be noted that a sampling phase offset of $t_{off} = T_s$ from the start of a symbol was used for these results; this gives best or near-best results in each case.

III. STATE DESCRIPTION AND REDUCED-STATE DECODING

The sequence estimator must account for ISI in the filtered signals. Filtered signal samples will depend on the current CPM signal state, the current symbol, and a number, L_{eff}, of previous

symbols, where L_{eff} is the effective memory caused by the filter response. A full trellis of $C * M^{L_{eff}}$ states could therefore be used for decoding, where C is the number of modulator states. To reduce the number of paths searched, however, a reduced-state Viterbi (RS-Viterbi) variant will be employed, in which decisions are forced between paths once they enter the same *modulator* state. Only C paths are kept at each depth, and signal sample values for path extension computations are found by table lookup using the past symbols of each survivor path. The procedure is equivalent to parallel decision feedback detection (PDFD) for coded linear modulations with ISI, as described in [9]. With this method, the small distance contribution of ISI occurring after a modulator-state merge is ignored.

For example, consider a 4-ary modulation having a frequency pulse of length $L_p = 2$ symbols (e.g. 2-RC), and 8 possible phase states, for a total of $C = 32$ modulation states. With two-samples-per-baud processing and an 8-pole filter, the filter response has $L_{eff} \simeq 2$. The lookup table would then need around $32 * 4^3 = 2048$ entries, which is large but reasonable.

For the same modulations of Table I, Table II shows the minimum distance between all path pairs that will be involved in forced decisions at state merges with the reduced-state (RS-Viterbi) decoding with 8-pole filtering and $S = 1, 2, 4$.

TABLE II

NORMALIZED SAMPLED MINIMUM SQUARED DISTANCE, $(\delta_{min}^2 T_s)/2E_b$, AND dB LOSS (BRACKETED) FOR OVERSAMPLED RS-VITERBI.

Modulation	$S = 1$	$S = 2$	$S = 4$
$M = 2, h = 1/2$ FSK	~ 2.0	1.98 (~ 0)	1.98 (~ 0)
$M = 4, h = 2/5$ FSK	~ 1.3	2.97 (-0.1)	3.03 (~ 0)
$M = 4, h = 1/3$ 2RC	~ 1.6	2.18 (~ 0)	2.18 (~ 0)
$M = 4, h = 1/3$ 3RC	~ 1.2	1.67 (-0.0)	1.67 (-0.0)

Note that there is very little loss associated with using the reduced-state purging rule with $S \geq 2$. It should be noted that these results are for the best sampling phase, which now occurs at an offset of $t_{off} = 2T_s$ from the start of a symbol in each case, that is, double the offset that was found best for Table I. This offset maximizes the accumulated distance at the decision time. In addition to these computations of filtered δ_{min}^2, simulations were used to check performance. For each of the modulations listed in the preceding tables, simulations show that losses are indeed negligible (less than 0.10 dB at error rates less than 10^{-1}), demonstrating that filtering does in fact not seriously degrade distances of the non-minimum distance events.

The reduction in complexity due to replacing the matched-filter bank is evident. The 4-ary 2RC, $h = 1/3$, modulation would require a a bank of 32 matched filters whose outputs are weighted by multiplications by $\cos \theta$ and $\sin \theta$, for each of the six phase states θ. The corresponding 3RC modulation would require 128 matched filters. In all cases the proposed simplified receiver, by comparison, requires only two simple filters, and no more decoder states than for the full-matched-filter receiver.

IV. OTHER EFFECTS ON PERFORMANCE

Errors in the sample phase timing can affect the error rate, both for the full-matched-filter (full-MF) receiver, and for the

oversampled RS-Viterbi receiver. Because the proposed receiver is a good approximation to the full-matched-filter receiver, it is not expected to be significantly more sensitive to errors in sampling phase. Fig. 2 shows that is indeed the case for the test $M = 4$, $h = 1/3$ 2RC modulation at 7 dB (error-event probability shown).
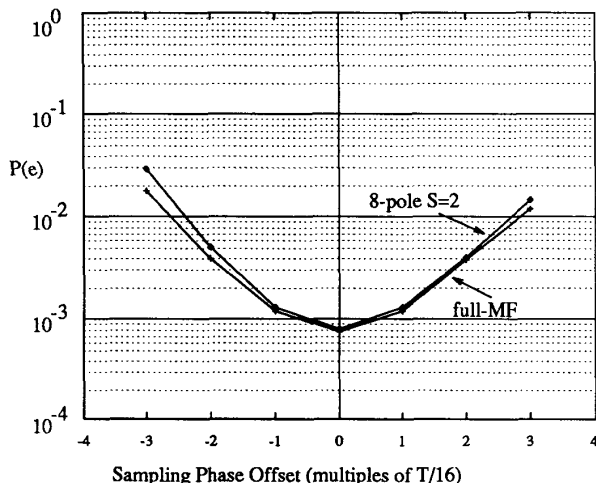

Fig. 2. Effect of sample timing error for $M = 4$, $h = 1/3$ 2RC at $E_b/N_o = 7$ dB.

Similarly, simulations show virtually no difference with respect to sensitivity to carrier phase error (the P(e) increase for every 5 degrees of phase error is almost equal to that for every T/16 of timing error).

It was noted earlier that with ideal filters, as $S \rightarrow \infty$, the sequence estimator would make the same decisions as an optimal matched-filter receiver, and this is true even in the presence of unwanted additive interfering (e.g. cochannel or adjacent-channel) signals $z(t)$, that is, even when $r(t) = s(t) + n(t) + z(t)$. At practical values of S, however, we need to consider possible increased receiver susceptibility to adjacent channel interference. In the proposed receiver with adjacent CPM signals, sampling will cause the filtered remnants of the adjacent signals to be aliased into the range of the desired signal's spectrum. Rejection of these adjacent signals is therefore primarily determined by the sharpness of the low-pass filter response. If the filter response falls off more slowly than the sidelobes of the modulation, the interference rejection will be inferior to that of a full-MF Viterbi receiver. This has been demonstrated by simulation, using the $h = 1/3$, quaternary 2RC modulation for illustration.

Fig. 4 shows the comparative performance of the proposed receiver, for 4-pole and 8-pole filters, in the presence of a $2/T$-spaced single adjacent interferer that is 10 dB, and 20 dB larger than the desired signal.

At 10 dB ACI, there is negligible performance loss for the full-MF Viterbi receiver. The 8-pole filter in the proposed receiver is sharp enough to avoid performance loss, although the 4-pole filter is not. It is therefore adjacent channel interference rejection that will be the criterion determining the minimum number of
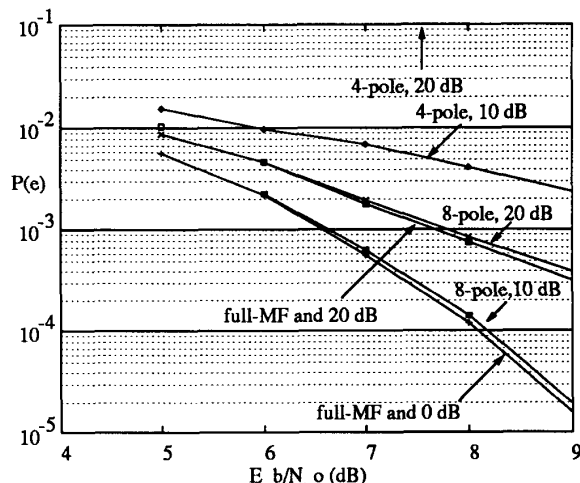

Fig. 3. Comparative performance in the presence of ACI signals with the $h = 1/3$, quaternary 2RC modulation, $S = 2$ samples per baud.

poles needed for the lowpass filters of the proposed receiver.

V. CONCLUSIONS

A simplified receiver structure that avoids a complicated matched filter bank has been demonstrated. Requirements on adjacent channel interference rejection will determine how sharp the lowpass filters must be. From the modulations studied, the proposed receiver does not appear to be significantly more sensitive to errors in sample timing or carrier phase reference than a receiver employing full matched filtering.

REFERENCES

[1] A. Svensson, "Reduced state sequence detection of partial response continuous phase modulation," *IEE Proc. Part I*, vol. 138, pp. 256–268, Aug. 1991.

[2] G.K. Kaleh, "Simple coherent receivers for partial response continous phase modulation", *IEEE JSAC*, vol. 7, pp. 1427–1436, Dec. 1989.

[3] J. Huber and W. Liu, "An alternative approach to reduced-complexity CPM receivers", *IEEE JSAC*, vol. 7, pp. 1437–1449, Dec. 1989.

[4] A. Svensson, C-E. Sundberg, and T. Aulin, "A class of reduced-complexity Viterbi detectors for partial response continuous phase modulation," *IEEE Trans. Commun.*, vol. COM-32, pp. 1079–1087, Oct. 1984.

[5] T. Maseng, "Digitally phase modulated (DPM) signals," *IEEE Trans. Commun.*, vol. COM-33, pp. 911–918, Sept. 1985.

[6] T. Palenius and A. Svensson, "A noncoherent CPM-detector which uses a reduced set of basis functions," *Proc. Globecom'91*, Phoenix, Arizona, pp. 2.5.1–2.5.5, Dec. 1991.

[7] Wozencraft and Jacobs, *Principles of Communication Engineering*. John Wiley & Sons, 1965.

[8] S.J. Simmons, "Simplified Coherent Detection of Bandwidth Efficient CPFSK/CPM Modulations", *Proc. IEEE PacRim*, Victoria, May, 1991, pp. 174–177.

[9] M. Eyuboğlu and S.U.H. Qureshi, "Reduced-state sequence estimation for coded modulation on intersymbol interference channels," *IEEE JSAC*, vol. 7, pp. 989–995, Aug. 1989.

[10] S.J. Simmons, "ACI Susceptibility of Reduced-State Decoding for CPM," submitted to *IEEE Trans. Commun.*, April 1994.